Lab 7: Low Reynolds numbers and Tube Flows

Bio427 — Biomechanics

In this lab, we explore two issues related to biological fluid dynamics: flows at low Reynolds numbers and flows within tubes. The former corresponds to the world of tiny organisms such as bacteria, invertebrate larvae, spermatozoa and many other small scale swimming creatures. We can regard tube flows as essentially low Reynolds number problems as well since the pressure stresses for flow in straight tubes is dominated by the viscous stresses associated with fluid shearing against the tube wall. Tube flows are ubiquitous in biological systems and include blood flow, some aspects of mucociliary transport, respiratory flows and many other examples.

Figure 1: Life at low Reynolds numbers with swimming bacteria from `http://physics.aps.org/story/v25/st9`

Goals

- Understand the influence of wall effects at very low Reynolds numbers.
- Use the Hagen-Poiseuille relationship to measure the viscosity of a fluid.
- Examine how suspensions of particles in fluids (like blood) interact with each other and the walls of tubes to affect the apparent viscosity of the suspension.

Some useful numbers of this lab

- Large red-capped tube diameter: 2.66 cm
- Small red-capped tube diameter: 1.5 cm
- Yellow air-soft pellet diameter: 0.6 cm
- Yellow air-soft pellet mass: 0.12 g
- Density of mineral oil: 0.9 g cm^{-3}

Conceptual Basis

Low Reynolds number flows are characterized by the overwhelming dominance of viscous stresses in determining the patterns of fluid motion and forces. One consequence of life at low Reynolds numbers is the importance of nearby boundaries. This so called *wall effect* dictates that disturbances caused by a moving body are manifest at relatively large distances from that body. A bacterium swimming within a body length of a fixed boundary may experience nearly 10 times (or more!) the resistance it might experience far from that boundary. Recall that the Reynolds number measures the relative importance of inertial to viscous stresses which for a sphere of diameter D is:

$$Re = \frac{\rho UD}{\mu} = \frac{UD}{\nu} \tag{1}$$

where ρ is the fluid density, μ is its dynamic viscosity and ν is the kinematic viscosity.

Falling spheres and wall effects

For a sphere falling in a fluid at low Reynolds numbers we had previously indicated that its drag force was directly proportional to the viscosity of the fluid (μ), its falling velocity (U) and its radius (r):

$$\text{drag} = 6\pi r \mu U \tag{2}$$

When we used this equation in prior lab to measure the viscosity of a fluid, we used very large diameter vessels (cups) and very small spheres. We mentioned as well that the sphere should be dropped in the center of the cup to avoid wall effects: if you drop the sphere near the wall of the container, it sinks much more slowly.

For a sphere of radius r falling axially in a cylindrical tube of radius R, wall effects become important even when the sphere radius is only 20% of the cylinder radius. To account for this, there are some approximate correction factors that have been derived. The drag on spheres falling in cylinders is increased by a factor K:

$$K = \frac{1}{1 - 2.104(r/R) + 2.089(r/R)^3 - 0.948(r/R)^5} \tag{3}$$

which works well for radius ratios (r/R) less than about 0.6. This approximation fails for situations where the sphere radius is close to the cylinder radius.

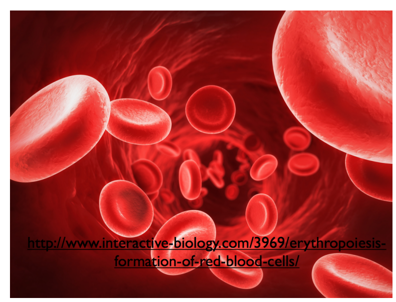


Figure 2: Life at low Reynolds numbers blood cells moving in an arteriole (at a disturbingly low hematocrit!)

Tube flows

A bacterium moving in a channel that is 20% wider than the cell experiences nearly 40 times the resistance it would be experience if it were swimming in an open fluid.

The movement of fluid in a tube is similarly dominated by viscous wall stresses. That is reflected in the Hagen-Poiseuille relationship which relates volumetric flow rate (Q) to the tube geometry and fluid properties:

$$Q = \frac{\pi R^4 \Delta P}{8 \mu L} \tag{4}$$

where R is the tube radius, and ΔP is the pressure difference across the length (L) of the tube. Due to the 4^{th} order dependence of Q on

R, very small changes in tube radius can profoundly affect the flow rate.

The average velocity of flow in tubes is the volume flow rate divided by the area:

$$U_{\text{tube}} = \frac{Q}{A} \tag{5}$$

and the Reynolds number for tube flow uses the tube diameter as the characteristic length.

We can use the relationship above to estimate fluid viscosity by measuring flow rate, pressure drop, and tube dimensions: indeed, that is precisely what has been done to measure blood viscosity. The challenge here is that blood is a suspension and, as you learned above, wall effects are important – and that includes wall effects between adjacent cells in suspension as well as between those cells and the tube walls. That shear stress makes the suspension more difficult to pump and, in turn, gives the fluid an apparent viscosity that is greater than that of the plasma itself.

Setting up affordable graduated cylinders

You will note that mineral oil is difficult to clean – so be careful with spilling and your clothes! Additionally, we are going to use plastic cups to measure volumes where needed. Each team will have to set up their own (and dispose of them at the end of lab). To do so use a permanent marker to put lines at the following heights from the bottom of the cup:

Height (cm)	Volume (cm^3)
1.7	50
2.5	100
4	150
5.5	200
6.5	250
7.5	300
8.4	350
9.3	400
10.2	450
11.0	500

The spacing between lines is uneven since the cup expands from the bottom.

Materials and methods

Wall effects and estimating viscosity

There are two cylinders filled with a high viscosity mineral oil. You will measure the apparent viscosity of the fluid using the falling ball method that we had done before. However, because the ball density is close to that of mineral oil, you need to compute the buoyant force as well. The balance of forces in this situation is

$$\text{weight} = \text{drag} + \text{buoyancy} \tag{6}$$

$$mg = 6\pi r\mu U + \rho_m V g \tag{7}$$

where ρ_m is the density of mineral oil and V is the volume of the sphere. The dimensions of the cylinders and sphere are given above. With the volume of a sphere as $\frac{4}{3}\pi r^3$ we can rearrange Eqn. **??** to provide an estimate of the viscosity:

$$\mu = \frac{mg - \frac{4}{3}\pi r^3 \rho_m g}{6\pi r U} \tag{8}$$

With appreciable wall effect, your estimate of the fluid viscosity will change.

In each cylinder, drop an airsoft pellet (aka sphere) and time its passage over a known distance. Use the worksheet to estimate the viscosity of the fluid and the Reynolds number of the sphere.

In the larger of the two cylinders, drop the sphere near the wall (not in the axial center of the cylinder) and note whether the presence of that closer boundary retards the motion of the sphere.

Using tube flow to estimate viscosity

We have provided you with a tube flow apparatus that you will use to estimate the viscosity of the same fluid using the Hagen-Poiseuille relationship (Eqn. **??**) which is rearranged below:

$$\mu = \frac{\pi R^4 \Delta P}{8QL} \tag{9}$$

and

$$\Delta P = \rho_m g h \tag{10}$$

With stoppers placed in the end of the tubes, fill the reservoirs with the same fluid used above. Then use the worksheet and the following protocol:

- Measure the height of the fluid in the cup (h in Eqn 10)
- Place the measuring cylinder beneath the tube

- Release the stopper
- Measure the time required to fill a known volume
- Return the fluid to the reservoir
- Use the worksheet to compute the viscosity of the fluid

Simulating blood flow

Vessel	Speed ($\mathrm{cm\,s^{-1}}$)	Diameter (cm)	Reynolds number
Aorta	48	2.5	3400
Artery	45	0.4	500
Arteriole	5	0.005	0.7
Capillary	0.1	0.0008	0.002
Venule	0.2	0.002	0.01
Vein	10	0.5	140
Vena Cava	38	3.0	3300

Table 1: Approximate speeds, dimensions and Reynolds numbers for mammalian blood flows

As mentioned above, flows of suspensions in tubes (like blood flow) is strongly determined by wall effects. Not only do the spheres interact with the tube walls, they interact with each other. That viscous interaction requires theoretically yields lower flow rates for a given pressure difference. In this section you will fabricate Reynolds-scaled blood by creating a suspension of pellets in mineral oil.

Our goal is to create a mixture of spheres and mineral oil that is similar to a blood hematocrit of 0.5 (a bit over the normal value of 0.4). Spheres closely packed into a container occupy approximately 74% of the volume. As such, adding enough mineral oil to simply cover the volume of spheres would give a "hematocrit" of 0.74. Adding an additional 25% volume of "plasma" would create a hematocrit close to 0.5.

To prepare your simulated blood, partially fill a reservoir with spheres and then fill those with mineral oil to 25% more volume than that of spheres alone. Because the mineral oil is less dense than the pellets, they have a modest settling rate and you will have to keep the artificial blood stirred during a repeat of the above procedure.

Estimate the viscosity of this mixture using the same process as in the previous section.

Lab 7: Tube Flow

Lab Section: ____________________

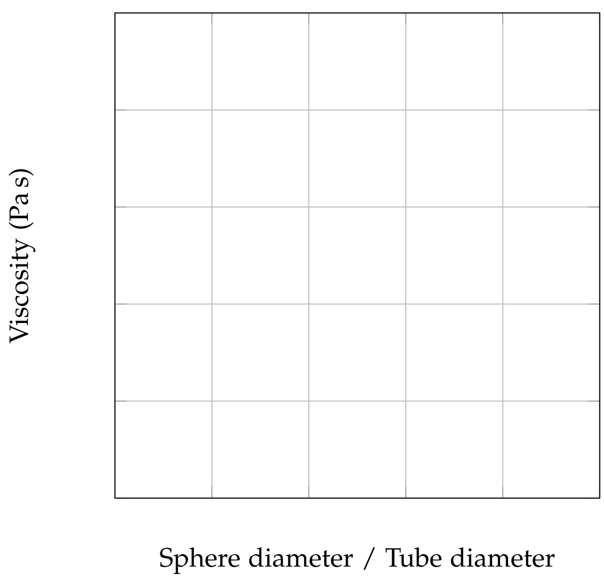
Name: ____________________ Partner's Name: ____________________

Wall effects and viscosity

Enter your data in the table below (remember to use SI units)

		1.5 cm tube	2.66 cm tube	Ball near wall of 2.66 cm tube
1	Distance			
2	Time			
3	Velocity			
4	Apparent viscosity			
5	Reynolds number			
6	Estimated drag on the sphere			

In the graph below plot the two estimates of the viscosity (from axially falling spheres) as a function of ratio of the sphere diameter to the tube diameter and provide a prediction of what the viscosity would be if the spheres were falling in an infinite fluid.



Bacterial drag: By what factor did the apparent drag on the sphere increase as it fell nearer to the side of the wall (the third column above)? What are the implications of this to sperm motility in oviducts?

Blood

Enter the appropriate values in the table below for the Hagen-Poiseuille apparatus.

Tube radius	Tube length

Enter the values (in SI units) in the table below.

		"Plasma"	"Blood"
1	Volume		
2	Time		
3	Flow rate		
4	Average velocity		
5	ΔP		
6	Apparent viscosity		
7	Reynolds number		

Blood flow: By what factor did the apparent viscosity increase with addition of cells?

Reynolds numbers: Refer to the table in the lab handout of Reynolds numbers for blood flow and estimate what part of circulatory system your model best represents.