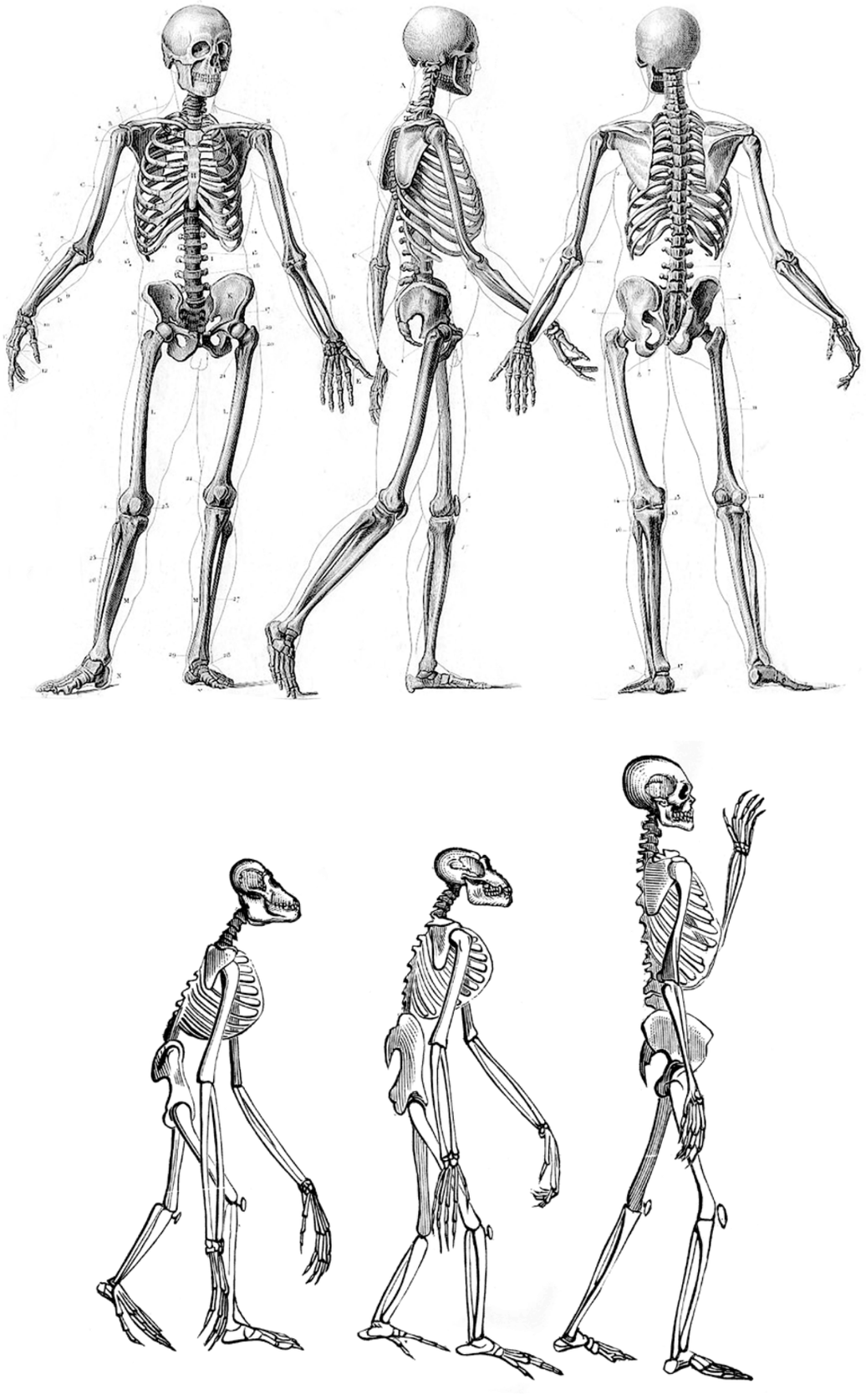
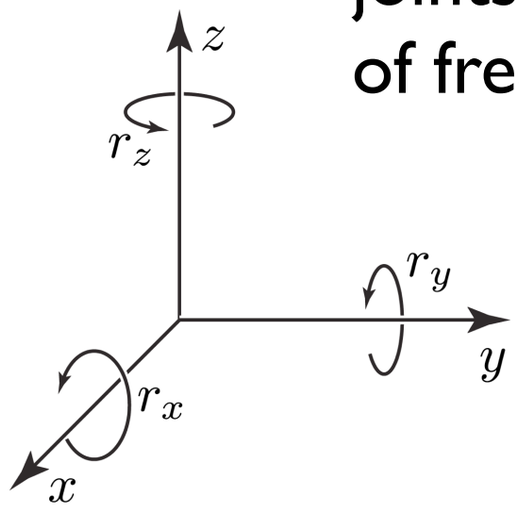
Translational and rotational degrees of freedom



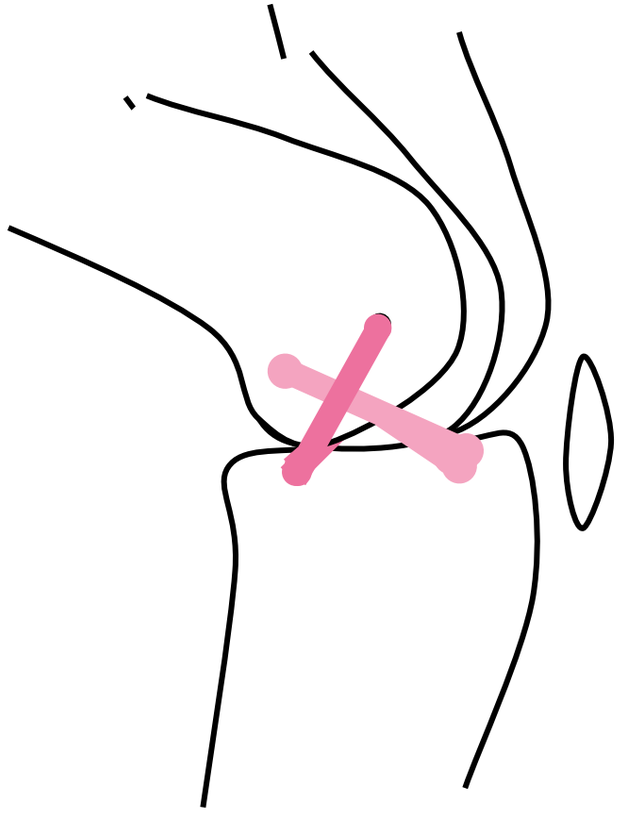


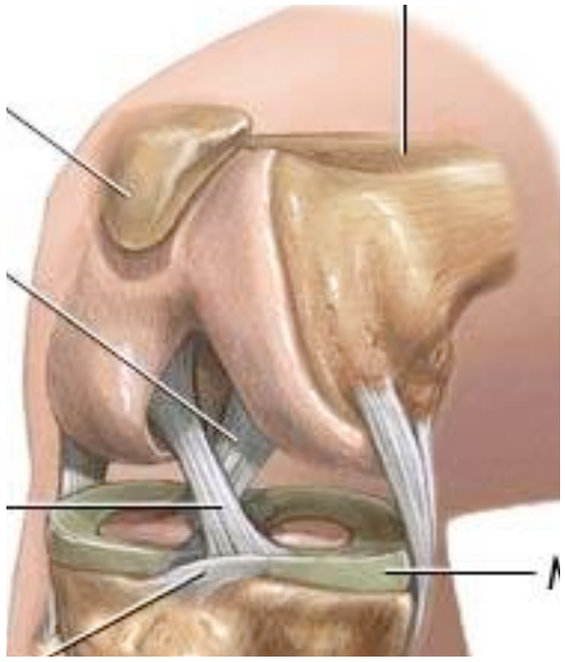
Rigid bodies connected at joints with up to 6 degrees of freedom.

Skeletal joints are largely rotational.

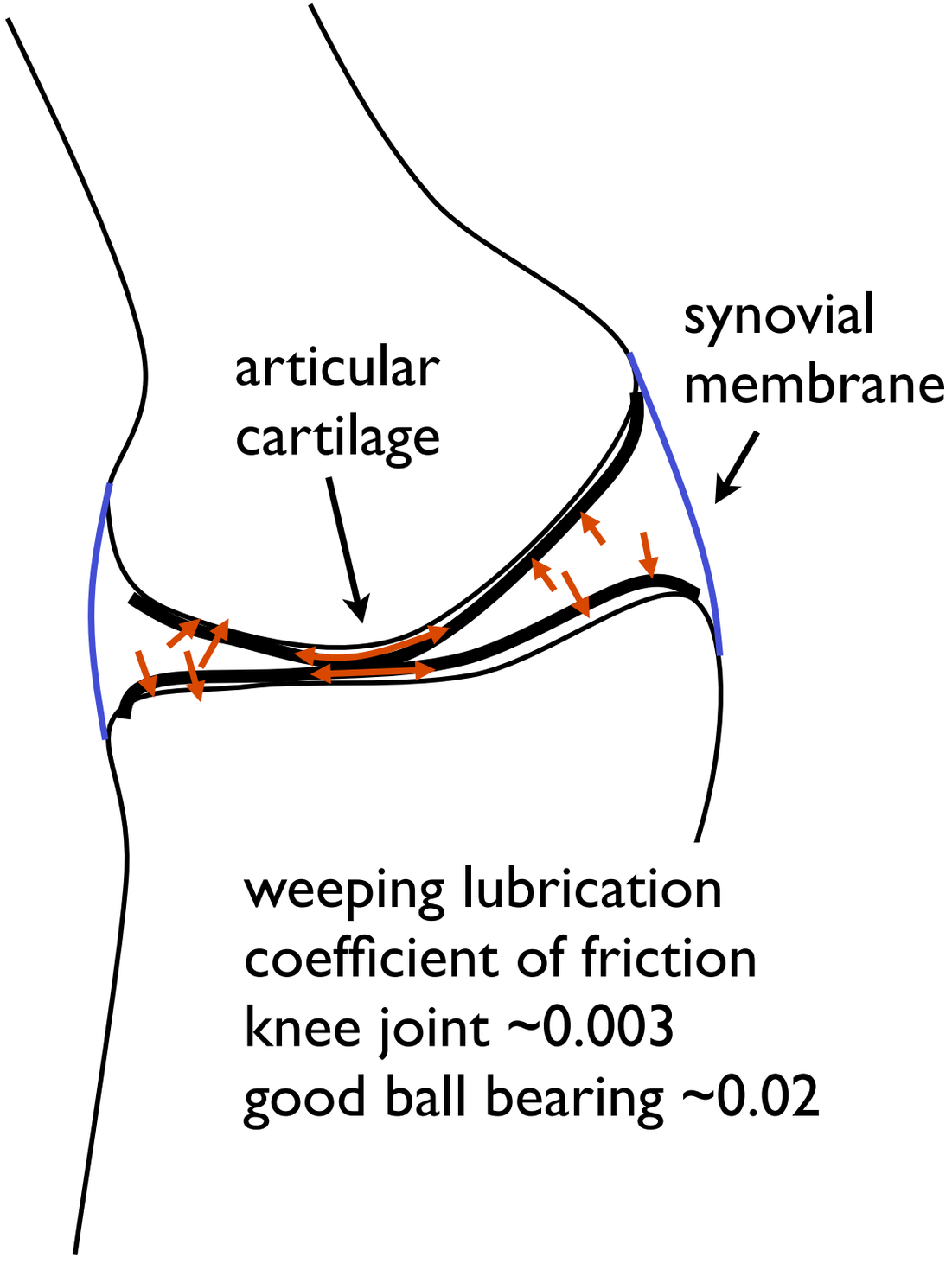
What are 1-, 2- and 3-DoF joints?
Any translational joints?

What restricts degrees of freedom?





cruciate ligaments constrain
knee motions

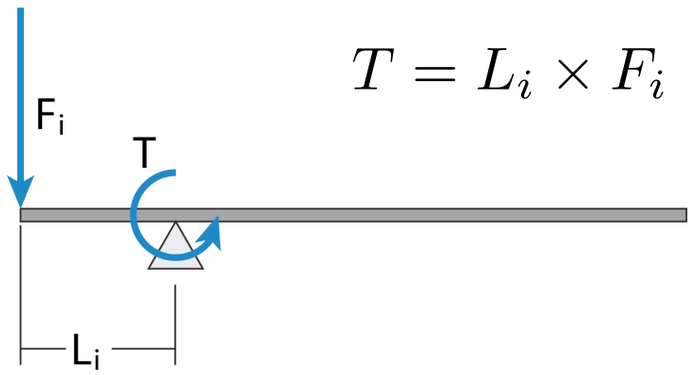


weeping lubrication
coefficient of friction
knee joint ~0.003
good ball bearing ~0.02

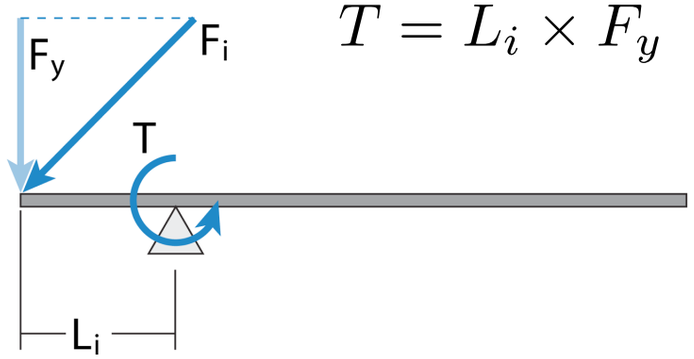
Rotational motion, torques and energy

angle	θ		$[rad]$
angular velocity	$\frac{d\theta}{dt}, \omega$	$\mathbf{T^{-1}}$	$[rad] \cdot s^{-1}$
angular acceleration	$\frac{d^2\theta}{dt^2}, \alpha$	$\mathbf{T^{-2}}$	$[rad] \cdot s^{-2}$
moment of inertia	I	$\mathbf{ML^2}$	$kg \cdot m^2$
angular momentum	$L, I \cdot \omega$	$\mathbf{ML^2T^{-1}}$	$kg \cdot m^2 \cdot s^{-1}$
torque	$T, x \times F$	$\mathbf{ML^2T^{-2}}$	$N \cdot m, kg \cdot m^2 \cdot s^{-2}$

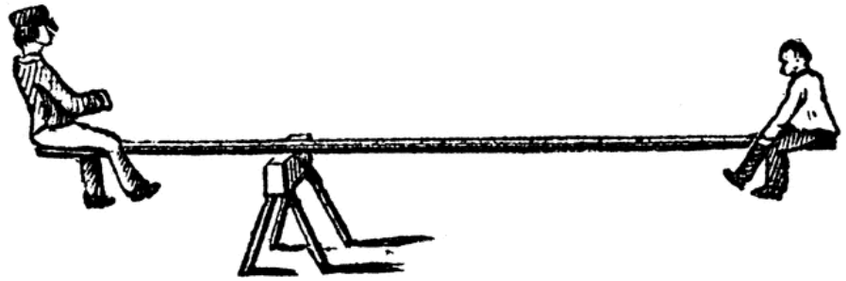
Rotational motion, torques and energy

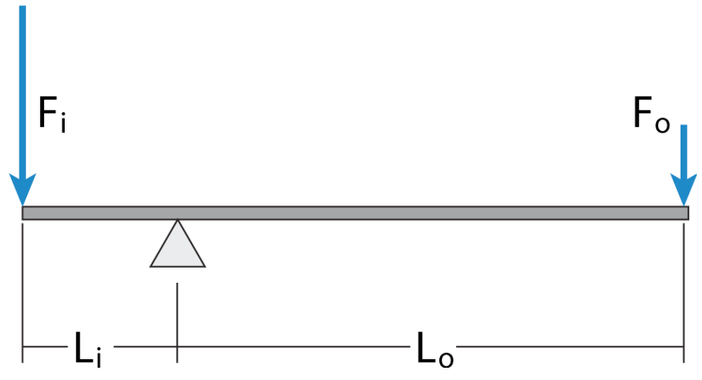


$$T = L_i \times F_i$$



$$T = L_i \times F_y$$





equilibrium

$$L_i \times F_i = L_o \times F_o$$

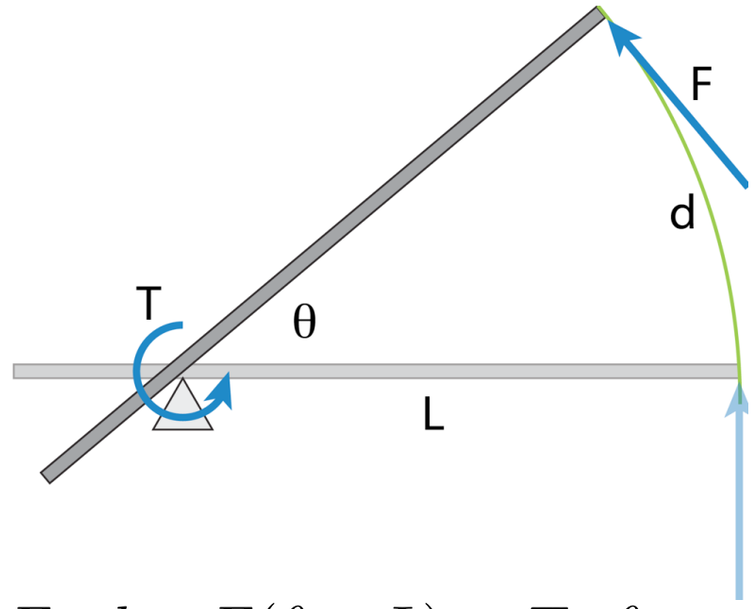
torque	$T, x \times F$	$\mathbf{ML^2T^{-2}}$	$N \cdot m, kg \cdot m^2 \cdot s^{-2}$

Torque and energy have the same units?
Are torque and energy the same thing?
No.

Rotational motion, torques and energy

Torque is force applied perpendicular to a lever arm.

Work is force applied along a distance.



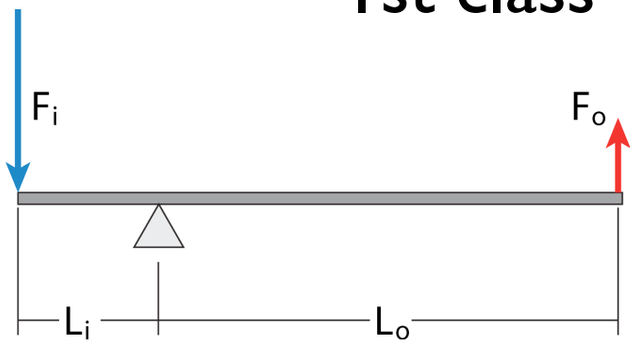
$$W = F \cdot d = F(\theta \times L) = T \cdot \theta$$

torque	$T,\ F \times x$	$\mathbf{ML^2T^{-2}}$	$N \cdot m, kg \cdot m^2 \cdot s^{-2}$

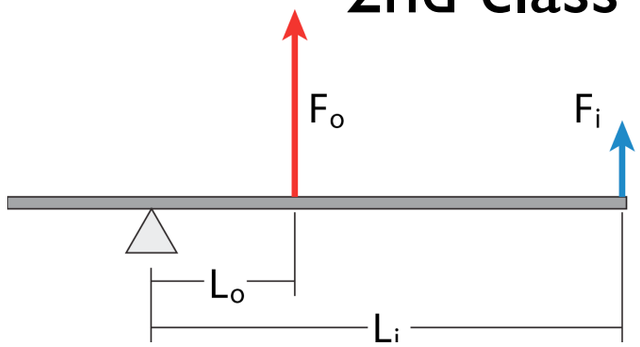
Torque and energy have the same units?
Are torque and energy the same thing?
No.

Types of levers

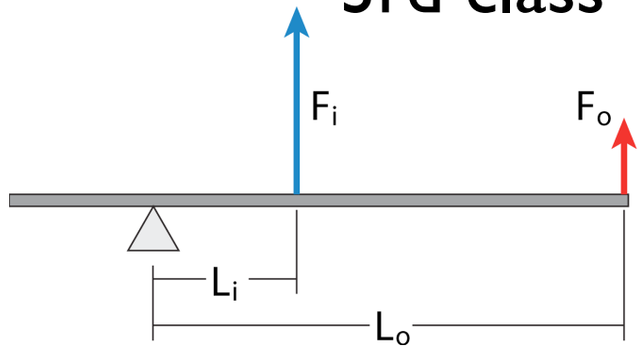
1st class

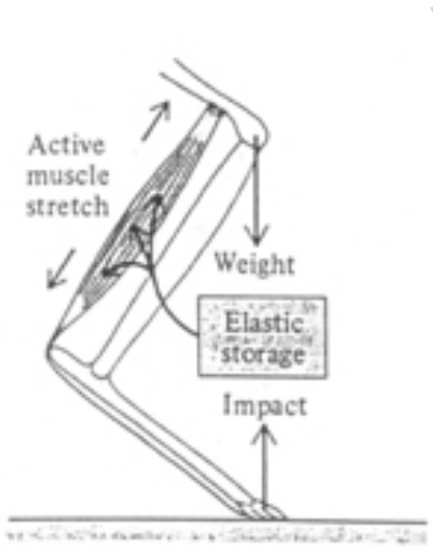


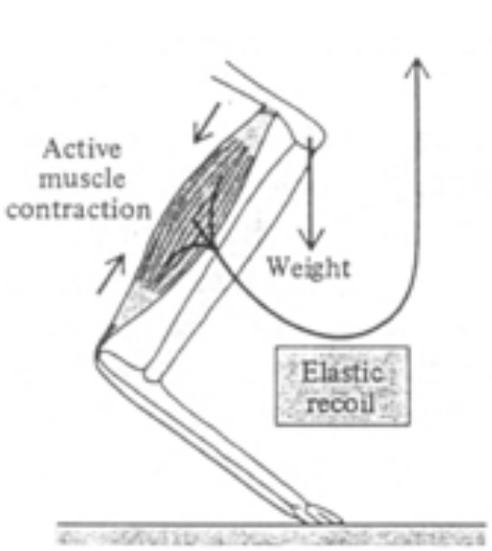
2nd class

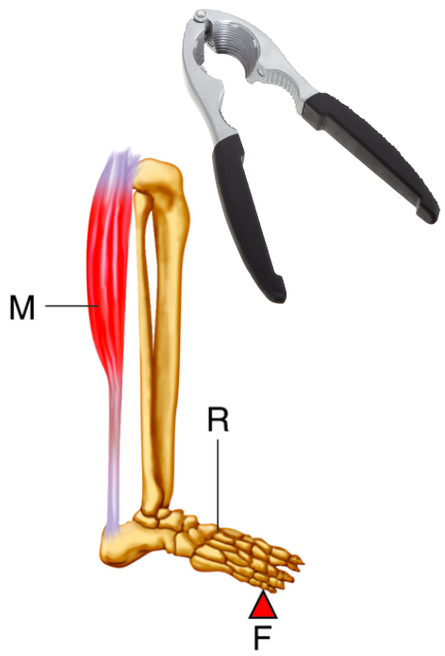


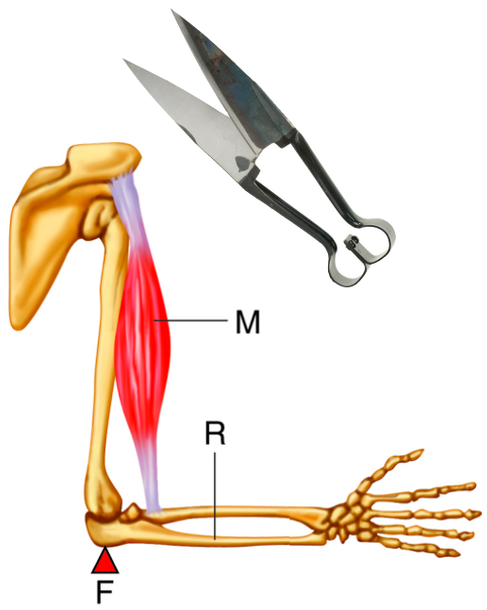
3rd class



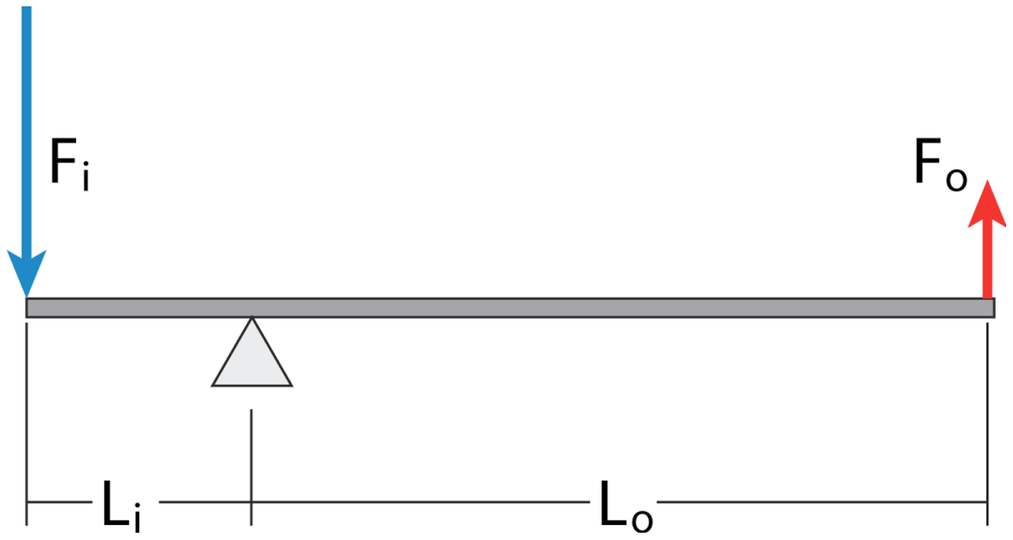


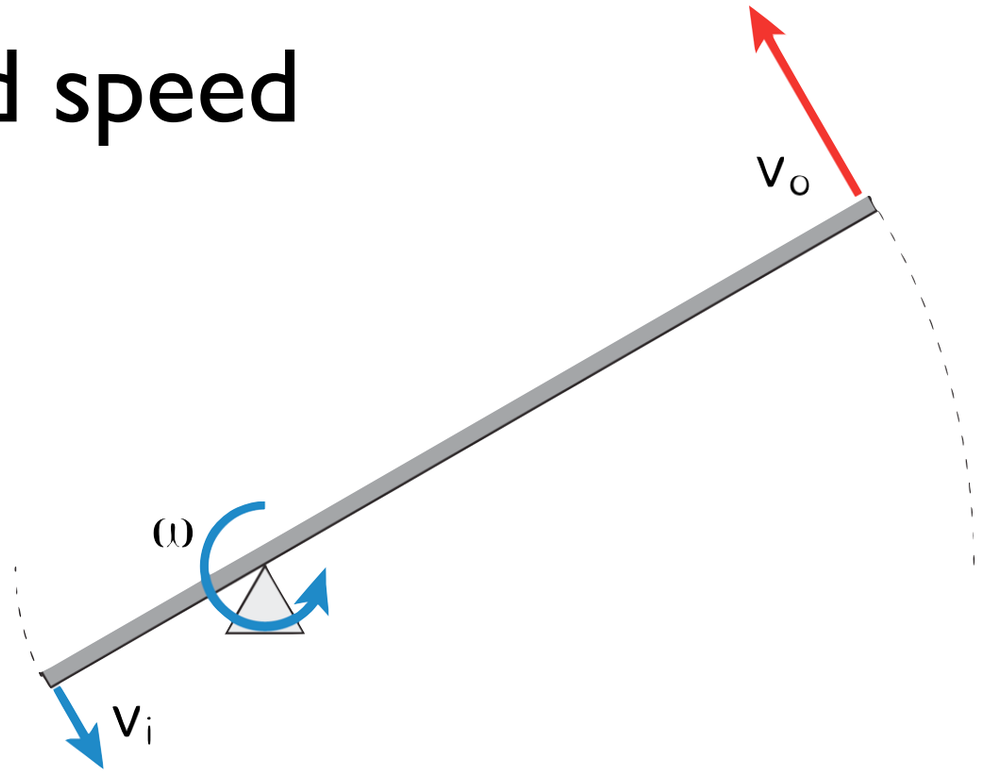






The trade-off between force and speed





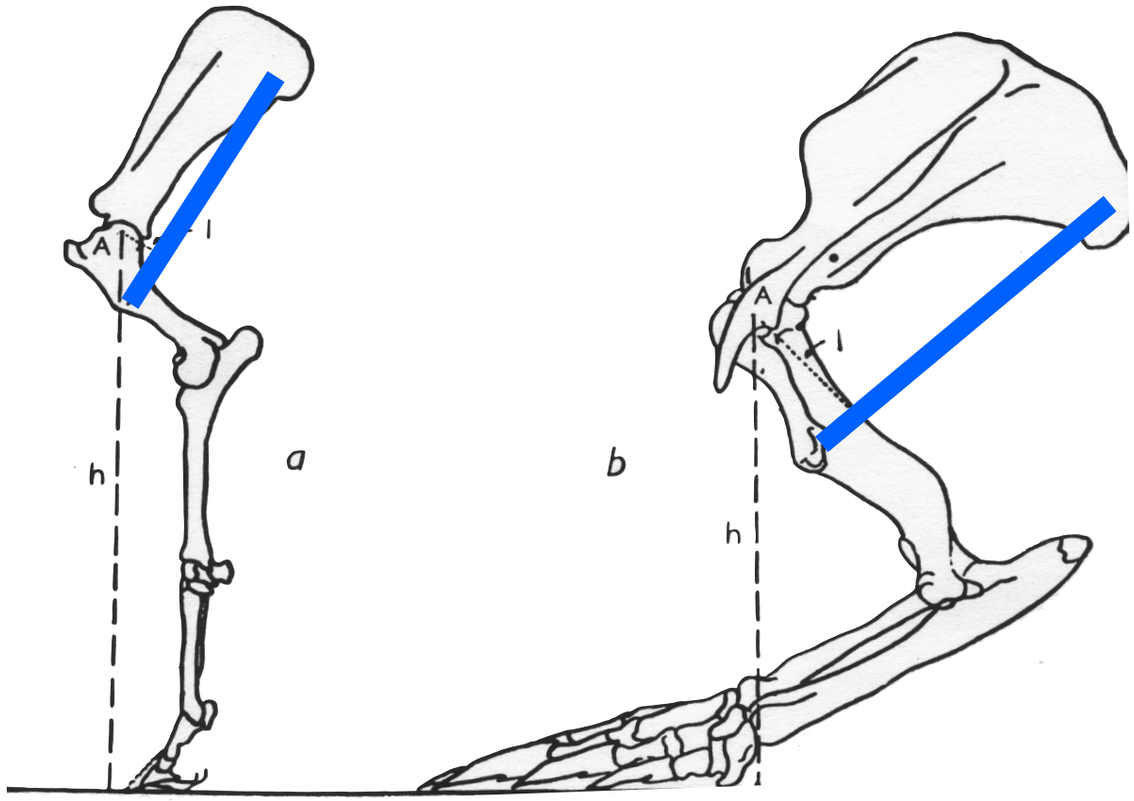
mechanical advantage

$$\frac{F_o}{F_i} = \frac{L_i}{L_o}$$

speed ratio

$$\frac{v_o}{v_i} = \frac{L_o}{L_i}$$

The trade-off between force and speed

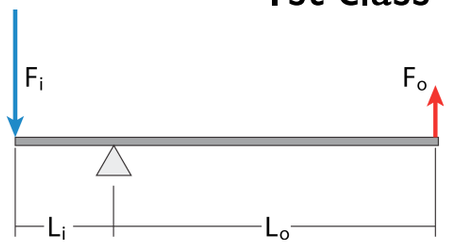


Which class of lever are these legs?

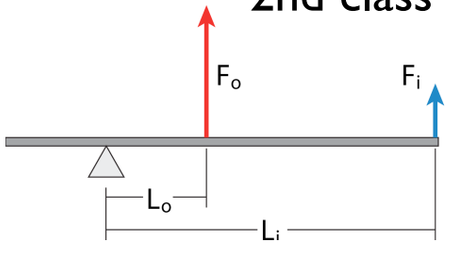
Which has a higher speed ratio? mechanical advantage?

What can you infer about the function of these limbs? Which animals might they belong to?

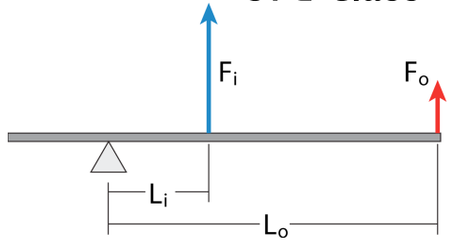
1st class



2nd class



3rd class



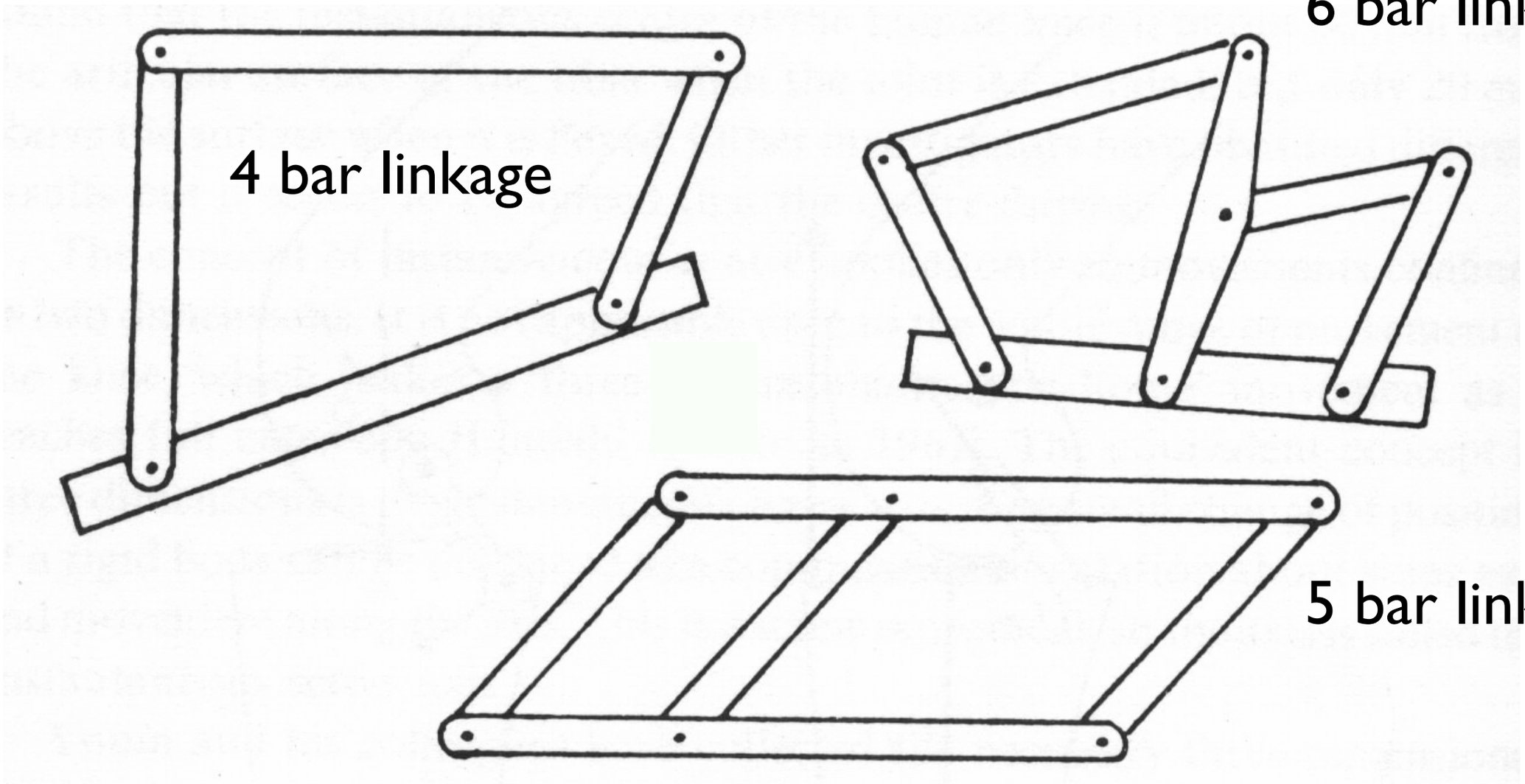
mech adv

$$\frac{F_o}{F_i} = \frac{L_i}{L_o}$$

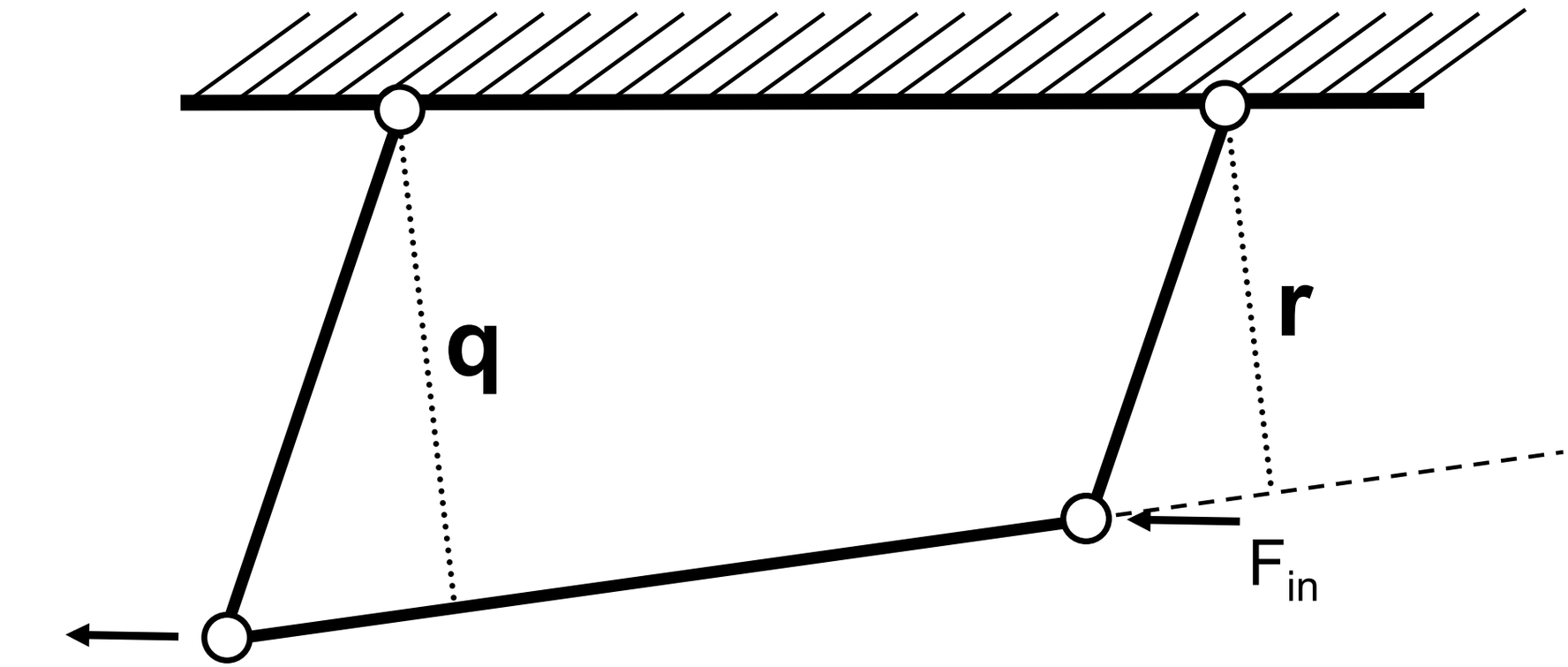
speed ratio

$$\frac{v_o}{v_i} = \frac{L_o}{L_i}$$

More complex levers: linkages



More complex levers: linkages



Force amplification ~ r/q
Speed Amplification ~ q/r

Linkages in nature

Cranial kinesis in birds, reptiles, and amphibians.