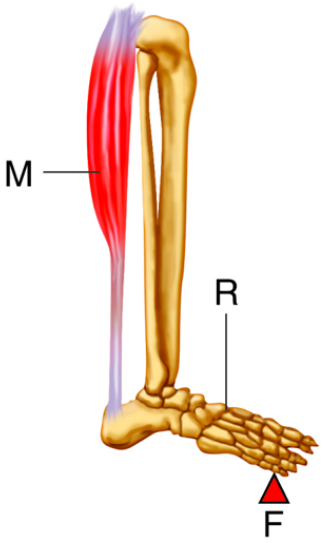
Biology 427 Biomechanics
Lecture 6. Everyday stress and strain and the stiffness of biological materials I: terms, definitions and other basics

- Recap: Loads basic concepts
- Stress -- the distribution of force over an area
- Strain -- a dimensionless measure of deformation
- Stiffness: a measure of how materials respond to loads -- the relationship between stress and strain
- Strength: the stress at breaking

What is the **force** in Gastrocnemius?
What is the **stress** in your Achilles tendon?
How much does the tendon **deform**?
How much energy is imparted into the tendon?
How much energy is returned by the tendon?
How close is the tendon to rupture (breaking)?



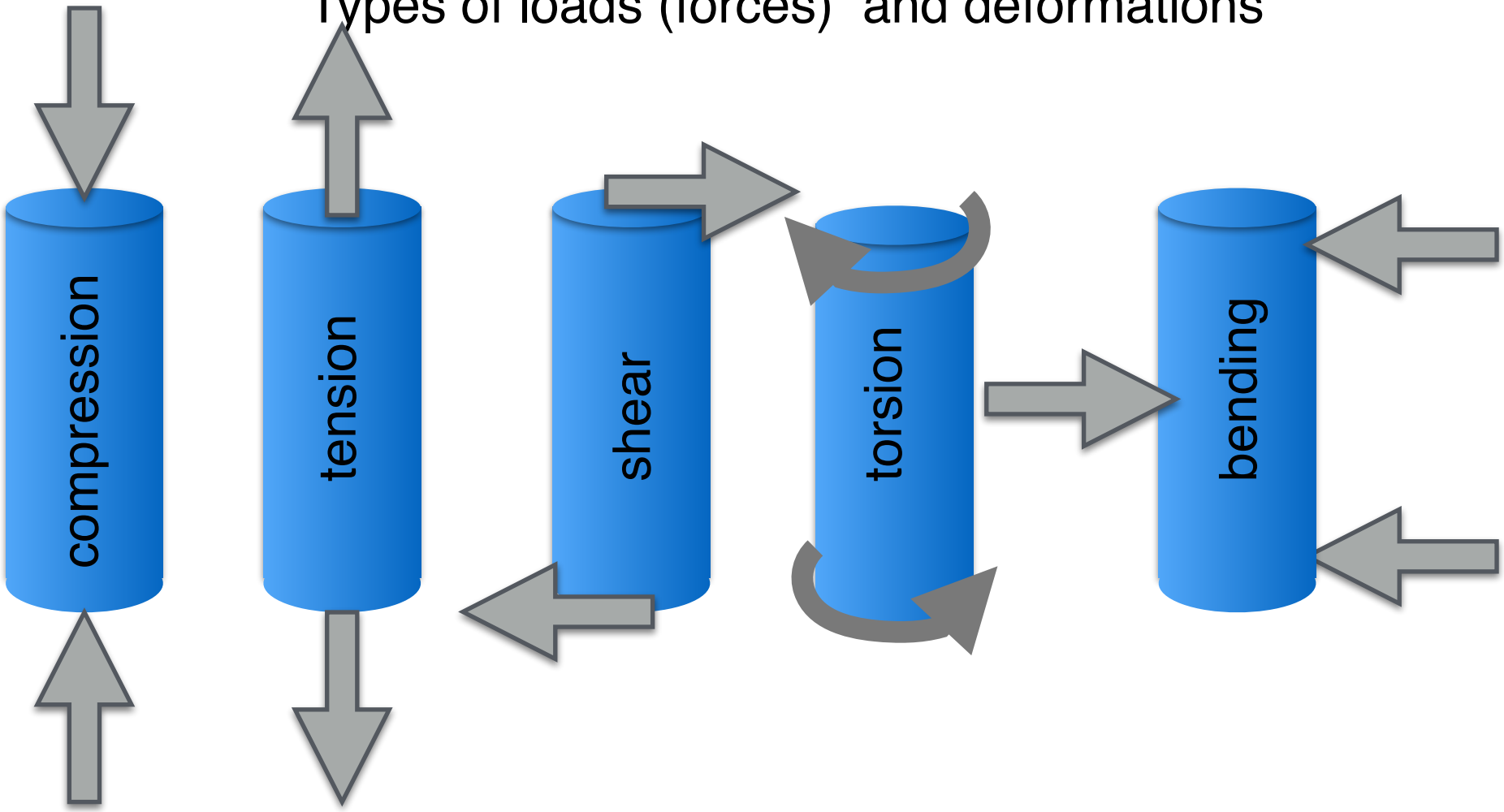
Principles for today

All (biological) materials respond to external forces by deforming.

We seek a way to understand and characterize ***material*** behavior as separate from **structural** behavior

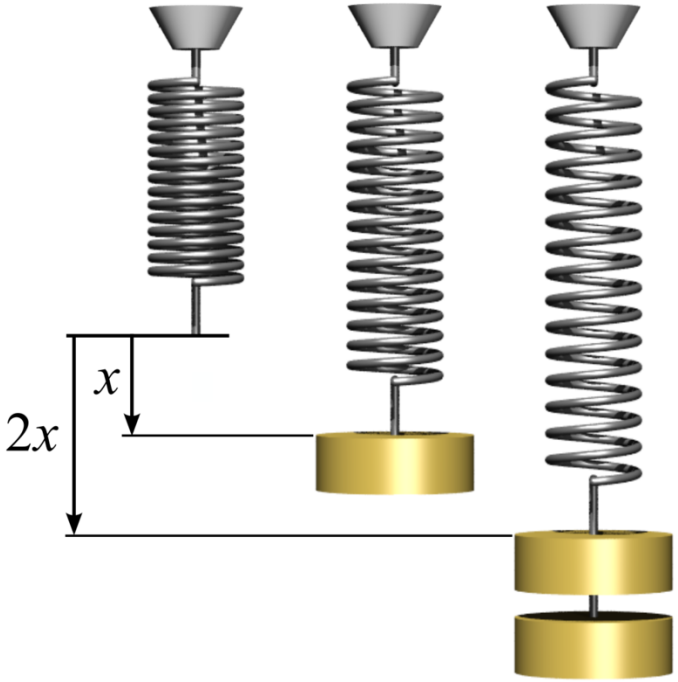
We will formalize the relationships between force and deformation.

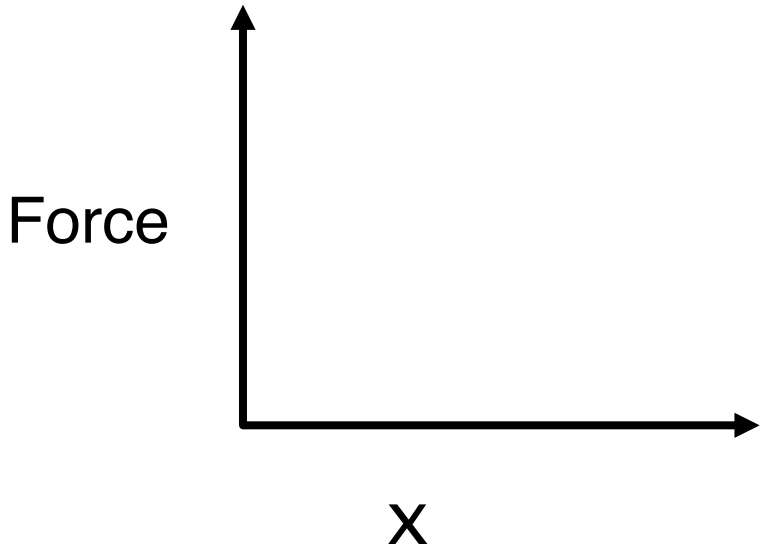
Types of loads (forces) and deformations



Hooke's Law

The displacement is proportional to the force

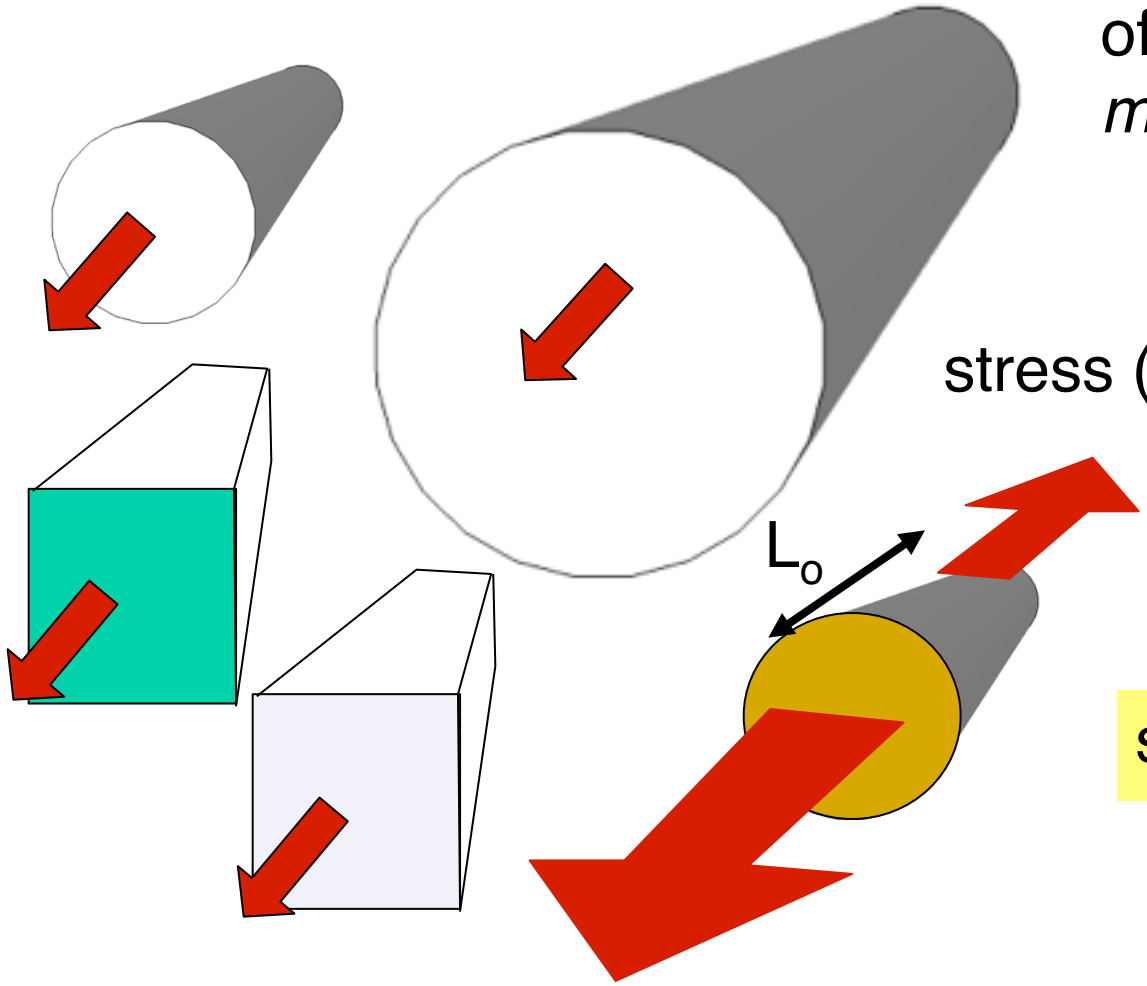




What factors influence this relationship?

Material vs. structural properties

Need to eliminate the effect of size and shape to define *material* properties.



stress (σ) = Force/Area

$\sigma = F/A$

Pa = N/m^2

strain (ε)= $(L - L_o)/L_o$

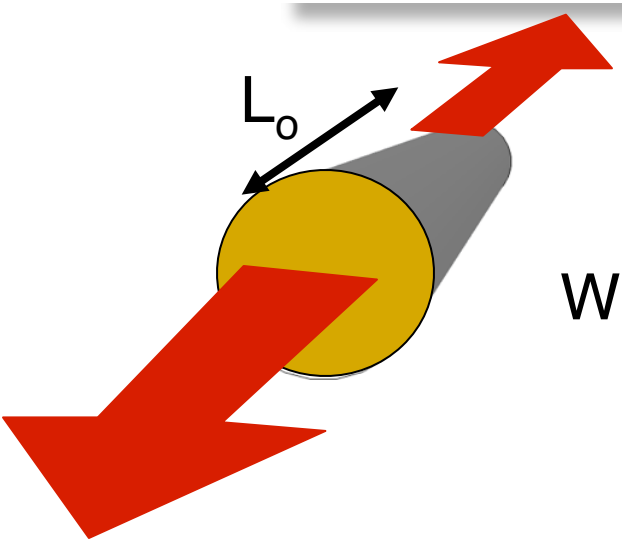
Material vs. structural properties

stress (σ) = Force/Area

strain (ε)= $(L - L_o)/L_o$

$\varepsilon_T = \int dL/L = \ln (L)$

$\lambda = L/L_o$



$\sigma = F/A$

Hooke's Law

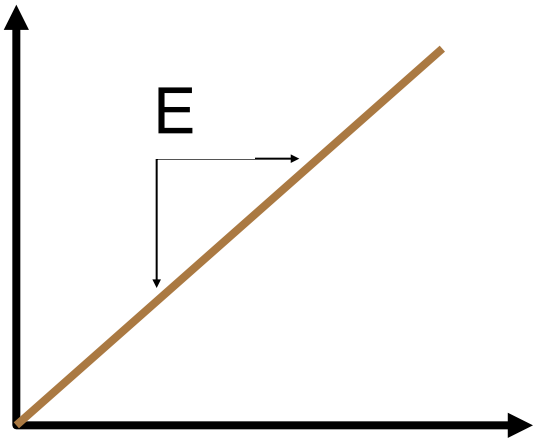
$F = k \Delta L$

Young's modulus (E) measures the stiffness of a material

$E = \sigma/\varepsilon$

What are the units of E?

$$\sigma = \frac{\text{Force}}{\text{Area}}$$



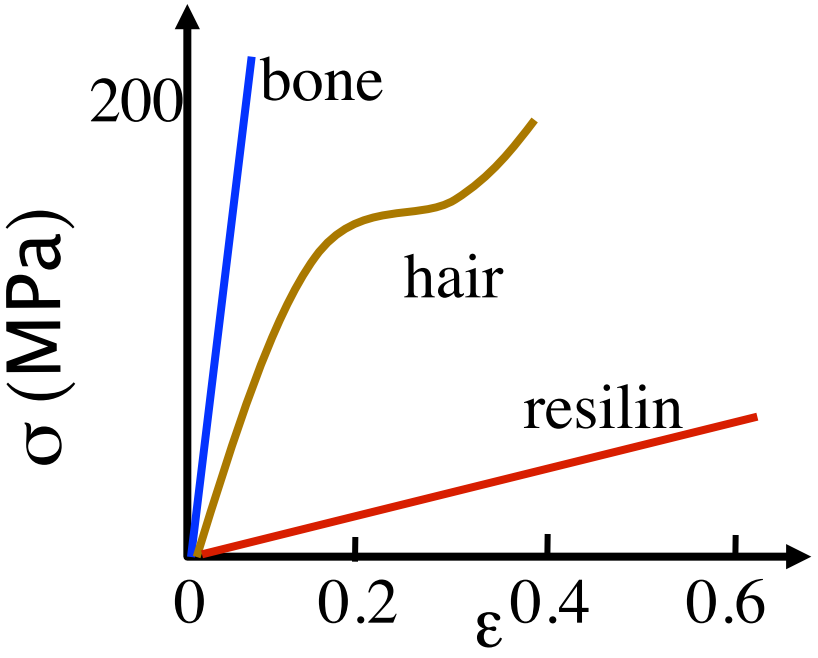
$\varepsilon = \Delta L/L_o$

stress (σ) : the distribution of force over an area

strain (ε): a dimensionless measure of length change

stiffness (E): the change in stress required for a change in strain
(the slope of a stress-strain curve)

Material	Youngs Modulus (Mpa)
Locust cuticle	0.2
Rubber	7
Human cartilage	24
Human tendon	600
Cheap plastic	1,400
Plywood	14,000
Human bone	21,000
Glass	70,000
Brass	120,000
Iron	210,000
Diamond	1,200,000



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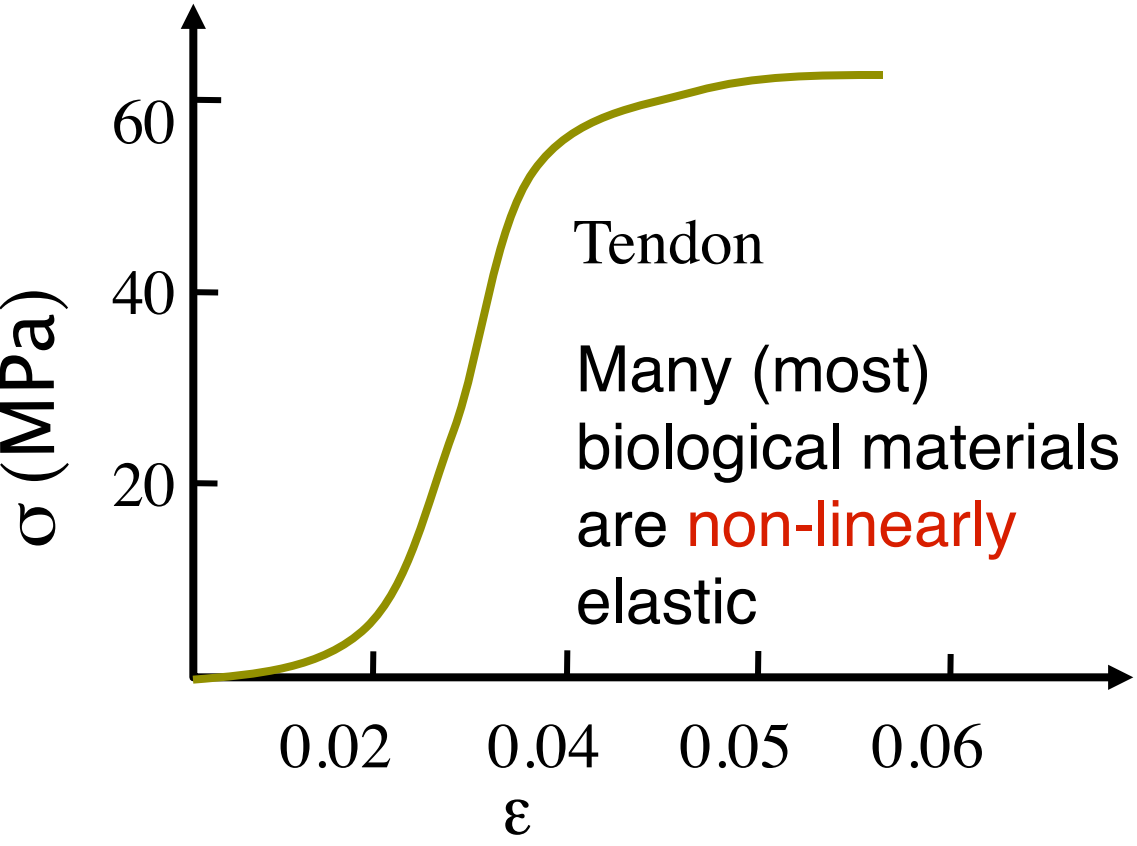
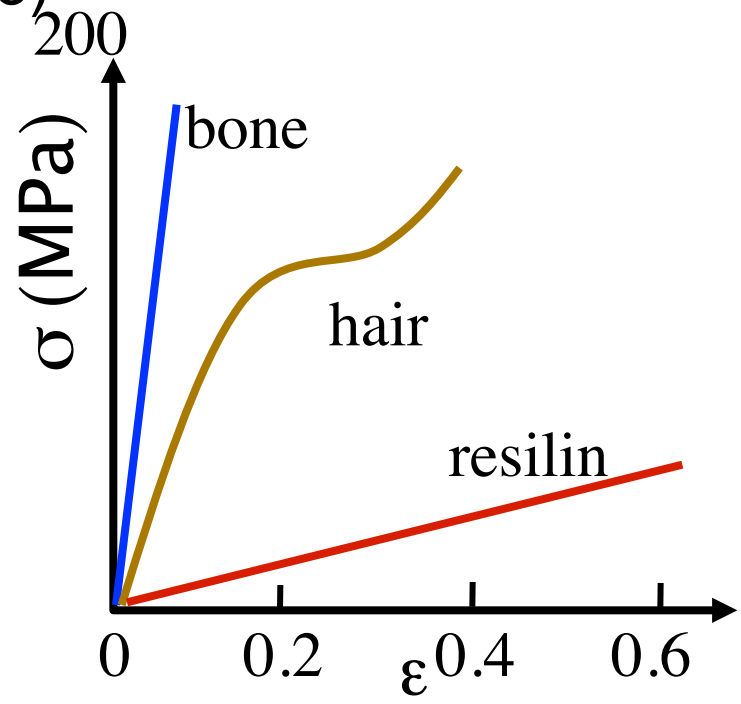
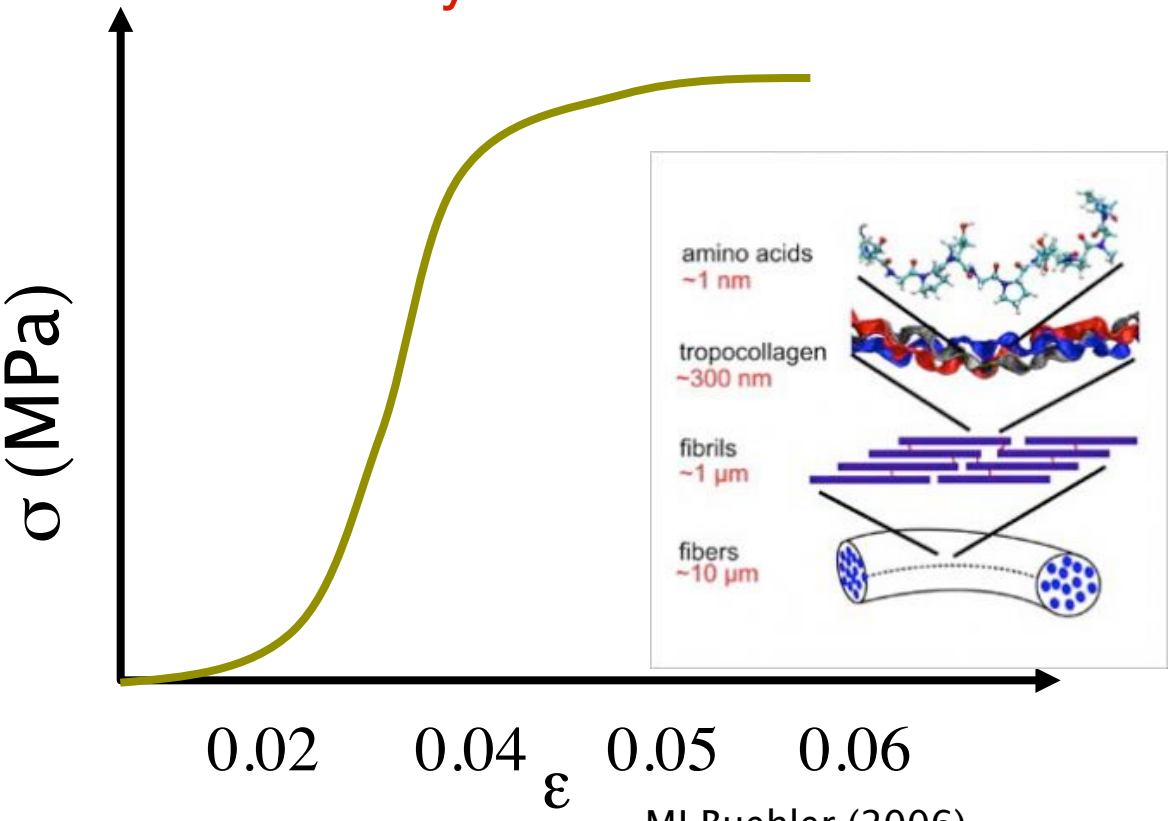
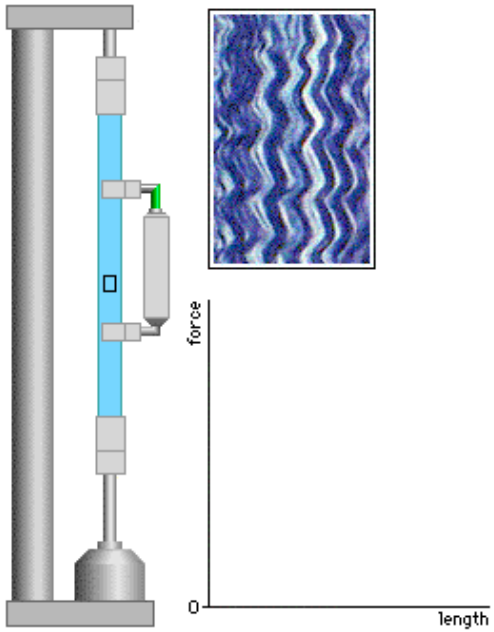




Many (most) biological materials are non-linearly elastic





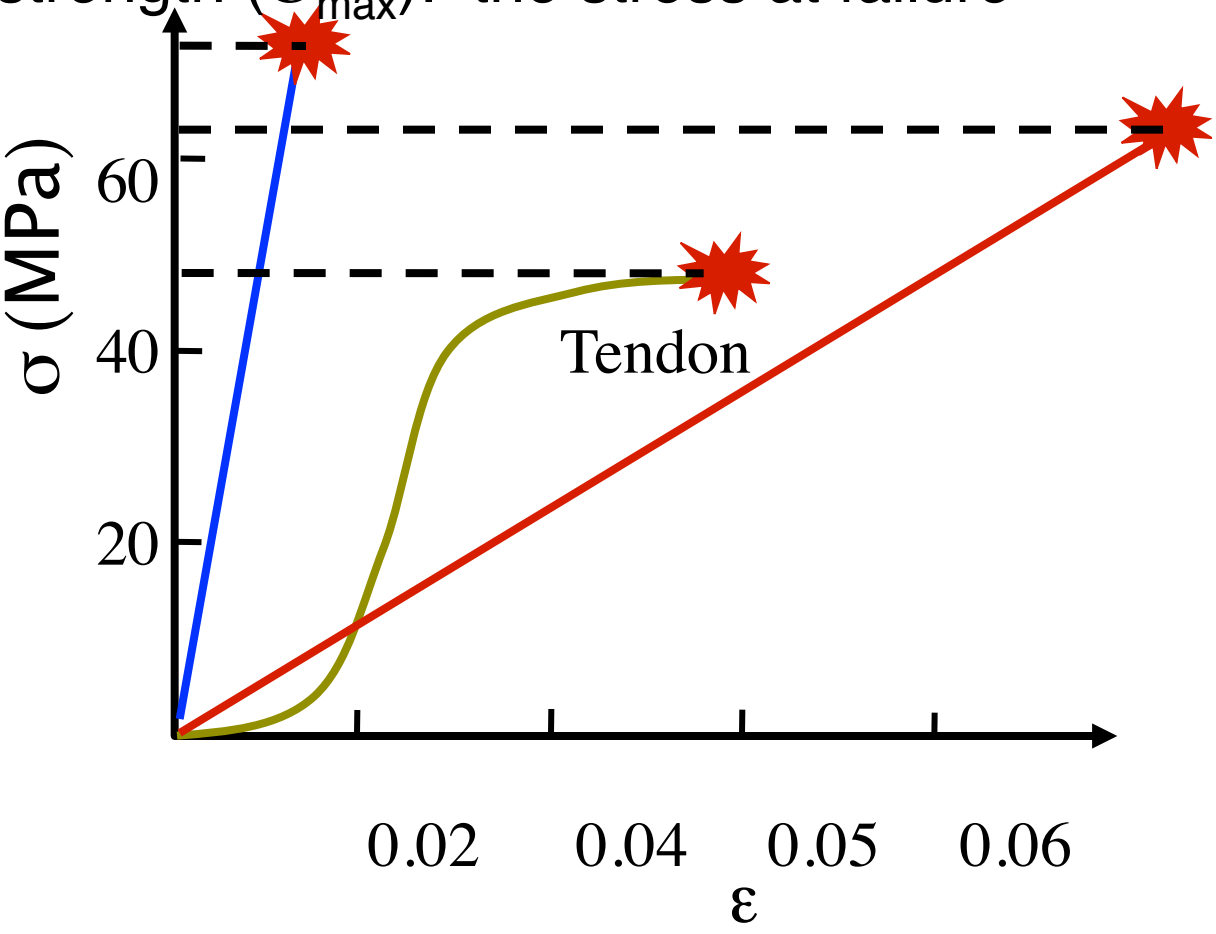
MJ Buehler (2006)
Nature designs tough collagen: Explaining the nanostructure of collagen fibrils PNAS 103:12285-12290

stress (σ) : the distribution of force over an area

strain (ε): a dimensionless measure of length change

stiffness (E): the change in stress required for a change in strain
(the slope of a stress-strain curve)

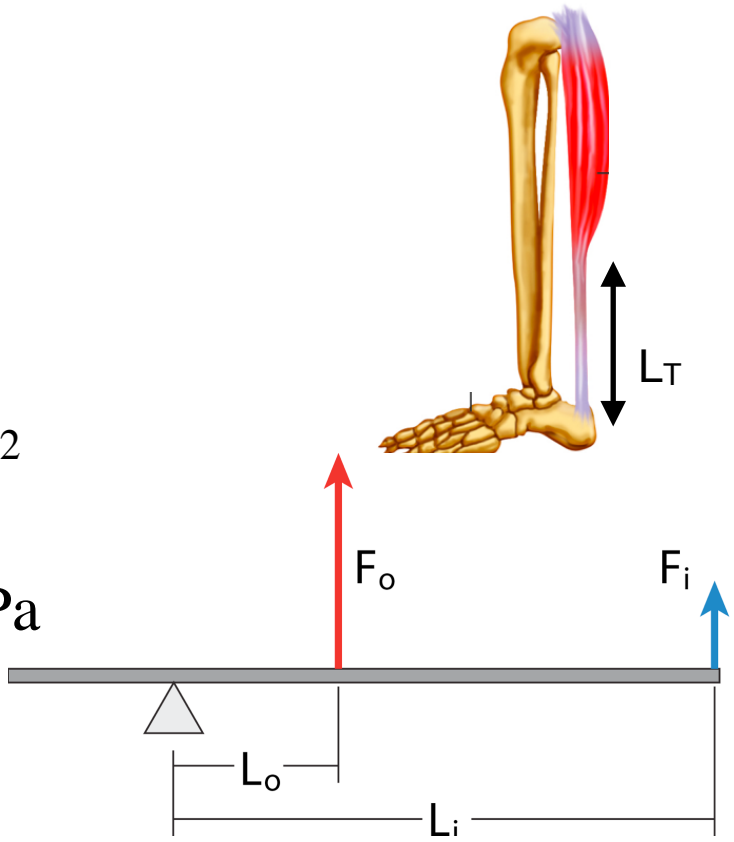
strength (σ_{max}): the stress at failure



material	strength (MPa)
arterial wall	2
human cartilage	3
cement	4
cheap aluminum	70
glass	100
human tendon	100
human bone	110
human hair	200
spider silk	350
titanium	1000
steel wire	3000

material	strength (MPa)	density (kg/m^3)	strength/density
arterial wall	2	1000	2000
human cartilage	3	1000	3000
cement	4	2000	2000
cheap aluminum	70	2700	26000
glass	100	2600	39000
human tendon	100	1000	100000
human bone	110	1200	92000
human hair	200	1000	200000**
spider silk	350	1000	350000**
titanium	1000	4500	222222**
steel wire	3000	8000	375000**

- m= 70 kg
- g = 9.8 m/s^2
- L_i = 20 cm
- L_o = 15 cm
- L_T = 10 cm
- $Area_T$ = 1 cm^2
- E = 600 MPa
- σ_{max} =100MPa



force on the tendon?
stress on the tendon?
strain on the tendon?