Biology 427

Lecture 7. Strength and toughness of biological materials

Recap stress, strain, stiffness and strength of biomaterials: measures of material properties

Strength revisited and the limits to the size of terrestrial vertebrates

Energy relations in biological materials: toughness and resilience

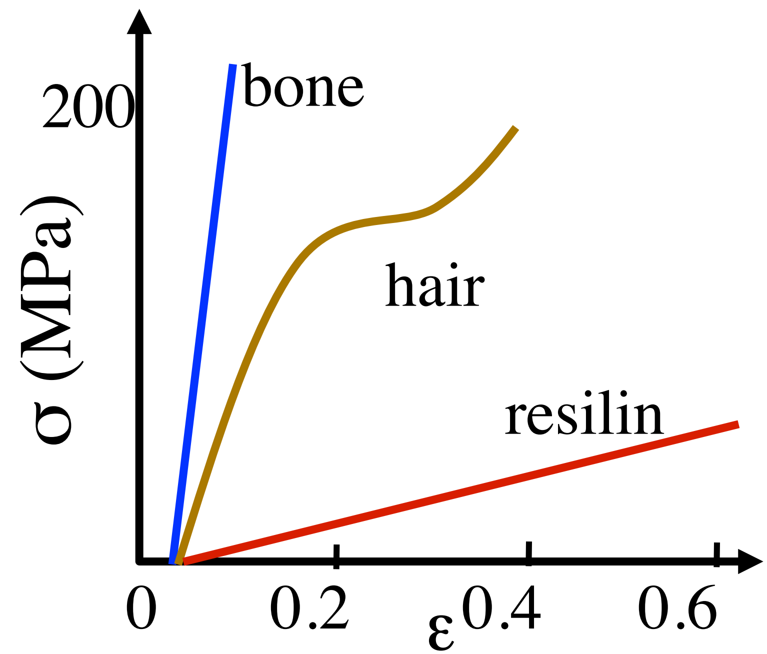
Plastic deformations: an introduction to time-dependent material properties.

stress (σ) : the distribution of force over an area

strain (ε): a dimensionless measure of length change

stiffness (E): the change in stress required for a change in strain (the slope of a stress-strain curve) a material property

strength (σ_{max}): the stress at failure



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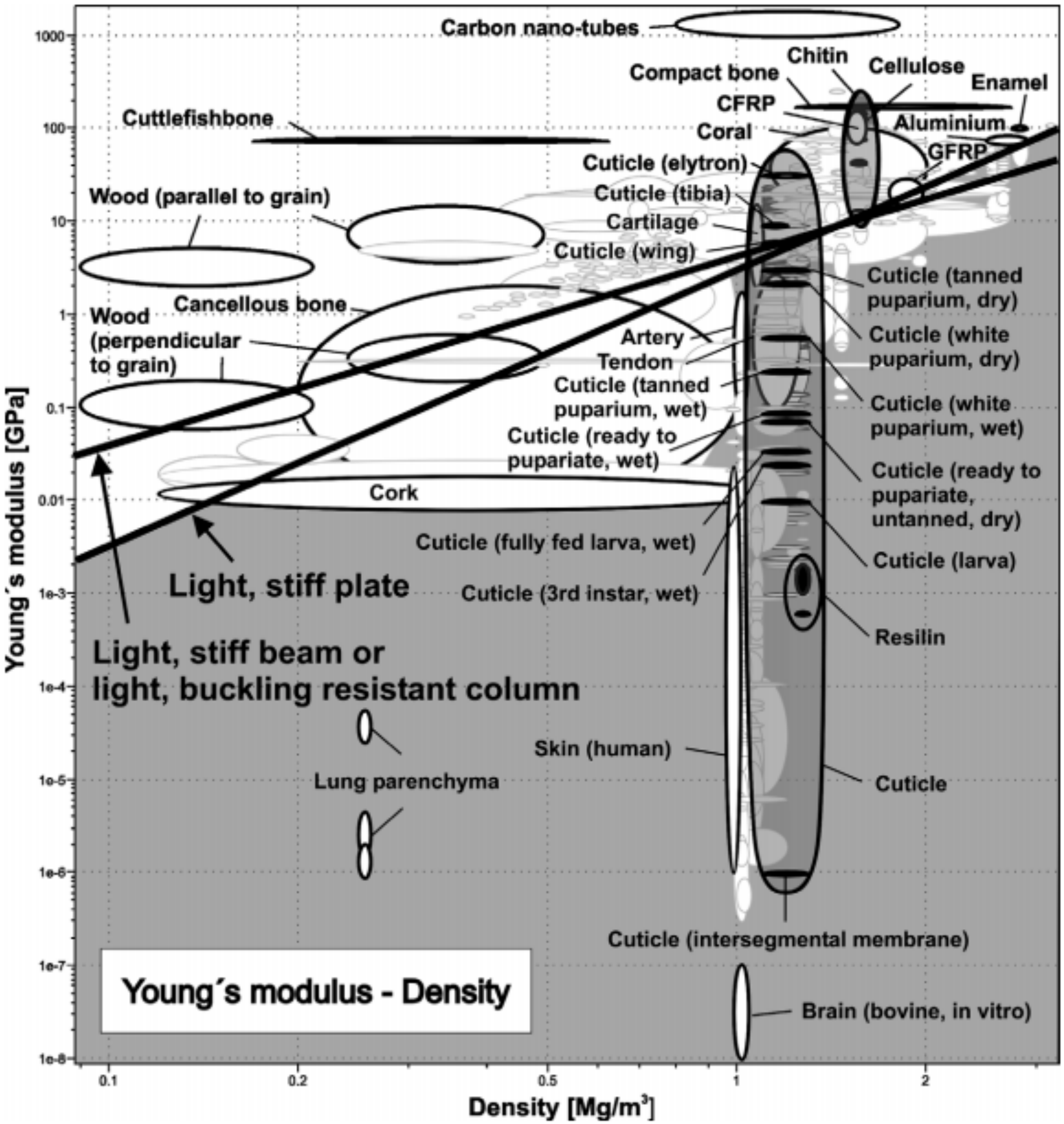
material	strength (MPa)	density (kg/m^3)	strength/density
arterial wall	2	1000	2000
human cartilage	3	1000	3000
cement	4	2000	2000
cheap aluminum	70	2700	26000
glass	100	2600	39000
human tendon	100	1000	100000
human bone	110	1200	92000
human hair	200	1000	200000**
spider silk	350	1000	350000**
titanium	1000	4500	222222**
steel wire	3000	8000	375000**

ε



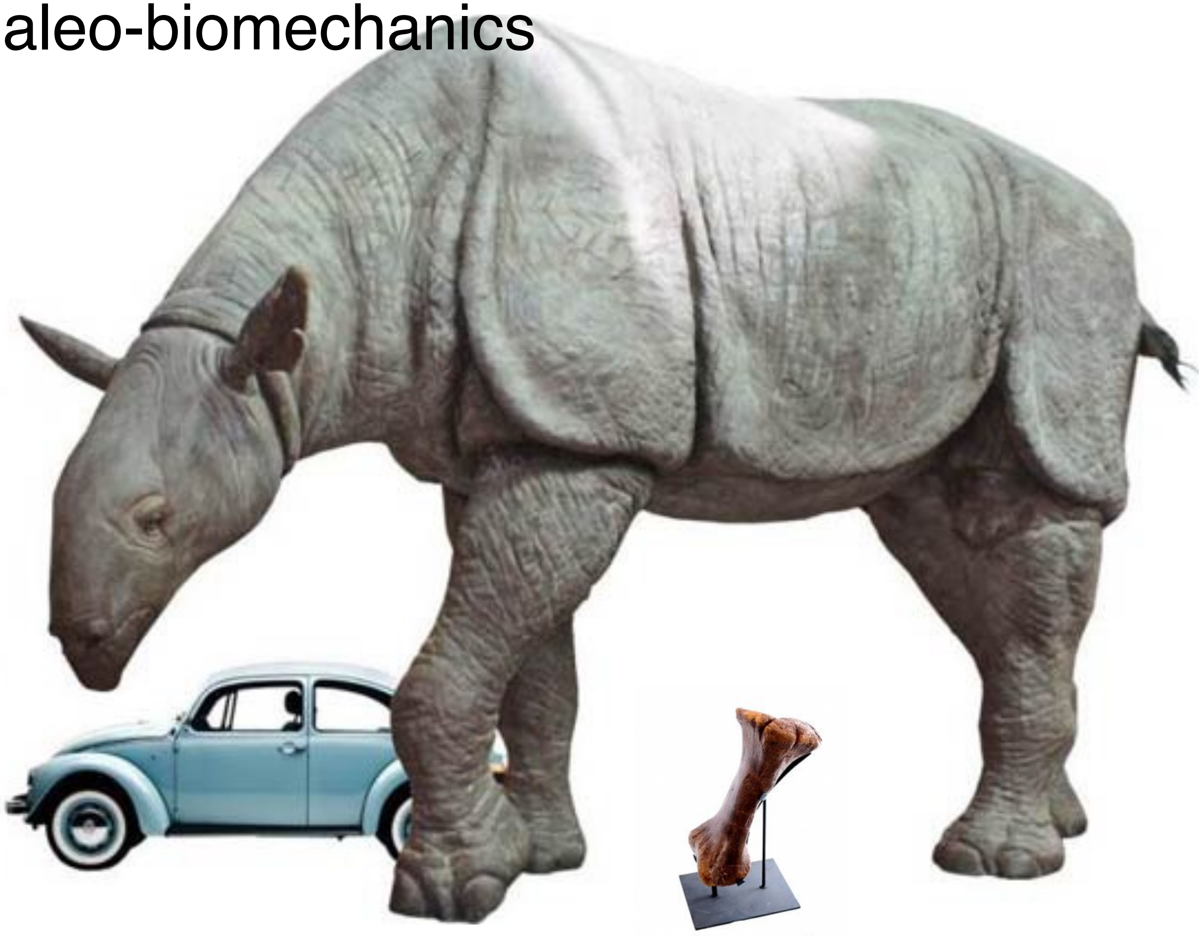
Young´s modulus - Density
Young´s modulus [GPa]
Density [Mg/m³]
1000
100
10
1
0.1
0.01
1e-3
1e-4
1e-5
1e-6
1e-7
1e-8
0.1
0.2
0.5
1
2
Carbon nano-tubes
Chitin
Cellulose
Compact bone
Enamel
CFRP
Cuttlefishbone
Aluminium
Coral
GFRP
Cuticle (elytron)
Wood (parallel to grain)
Cuticle (tibia)
Cartilage
Cuticle (wing)
Cuticle (tanned puparium, dry)
Cancellous bone
Wood (perpendicular to grain)
Artery
Cuticle (white puparium, dry)
Tendon
Cuticle (tanned puparium, wet)
Cuticle (white puparium, wet)
Cuticle (ready to pupariate, wet)
Cuticle (ready to pupariate, untanned, dry)
Cork
Cuticle (fully fed larva, wet)
Cuticle (larva)
Light, stiff plate
Cuticle (3rd instar, wet)
Resilin
Light, stiff beam or light, buckling resistant column
Skin (human)
Lung parenchyma
Cuticle
Cuticle (intersegmental membrane)
Brain (bovine, in vitro)

Paleo-biomechanics

Were sauropods too large to support their weight?

land animals were thought to be size constrained by the strength of bone

Paleo-biomechanics



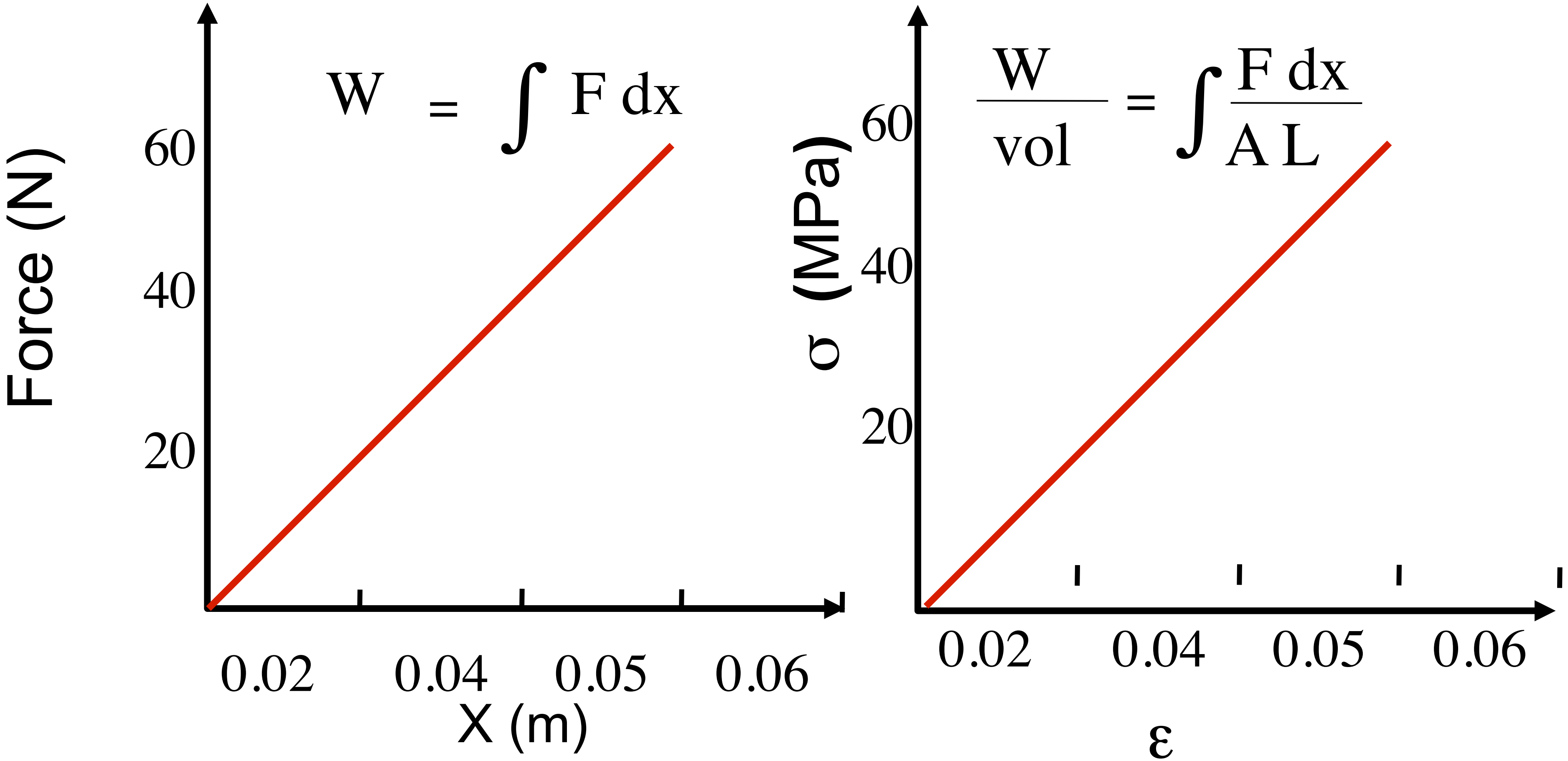
Baluchitherium:
about 30 Tons
Could the foot bones support its weight?

$\sigma_{max,bone} = 100$ MPa

diameter = 14 cm
Area = 151 10^{-4} m^2

Compute the mass that could be supported

Energy Basics for Materials



The energy imparted is the mechanical strain energy

$$U = \int \sigma \, d\varepsilon$$

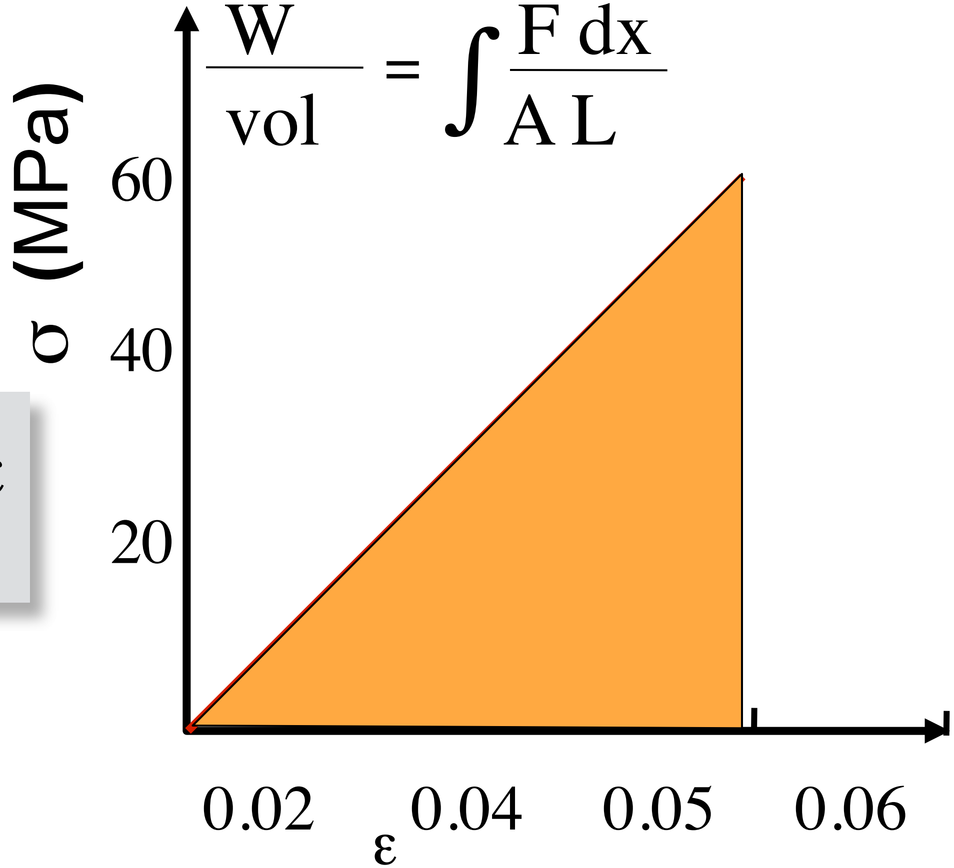
For Hookean materials

$$\sigma = E\,\varepsilon$$

$$U = \int \sigma \, d\sigma / E = \sigma^2/2E$$

$$U = \int E\,\varepsilon \, d\varepsilon = E\,\varepsilon^2/2$$

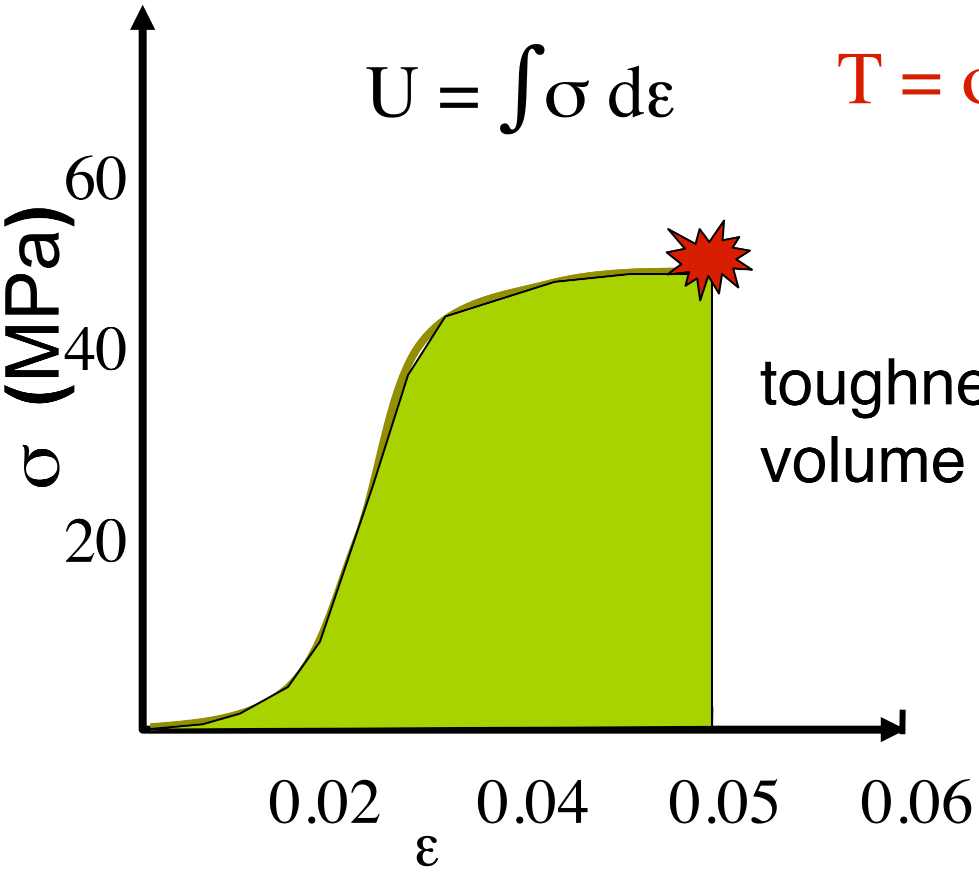
$$\frac{W}{vol} = \int \frac{F\,dx}{A\,L}$$



The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material

$U = \int \sigma \, d\varepsilon$

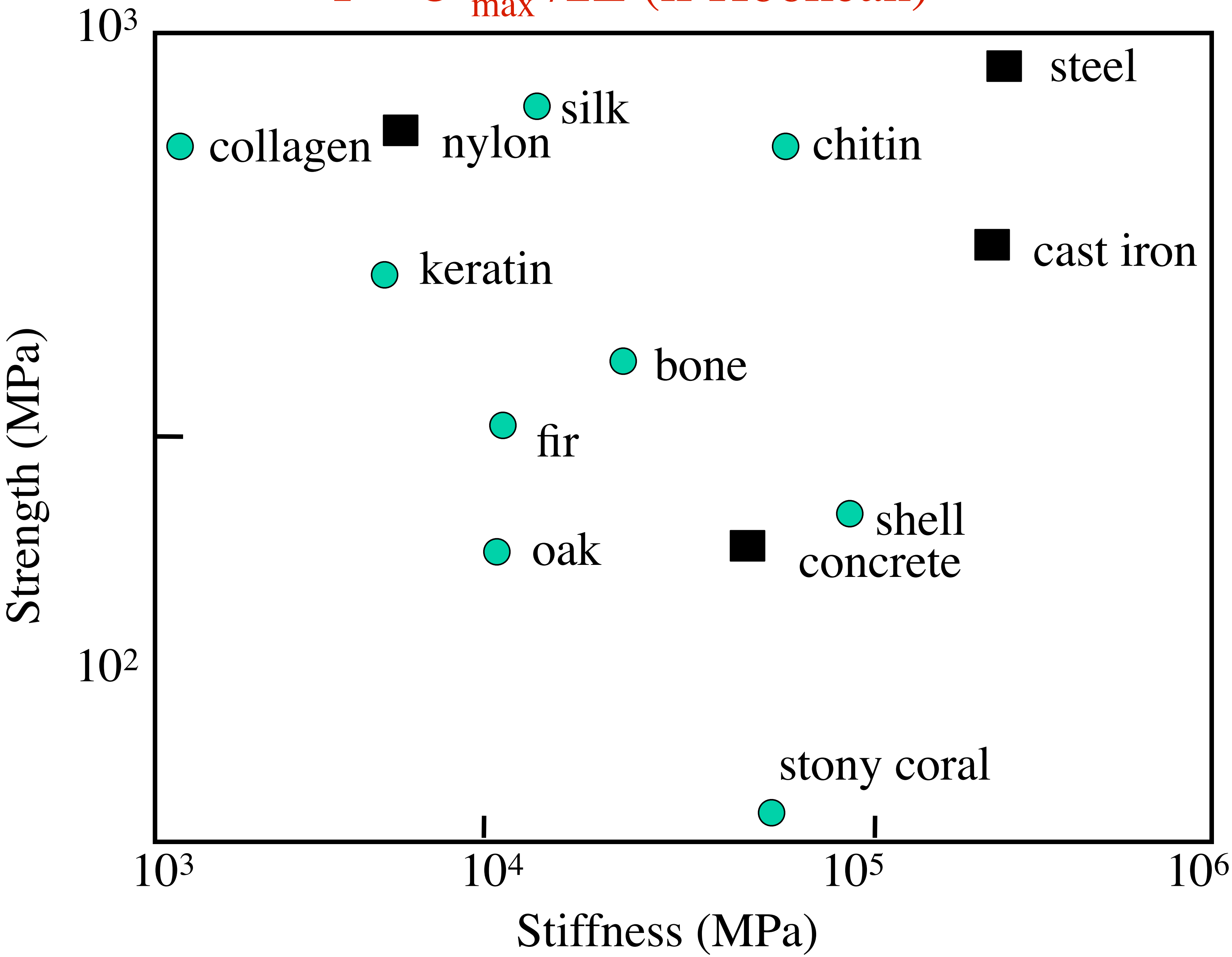
$T = \sigma^2_{max} / 2E$ (if Hookean)

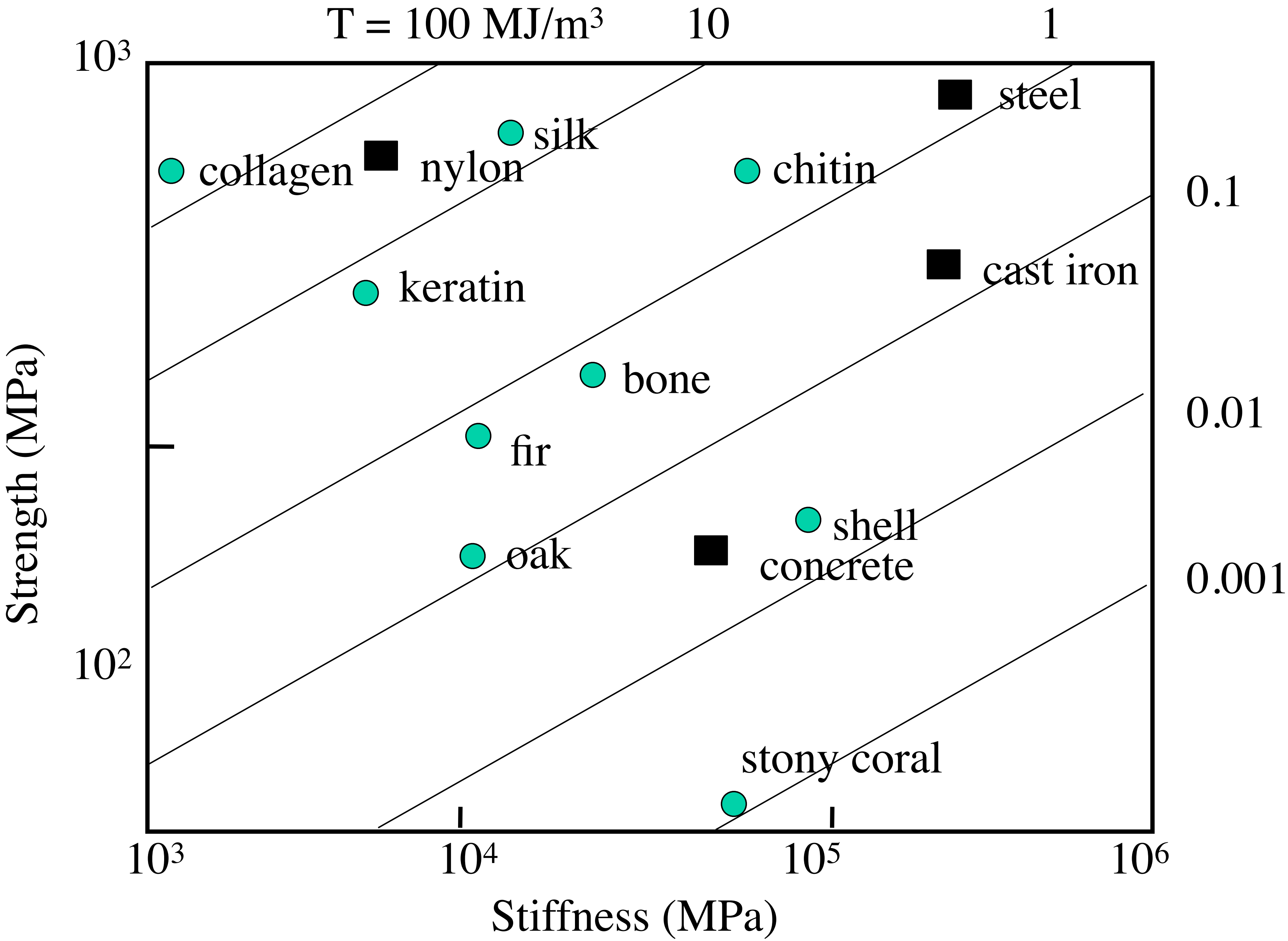


toughness: the energy per unit volume required to break a material

$T = \int \sigma \, d\varepsilon$

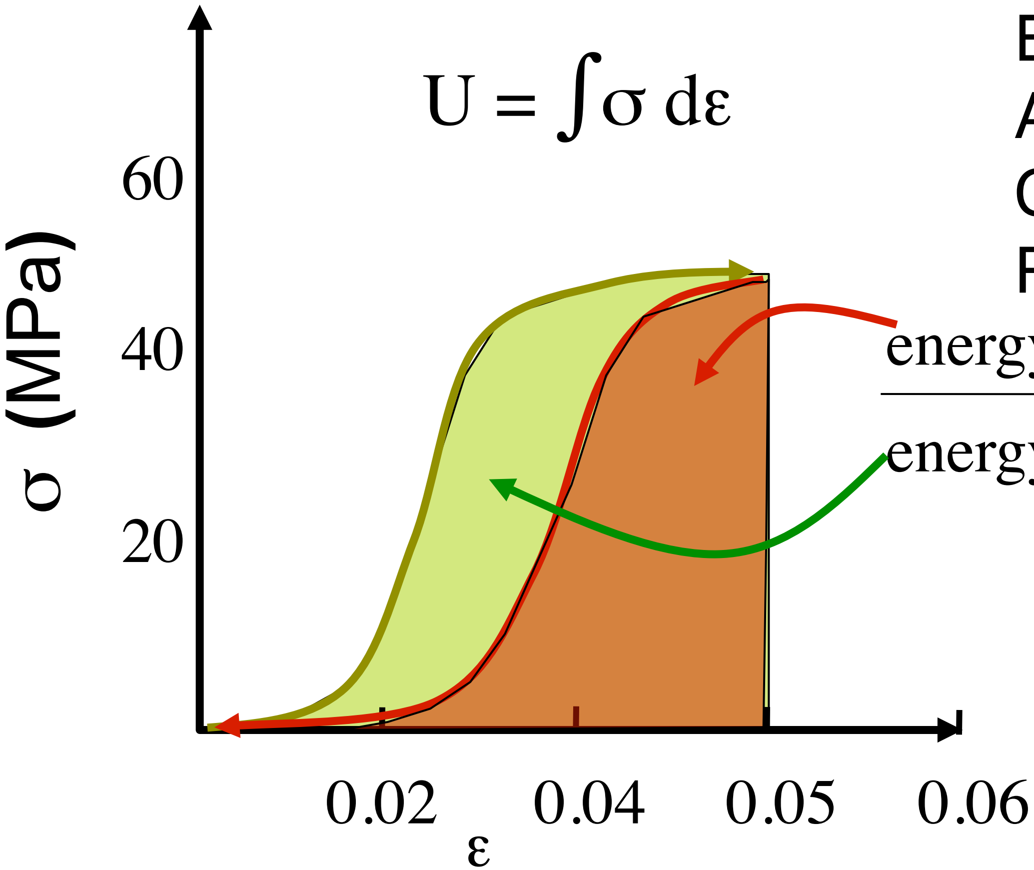
$T = \sigma^2_{max} / 2E$ (if Hookean)





T = 100 MJ/m³
10
1
10³
0.1
0.01
0.001
10²
Strength (MPa)
collagen
nylon
silk
chitin
steel
keratin
cast iron
bone
fir
shell
oak
concrete
stony coral
10³
10⁴
10⁵
10⁶
Stiffness (MPa)

Some of the mechanical strain energy may be returned

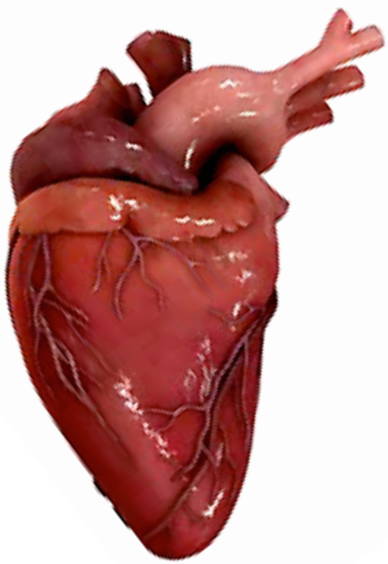


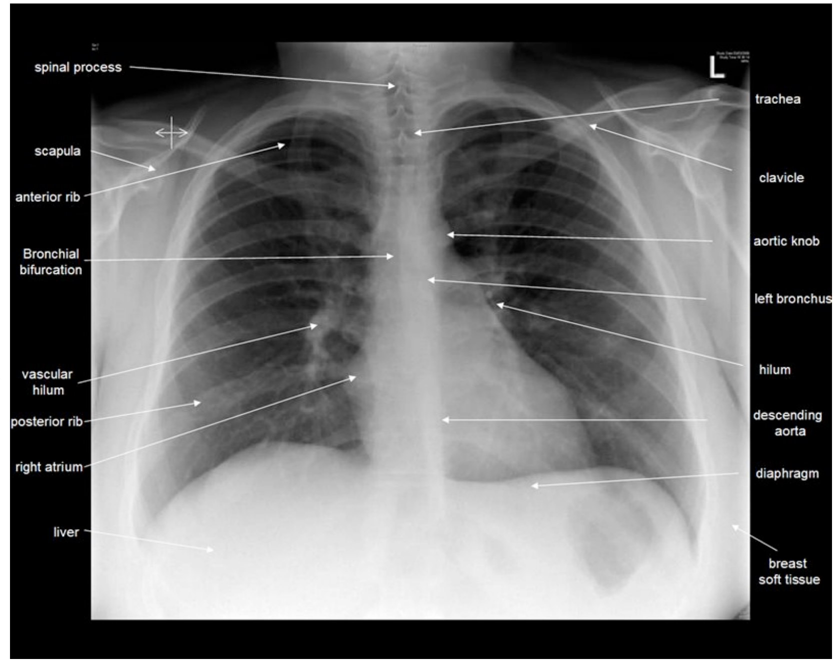
Silk	35%
Elastin	76%
Abductin	80%
Collagen (tendon)	93%
Resilin (wing hinge)	97%

$$\frac{\text{energy returned}}{\text{energy imparted}} = \text{Resilience (R)}$$

Recovery of elastic strain energy is critical in animal function and locomotion: resilin and elastin and collagen

Other physiological functions:



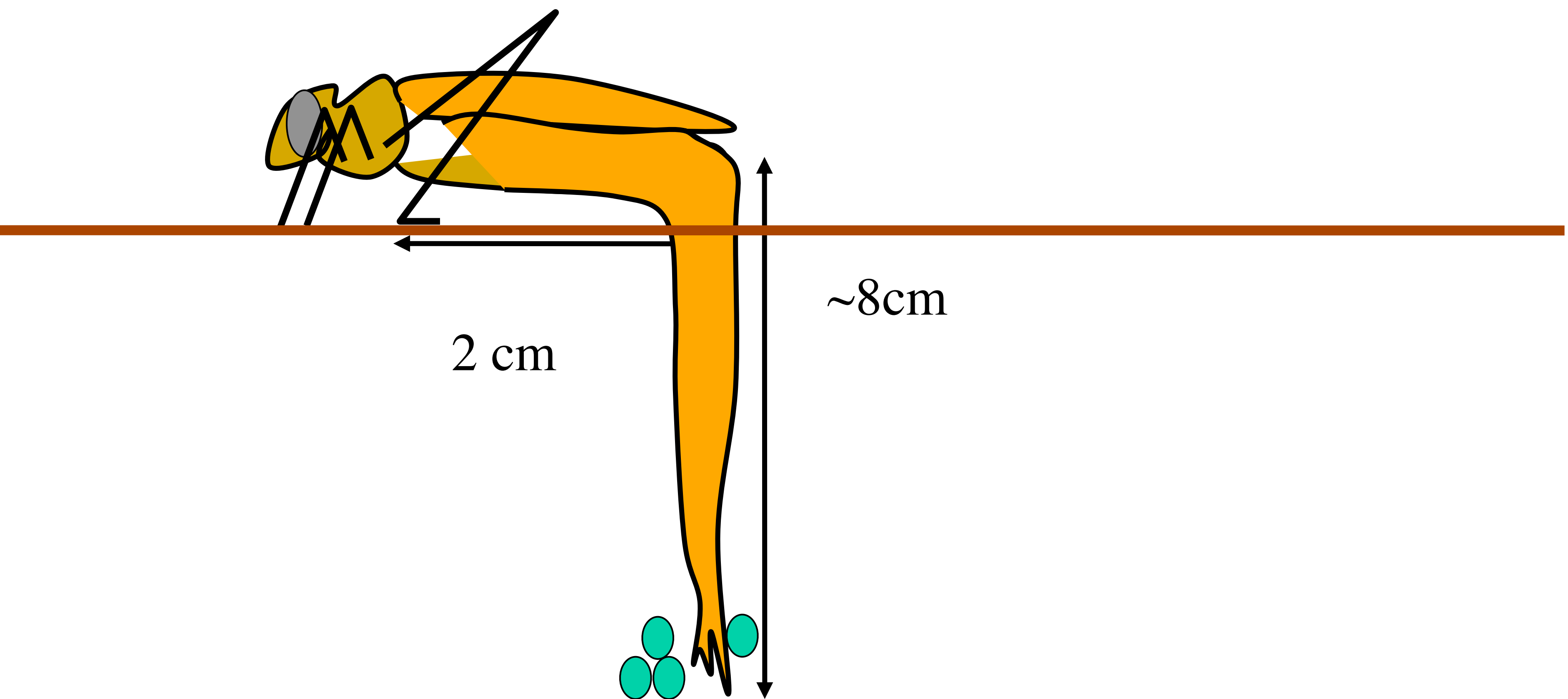


Sports??

The story of the pregnant (gravid) locust

- locusts are migratory
- light weight important in flight
- fertilized eggs in dehydrated state
- live in very arid climates
- trick: bury the eggs ~ 8 cm beneath surface

$E = 2\ 10^5$ Pa
$\varepsilon = 3$



The energy imparted is the mechanical strain energy that can be returned or be so great as to break the material or be lost as a permanent deformation (plastic deformation)

Any examples in humans?