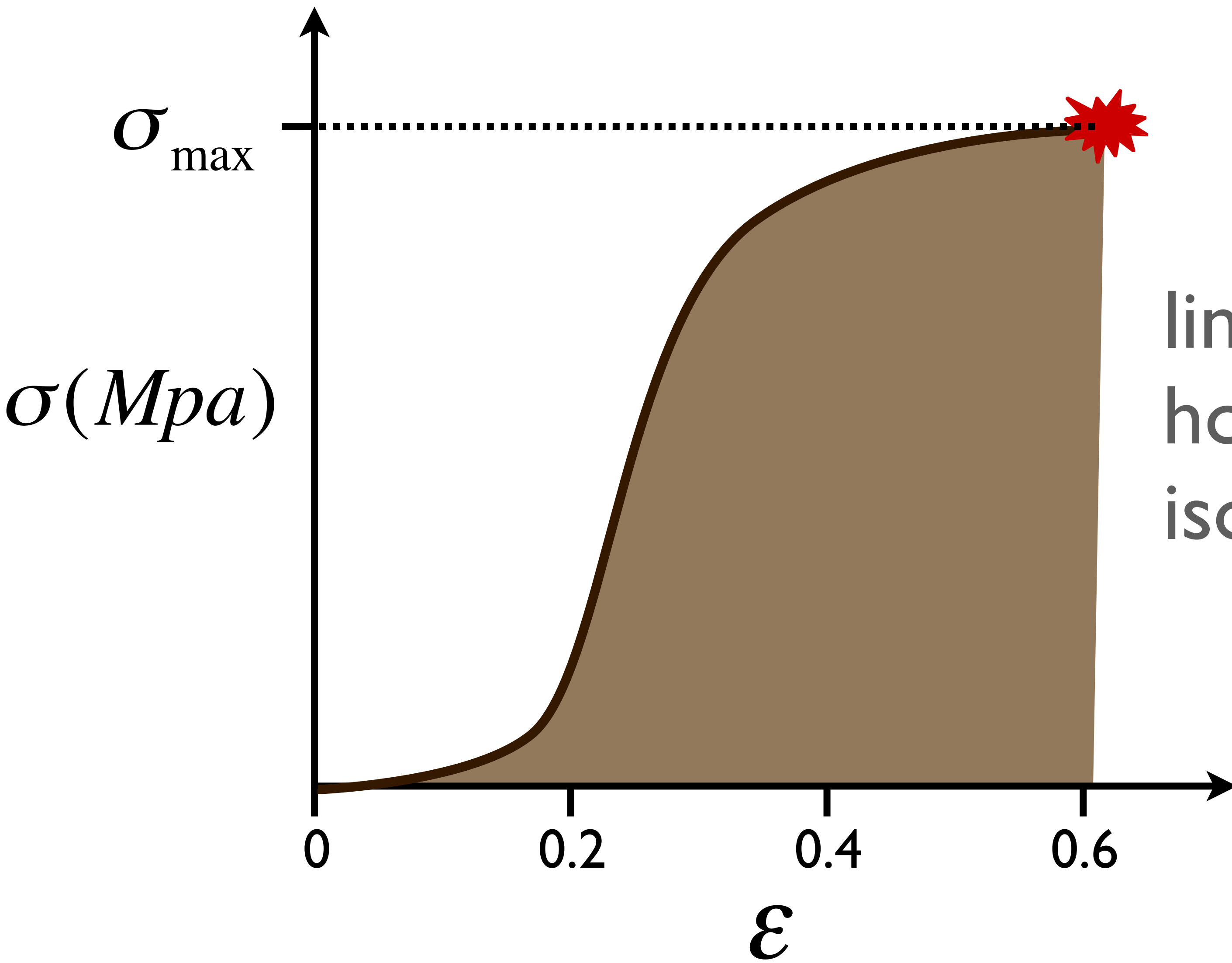
Biomechanics 427

Lecture 8: Visco-elasticity - time dependent properties of biological materials

- Recap **strength**, **toughness** and **resiliency** of biomaterials: measures of *material* properties and a few more notions about biomaterials
- Breaking up
- Introduce fluid (viscous) behaviors and differentiate from solid (elastic) behaviors
- Experimental visco-elastic results for some biomaterials
- Elemental descriptions of visco-elastic material properties

A few more ideas:



linear *vs*. non-linear

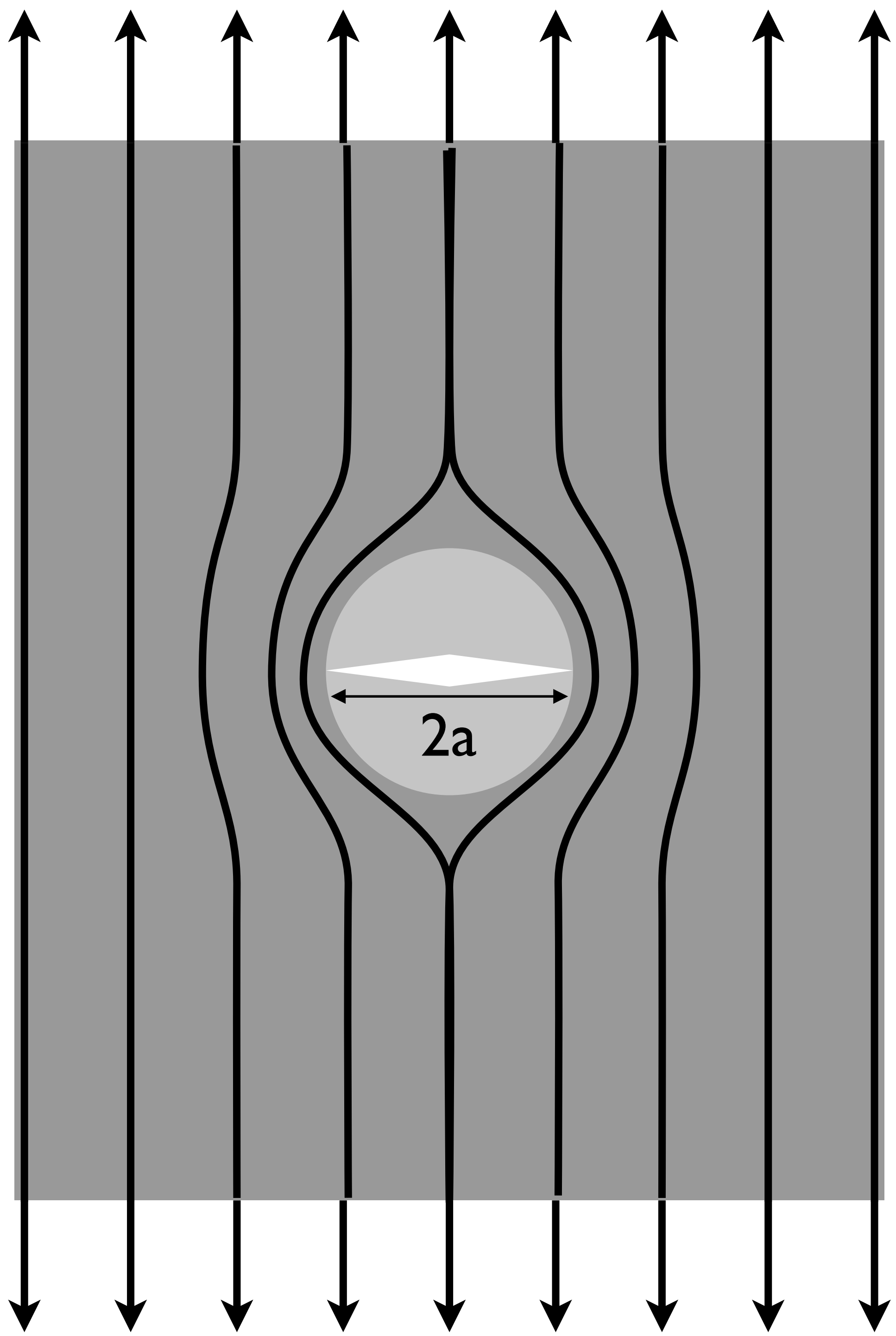
homogeneous *vs*. inhomogeneous

isotropic *vs*. anisotropic

The dynamics of fractures: Griffith fracture theory:

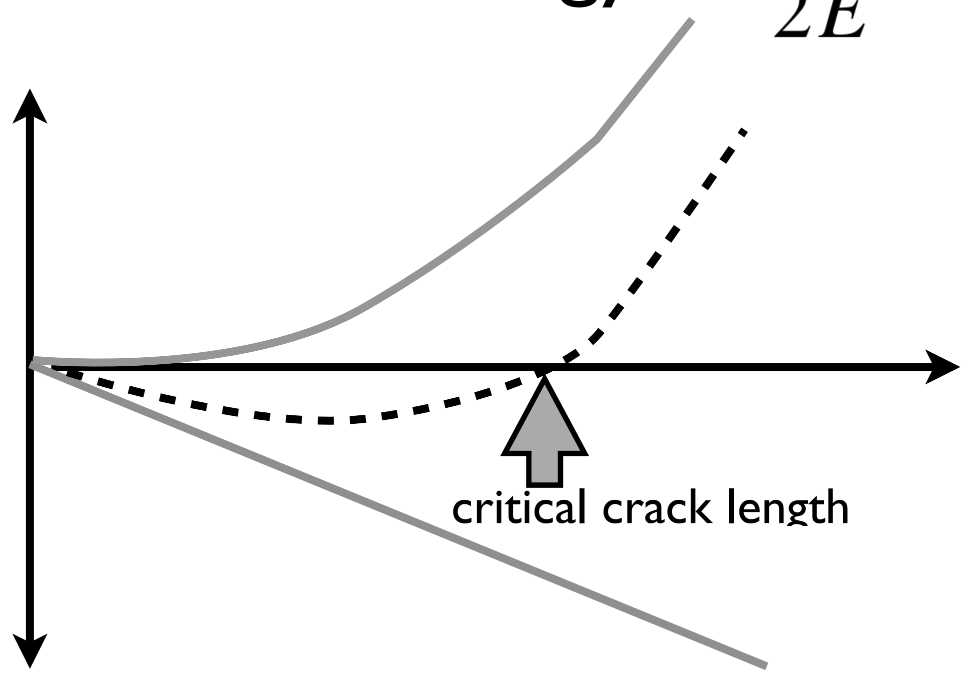
why do bones and shells break explosively?

why do cracks propagate?



strain energy $U = \int \sigma d\varepsilon = \int \frac{\sigma d\sigma}{E} = \frac{\sigma^2}{2E}$

released strain energy $\frac{\sigma^2}{2E}\pi a^2 t$



energy to make new crack = $4Sa\ t$

S= surface energy of bonds

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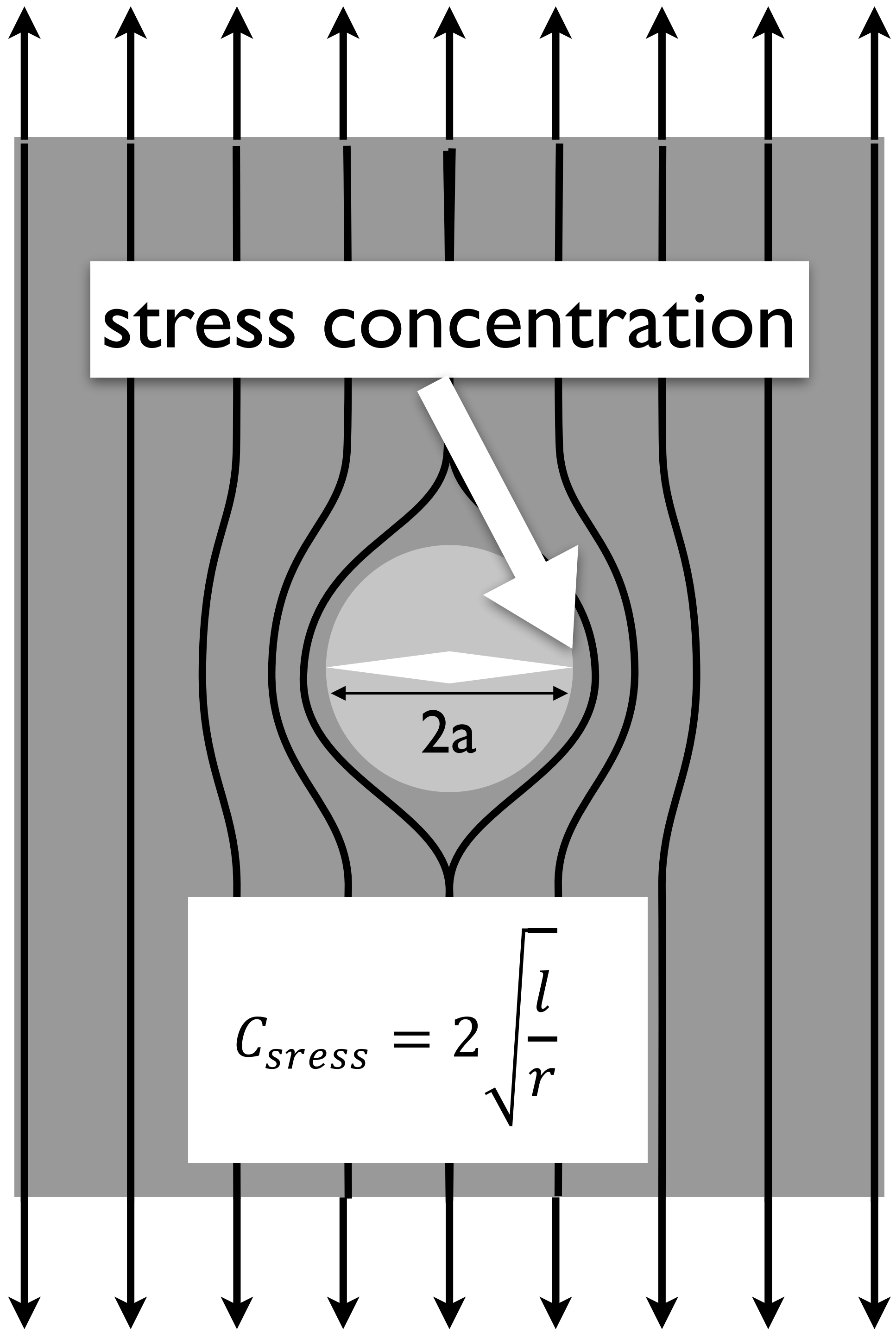

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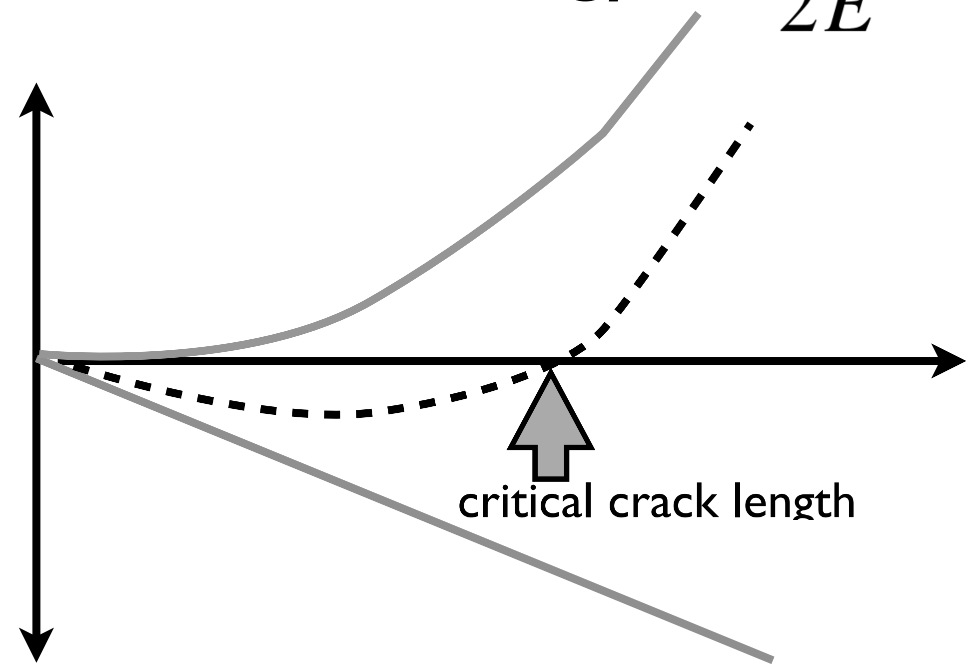

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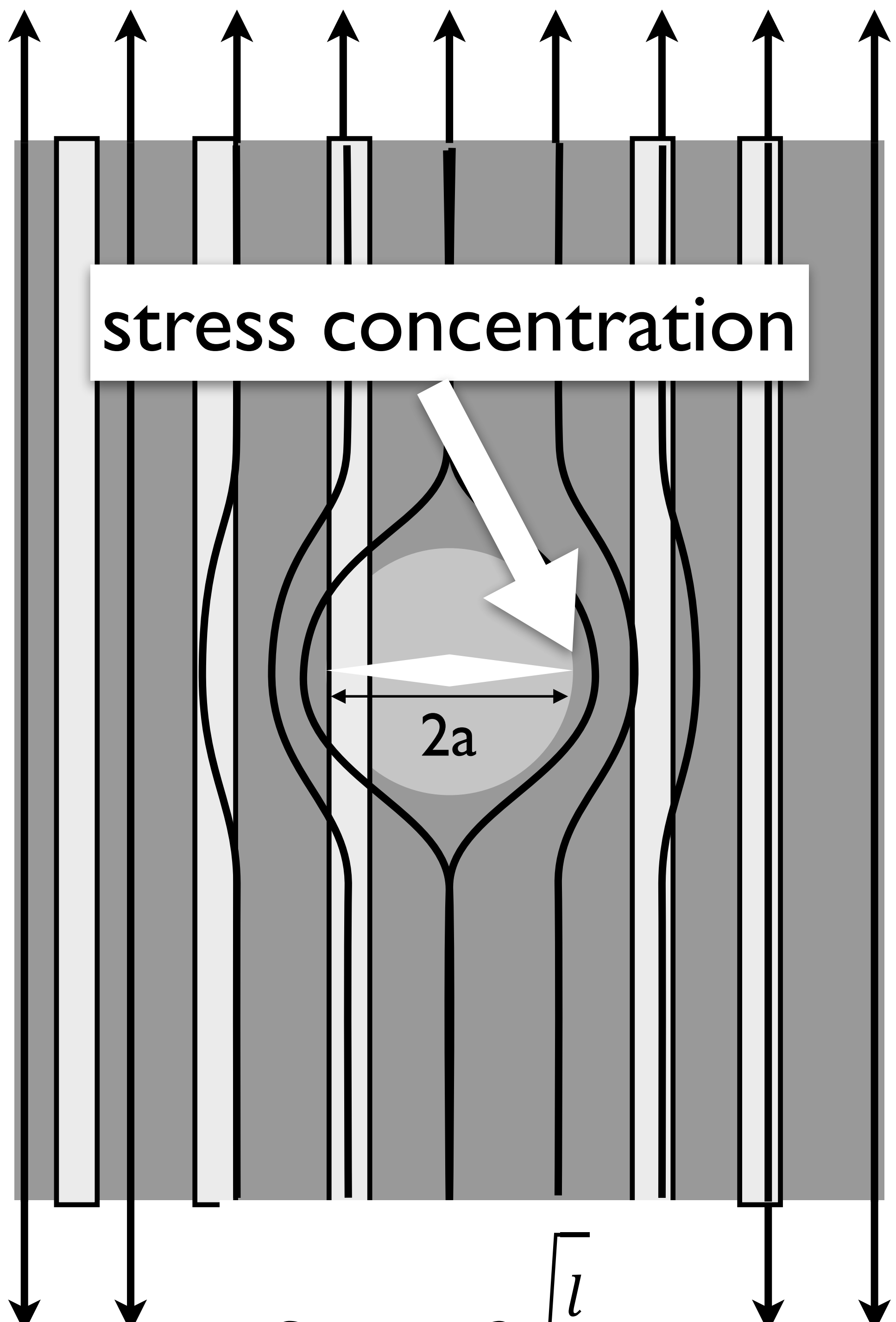

$$C_{sress} = 2\sqrt{\frac{l}{r}}$$

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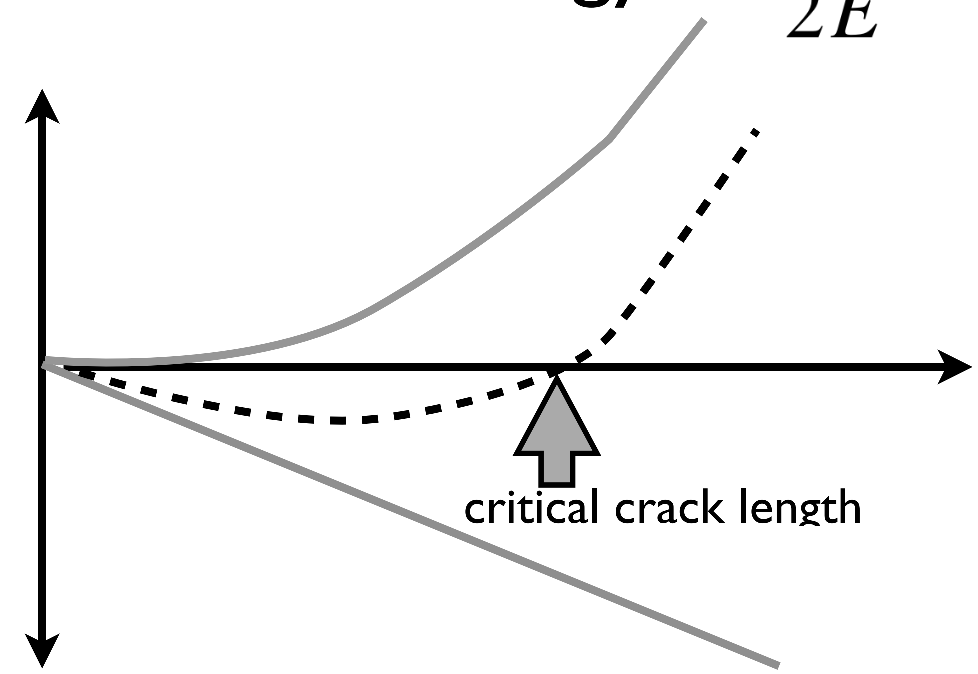

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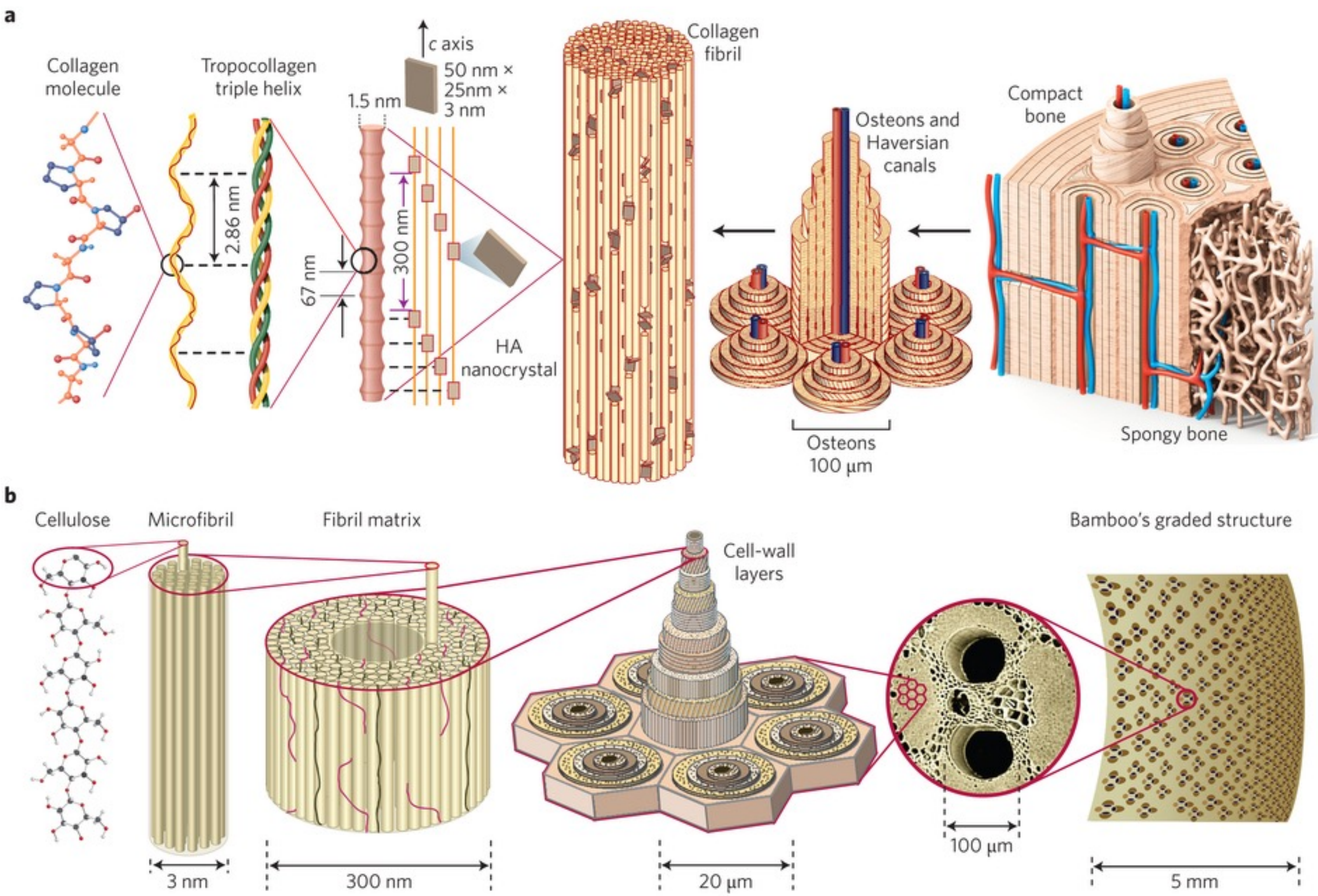
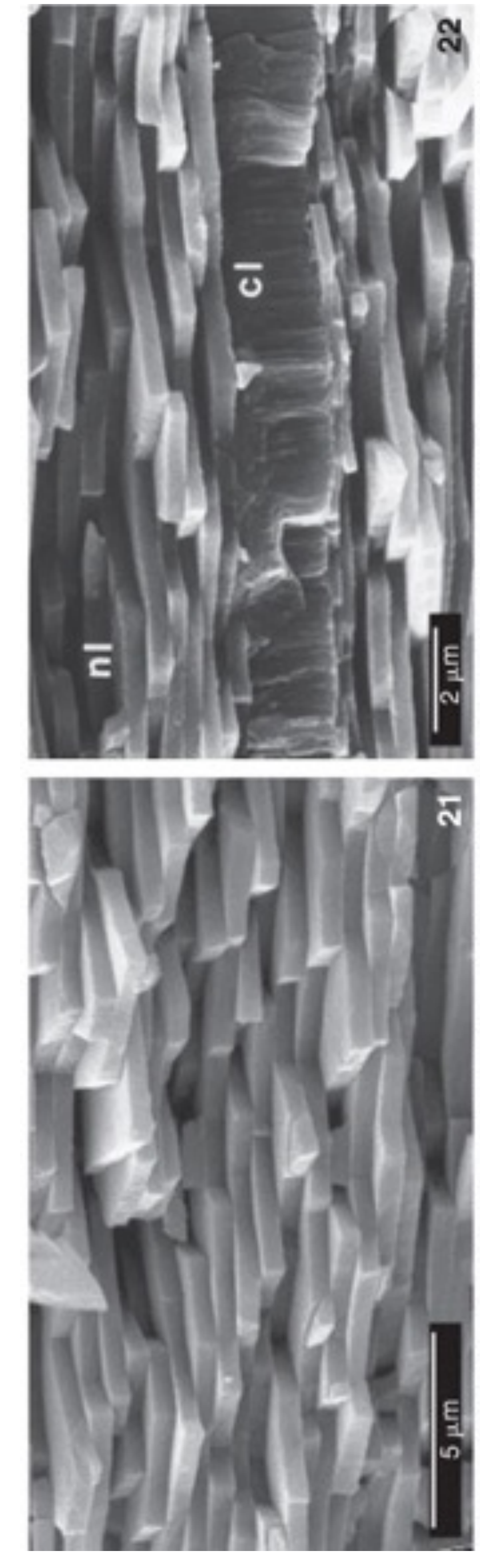
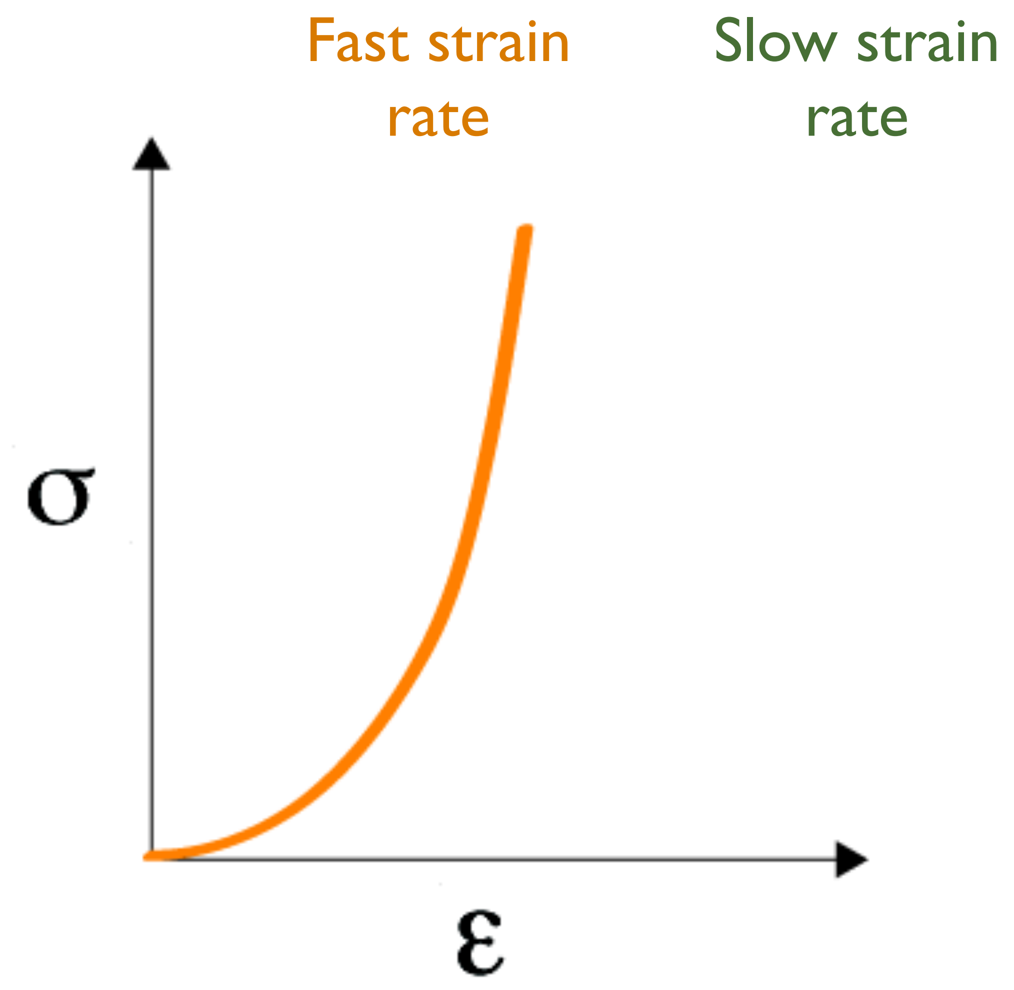


a
Collagen molecule
Tropocollagen triple helix
2.86 nm
1.5 nm
67 nm
c axis
50 nm × 25nm × 3 nm
300 nm
HA nanocrystal
Collagen fibril
Osteons and Haversian canals
Osteons 100 µm
Compact bone
Spongy bone
b
Cellulose
Microfibril
Fibril matrix
Cell-wall layers
Bamboo's graded structure
3 nm
300 nm
20 µm
100 µm
5 mm



22
cl
nl
2 µm
21
5 µm

Many biomaterials have time-dependent properties



Materials that show time-dependent properties

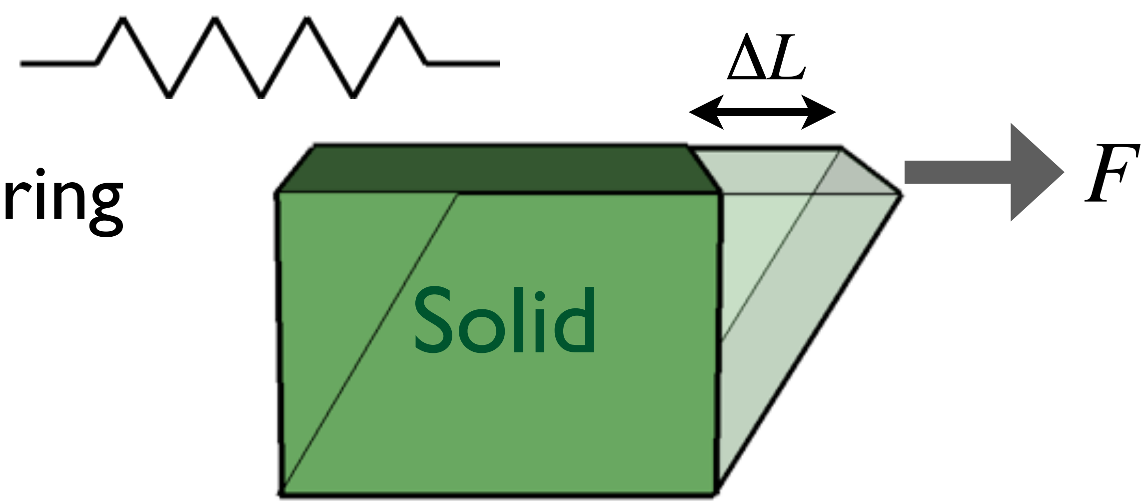
(ie: tendon, muscle, cuticle, cartilage, mucus, hair, mesoglea, skin...)

are visco-elastic

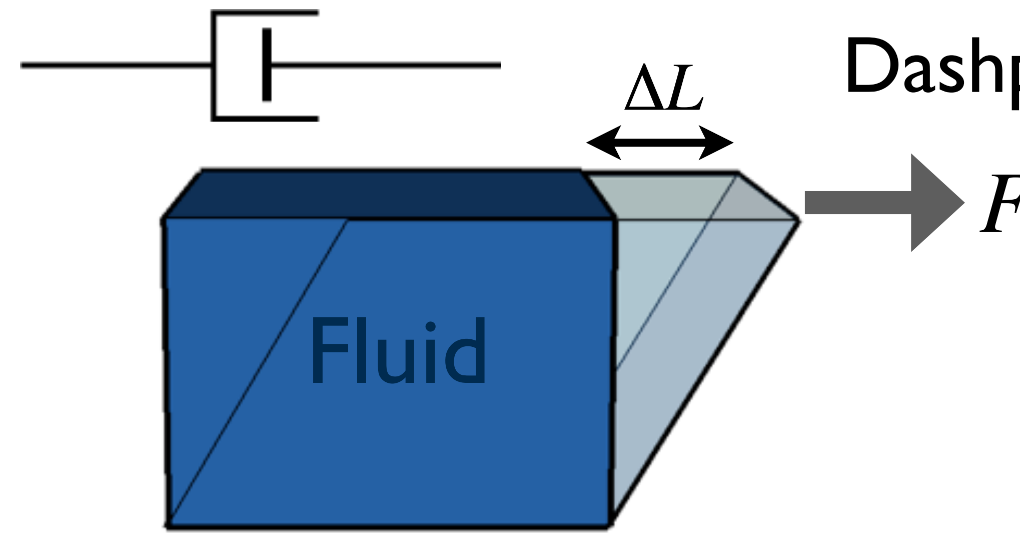
fluid behavior ← visco | elastic → solid behavior

*Solids and fluids respond differently to a shear force

Spring



Dashpot



$$F \sim \Delta L$$
$$\sigma \sim \varepsilon$$
$$\sigma = E\varepsilon$$

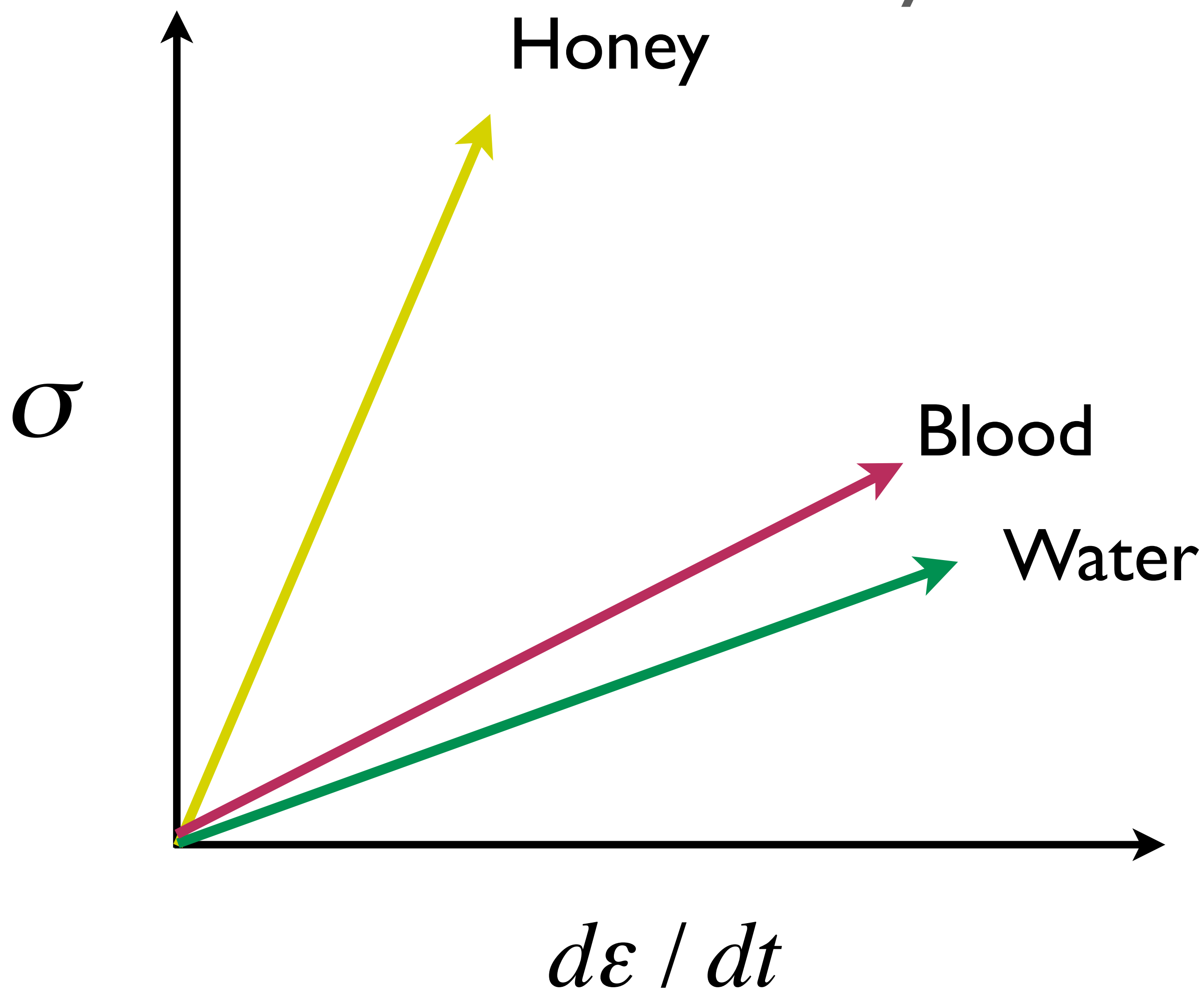
Young's modulus (E)

$$F \sim \Delta L / \Delta t \sim dL / dt$$
$$\sigma \sim d\varepsilon / dt$$
$$\sigma = \mu d\varepsilon / dt$$

Viscosity (μ)

Newton's Law of viscosity

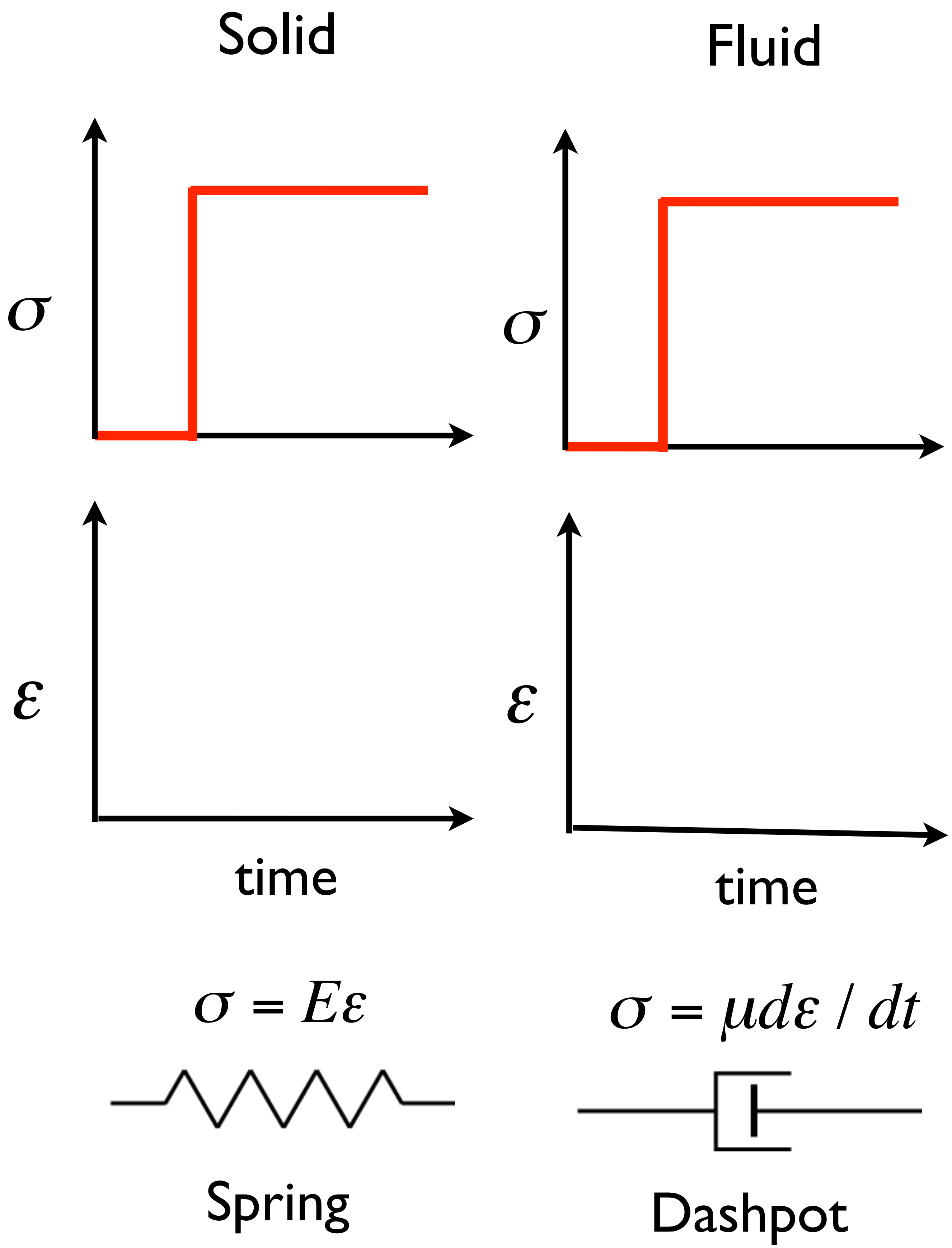
Fluid: a substance that deforms continuously under an applied force



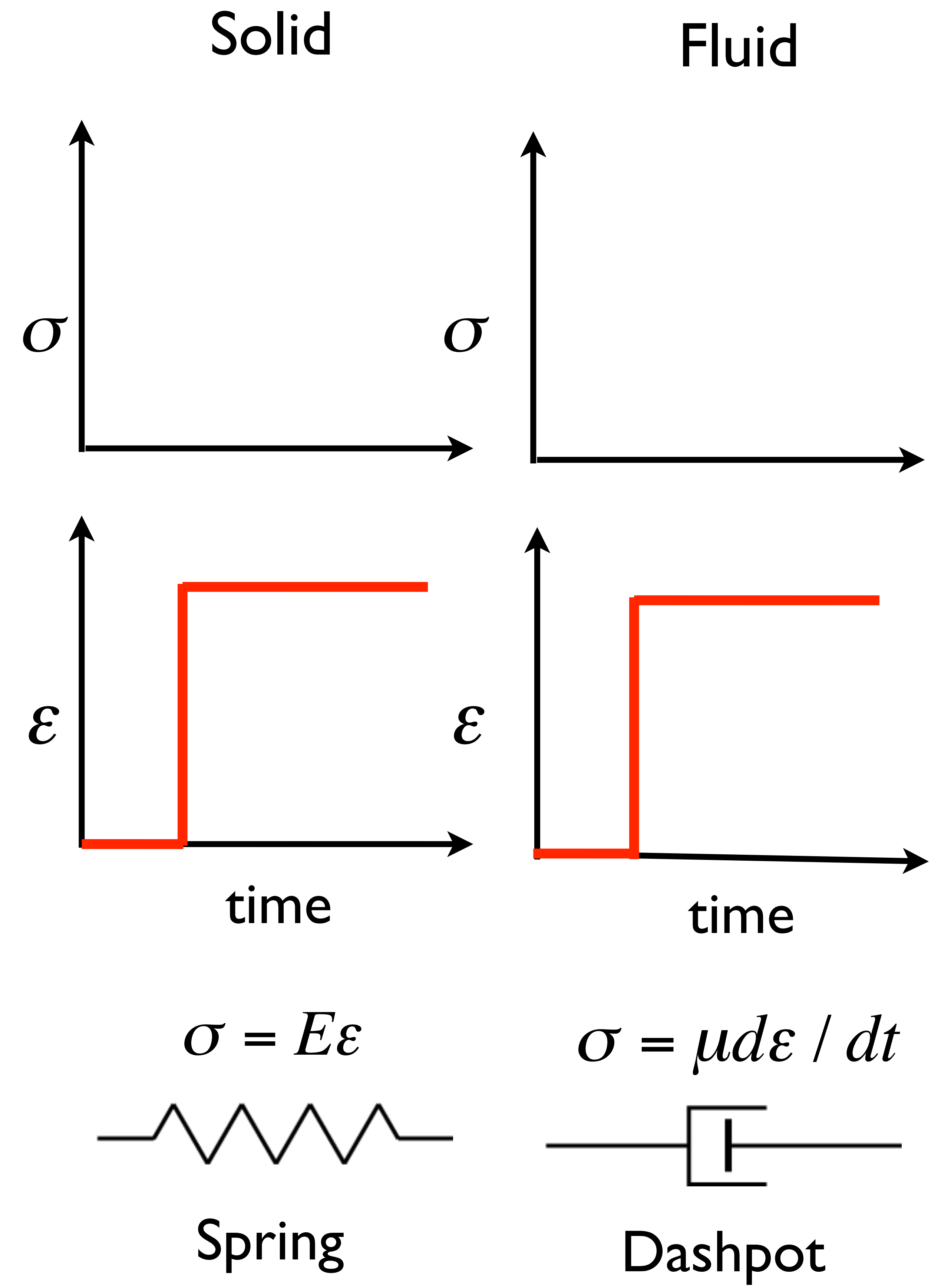
Hookean material: stress vs. strain is linear (constant stiffness)

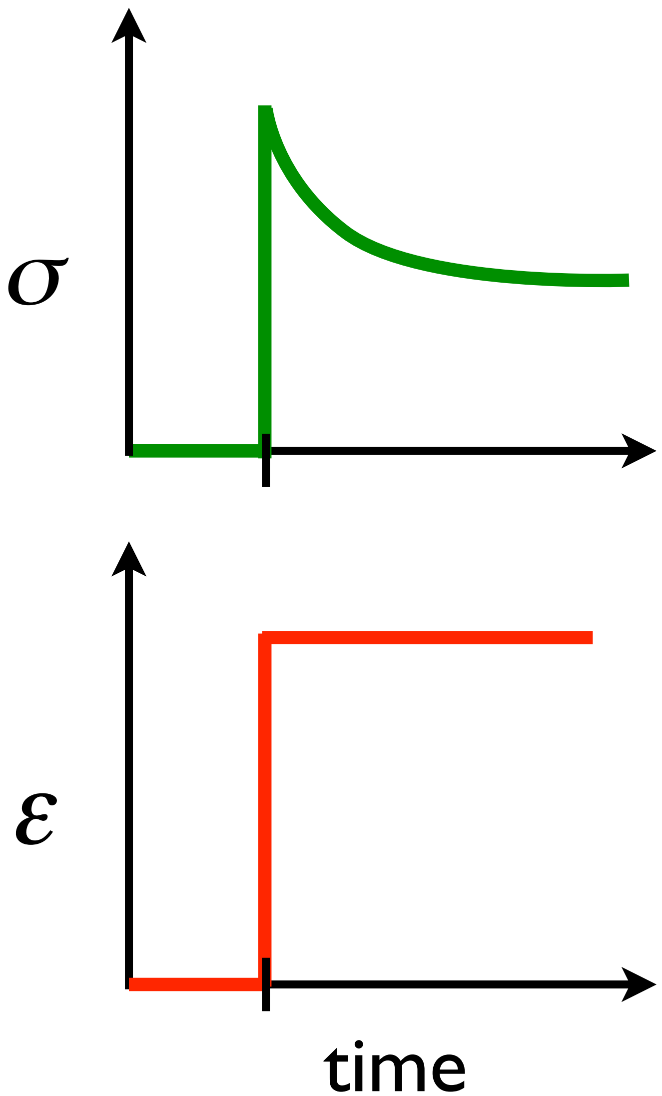
Newtonian fluid: stress vs. *strain rate* is linear (constant viscosity)

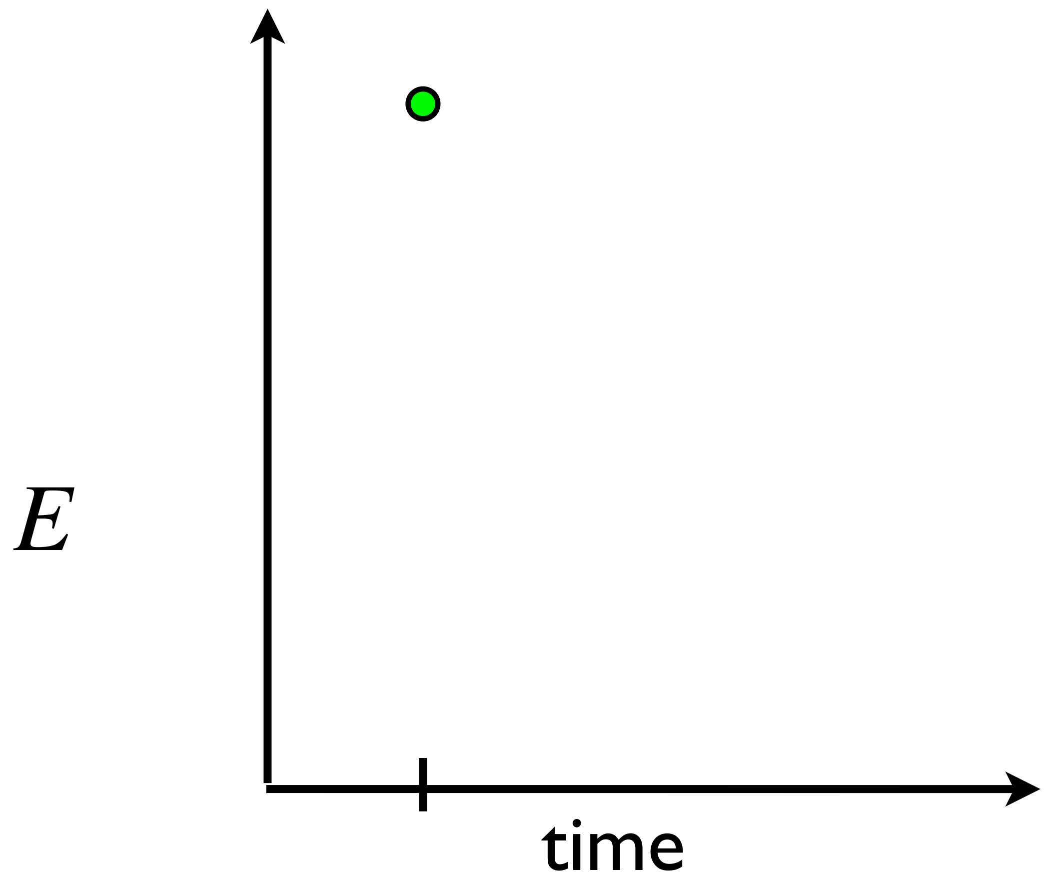
A creep test: apply a constant stress, measure strain



A stress-relaxation test: apply a constant strain, measure stress

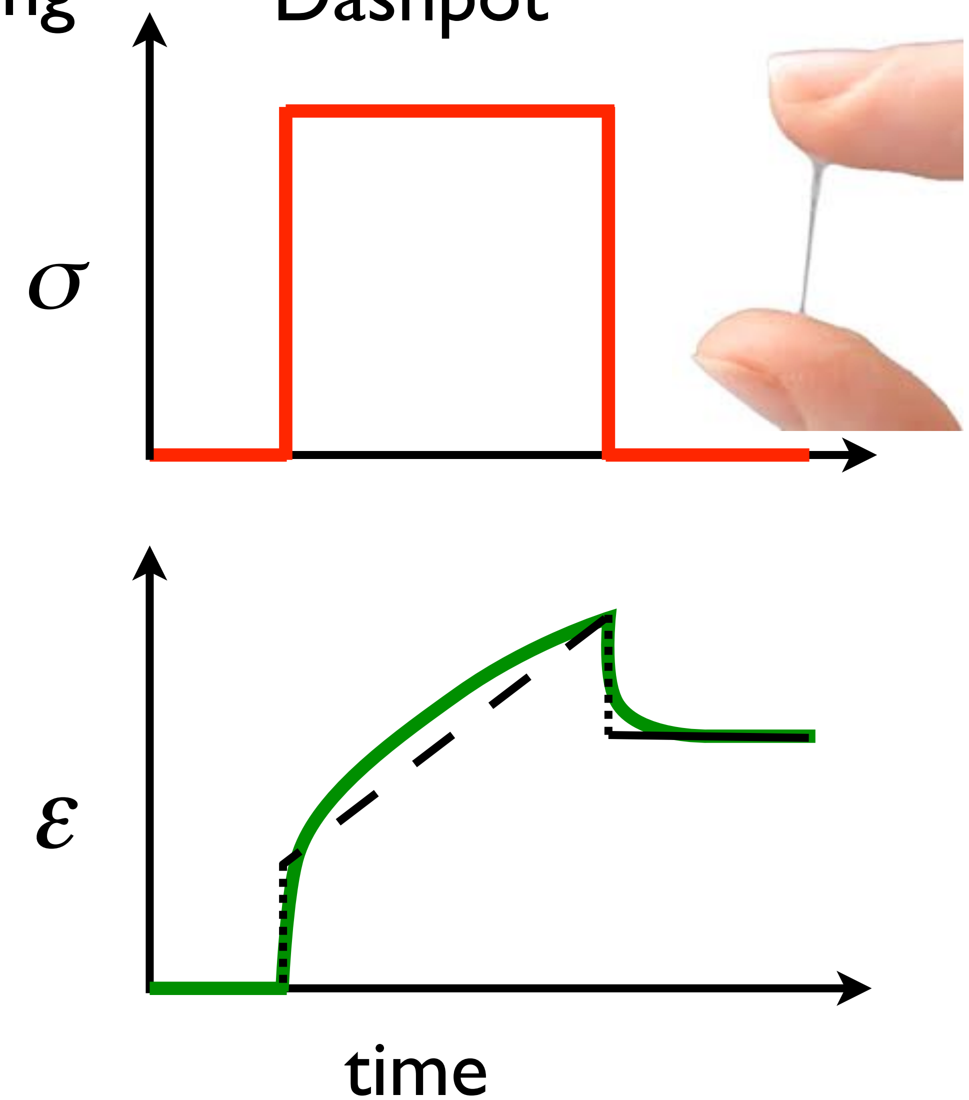


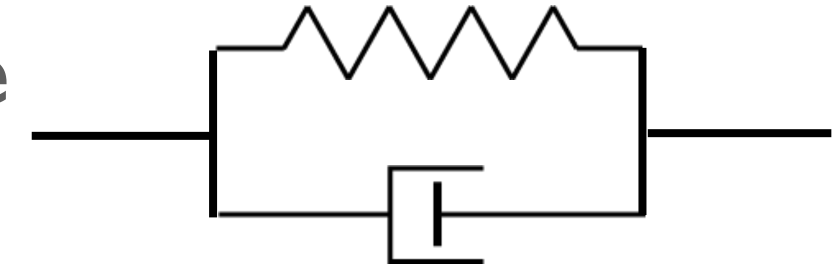


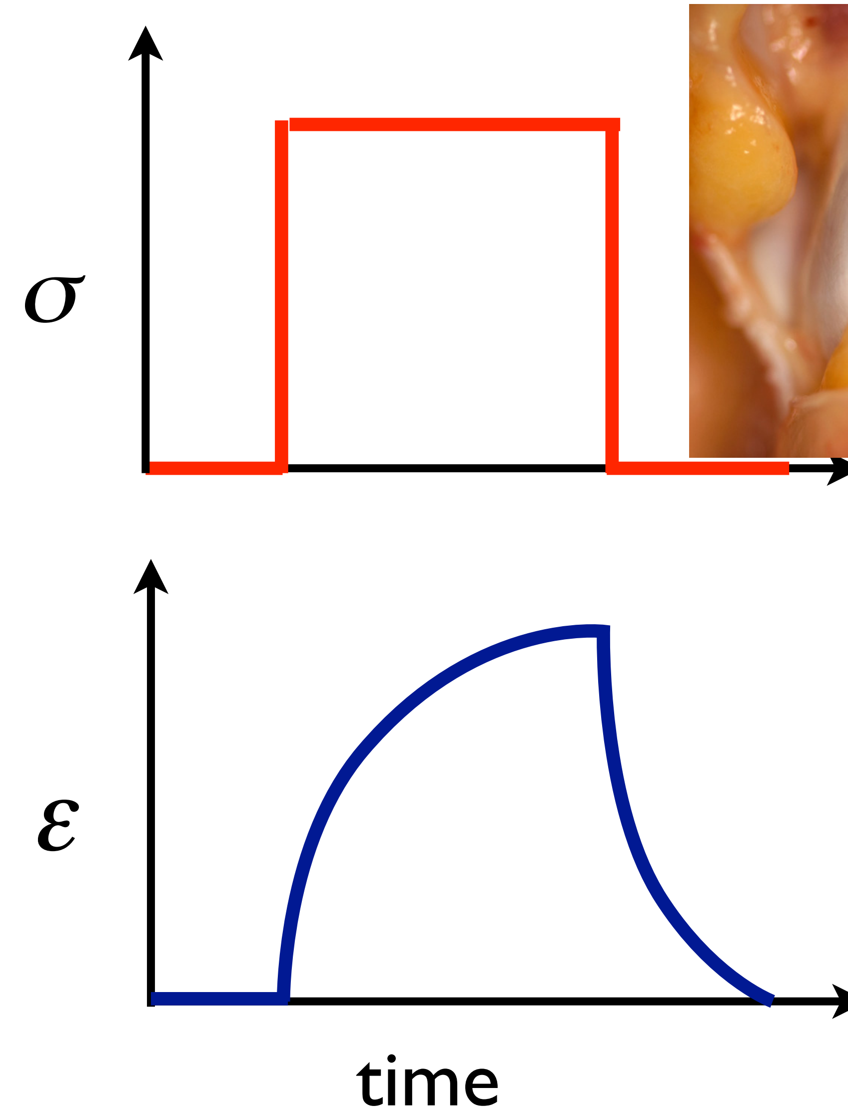


How does stiffness vary in time?
What is a good model for this?

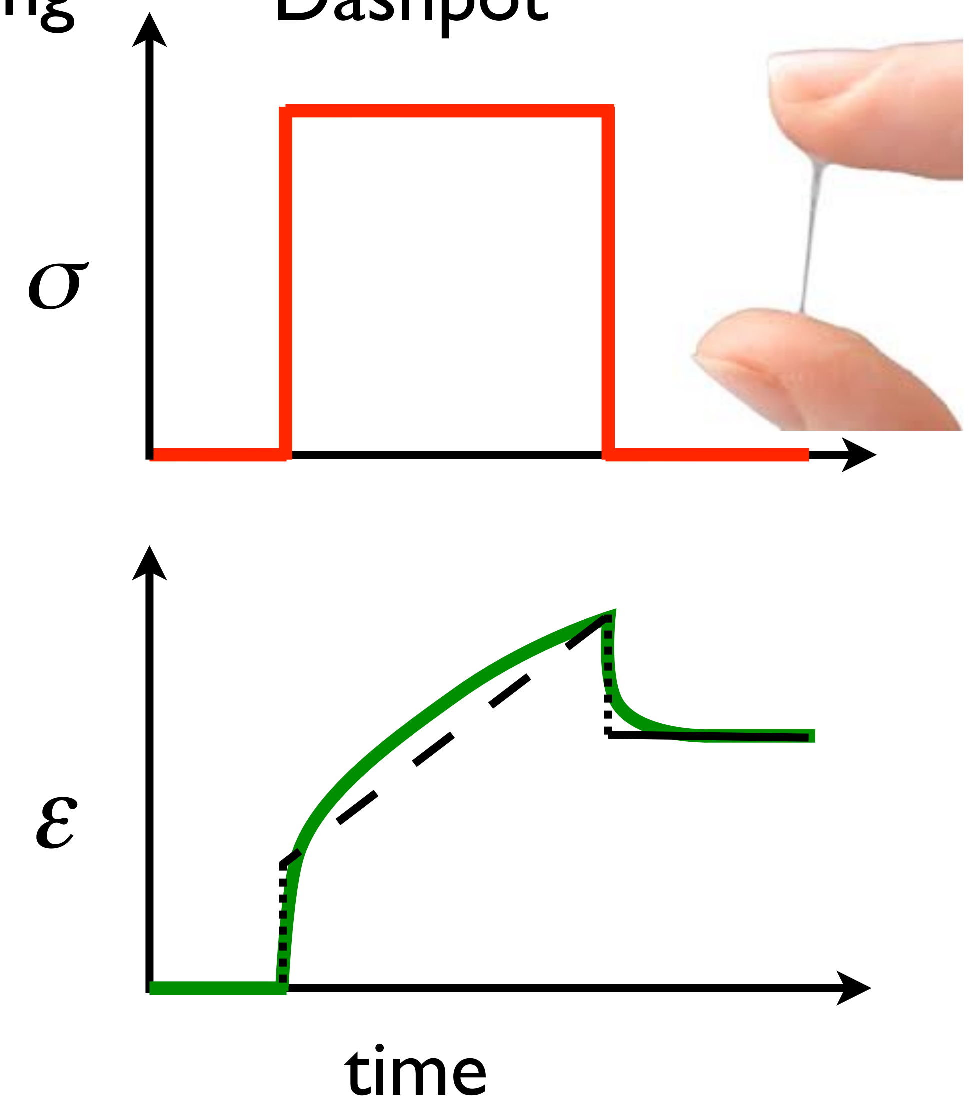
Creep characteristics of mucus and cartilage







Creep characteristics of **mucus** and



$$\sigma_s = \sigma_d = \sigma_{total}$$

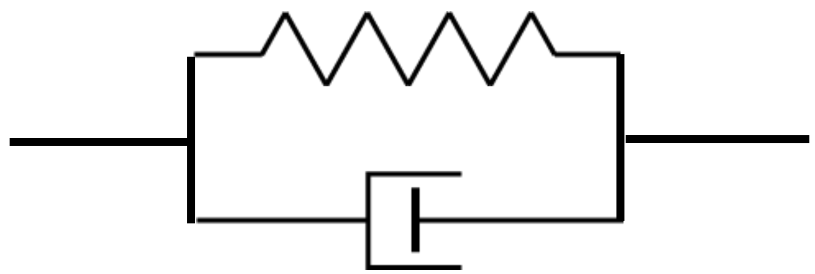
$$\varepsilon_s + \varepsilon_d = \varepsilon_{total}$$

$$\mu d\varepsilon_d / dt = \sigma$$

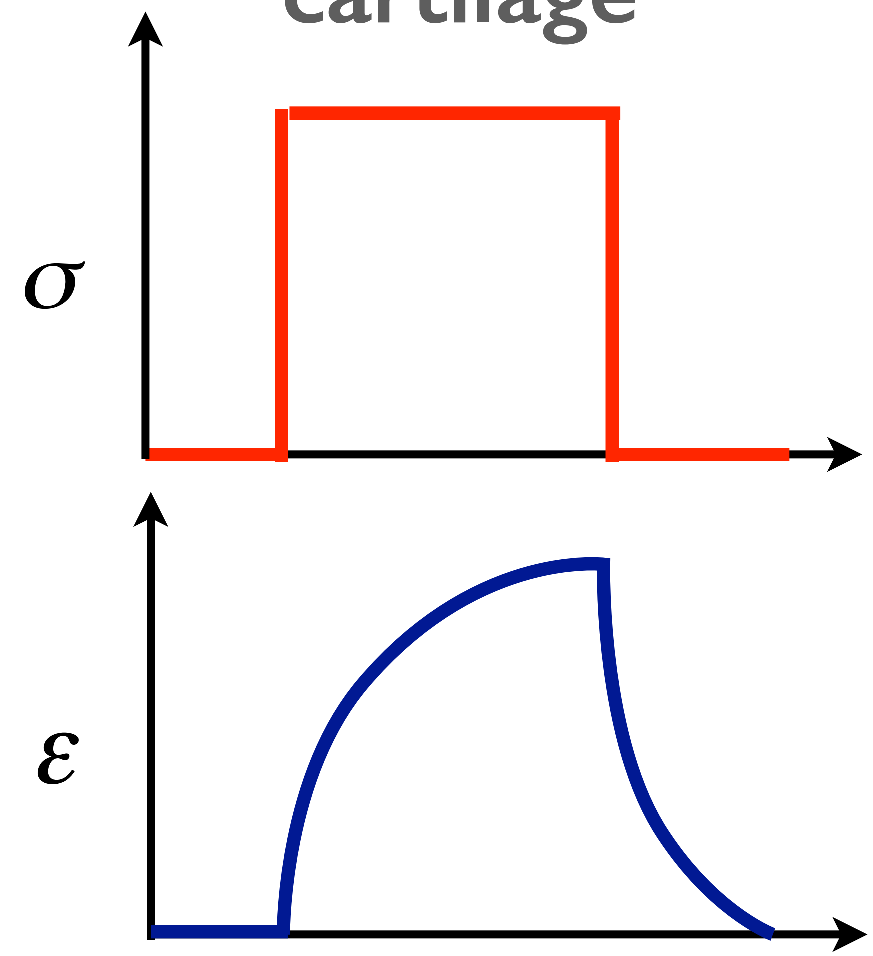
$$\varepsilon_d = \sigma t / \mu$$

$$\sigma / E + \sigma t / \mu = \varepsilon_{total}$$

Creep characteristics of **cartilage**



cartilage



$$\varepsilon_s = \varepsilon_d = \varepsilon_{total}$$

$$\sigma_s + \sigma_d = \sigma_{total}$$

$$E\varepsilon + \mu d\varepsilon / dt = \sigma_{total}$$

$$\varepsilon = \frac{\sigma}{E}(1 - e^{-(E/\mu)t})$$