Biology 427 Biomechanics
Lecture 10.

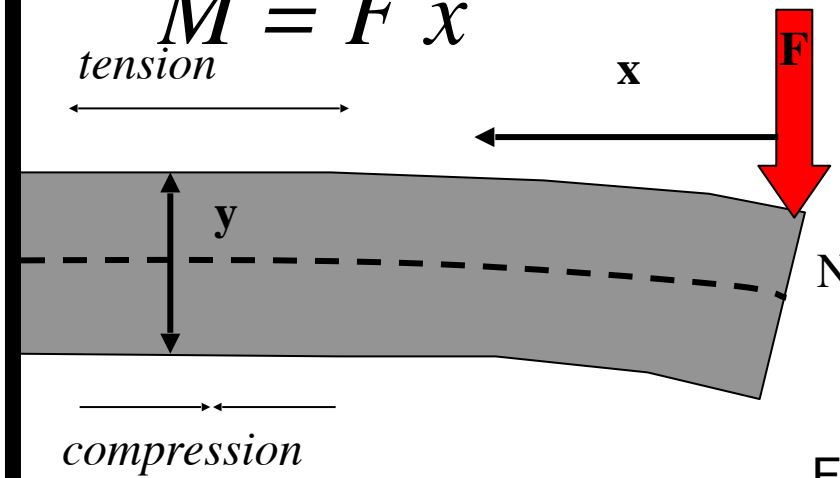
- Brief recap of beams, stress and the second moment of area.
- Flexural stiffness revisited
- The deflection of a simple beam -- both stiffness and shape determine deformation.
- A joint effort – analyses of femoral mechanics
- How hollow are bones? How hollow should they be?

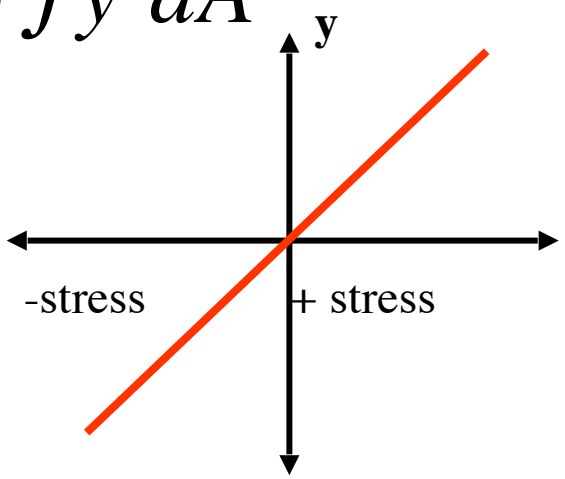
E I :

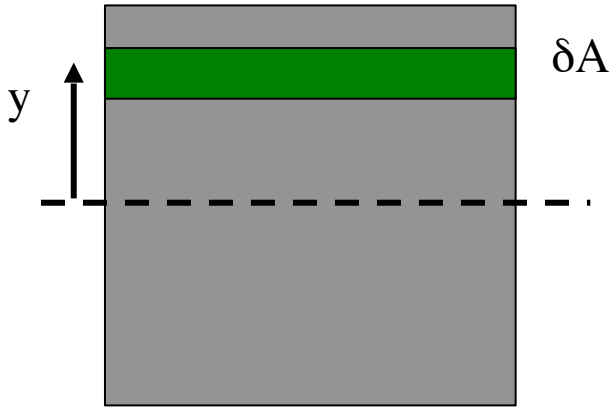
Flexural stiffness of a structure

$$M = (\sigma/y) \int y^2 dA$$

$$M = F\,x$$







Force on dA $F = \sigma_y\, dA$

Moment about neutral axis

$dM = F\,y = (\sigma_y\, dA)\, y$

The total moment about axis

$$M = \int dM = \int \sigma\, y\, dA = \int (k\, y)\, y\, dA$$

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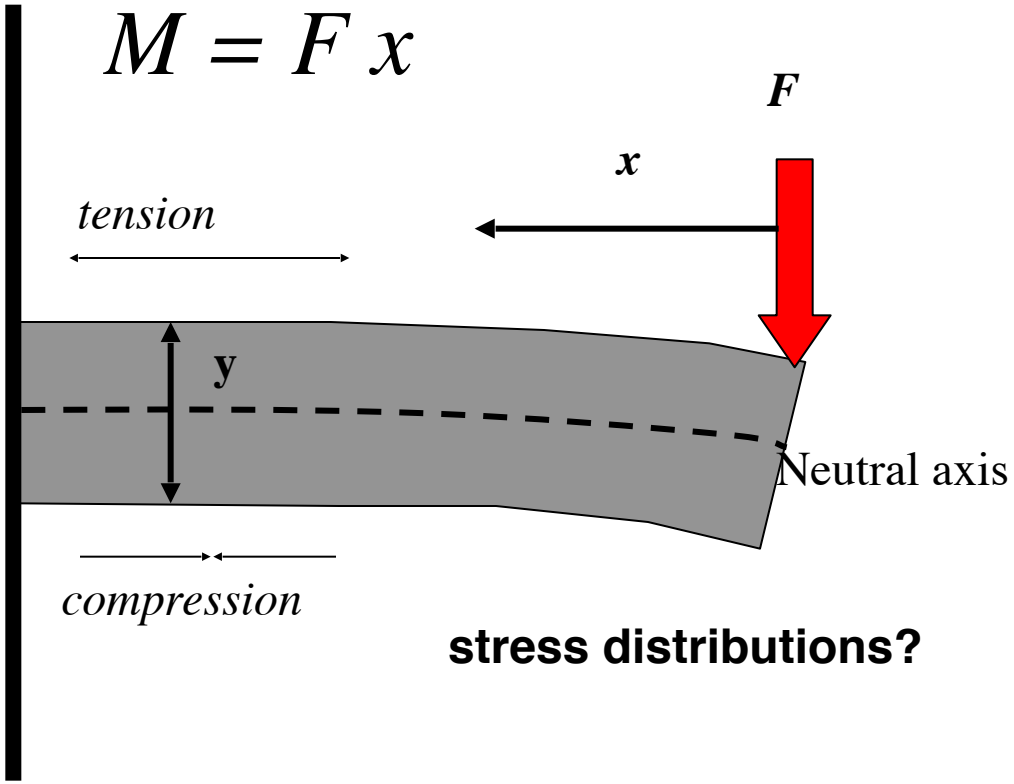


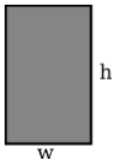
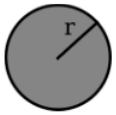
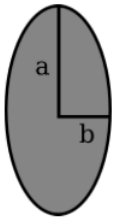
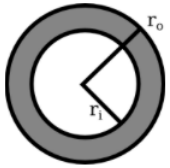
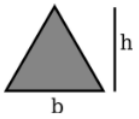
stress distributions?

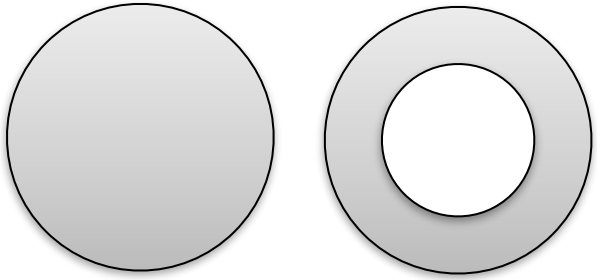
$$M = (\sigma/y)\, I$$

$$F\,x = (\sigma/y)\, I$$

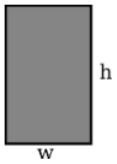
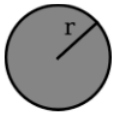
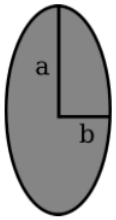
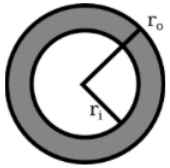
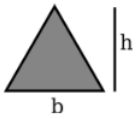
$$\sigma = F\,x\,y/\,I$$

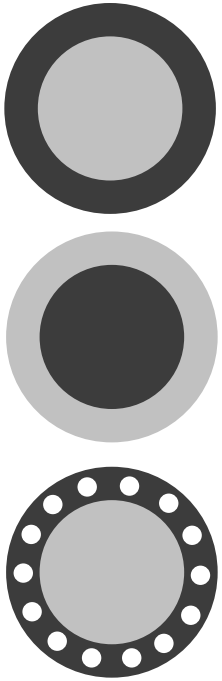
$$\varepsilon = F\,x\,y/\,E\,I$$

Shape	Area	I
	wh	$\frac{wh^3}{12}$
	πr^2	$\frac{\pi r^4}{4}$
	πab	$\frac{\pi a^3 b}{4}$
	$\pi(r_o^2 - r_i^2)$	$\frac{\pi(r_o^4 - r_i^4)}{4}$
	$\frac{bh}{2}$	$\frac{bh^3}{36}$



How does ***I*** change as we remove material from the inside of the beam?
(r_{inner} = 0.5 r_{outer})

Shape	Area	I
	wh	$\frac{wh^3}{12}$
	πr^2	$\frac{\pi r^4}{4}$
	πab	$\frac{\pi a^3 b}{4}$
	$\pi(r_o^2 - r_i^2)$	$\frac{\pi(r_o^4 - r_i^4)}{4}$
	$\frac{bh}{2}$	$\frac{bh^3}{36}$



For composites:

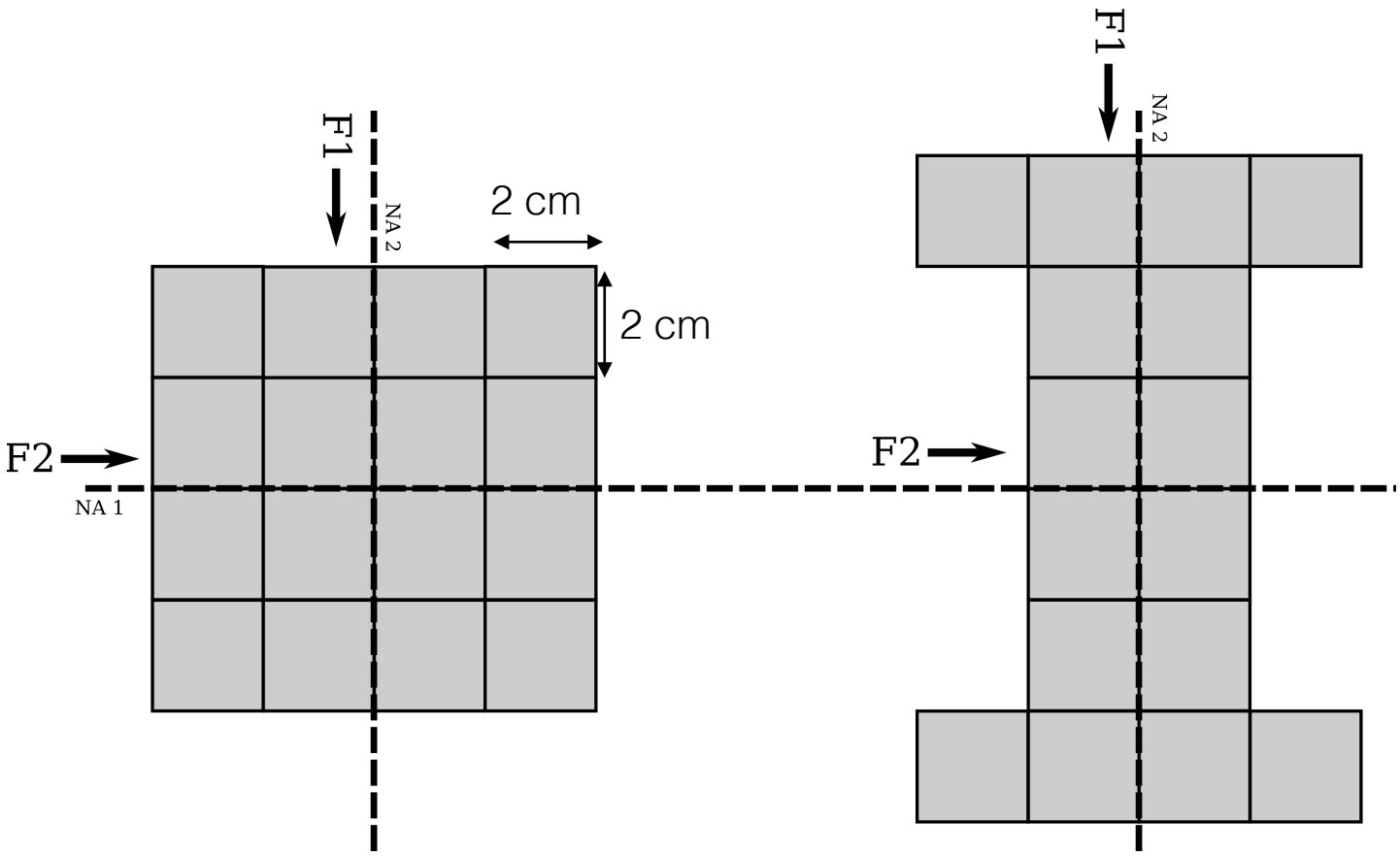
$$EI_{Total} = EI_1 + EI_2 + \ldots$$

For cavities:

$$EI_{Total} = EI_1 + EI_2 + \ldots - EI_{cav1} - EI_{cav2} - \ldots$$

https://en.wikipedia.org/wiki/List_of_area_moments_of_inertia

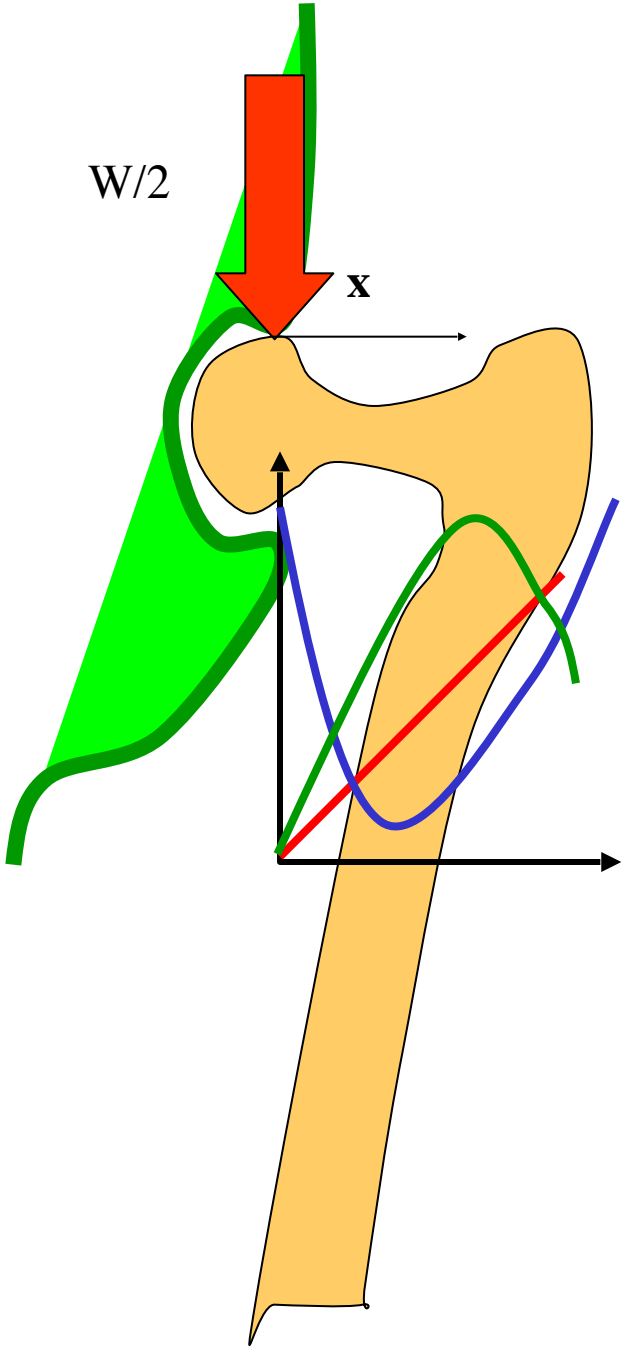
Area? Second Moment of Area?



What additional arrangement of squares can maximize I?
Any examples of I-beams in biology?

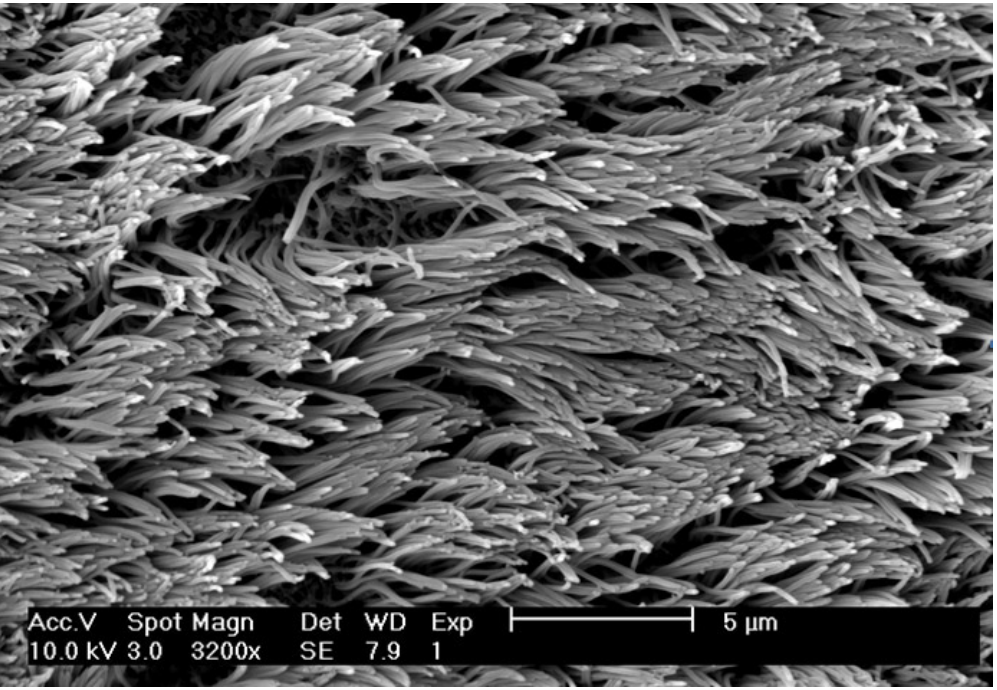
Where do you think the tensile stress is greatest?

Where is the most likely zone for failure?

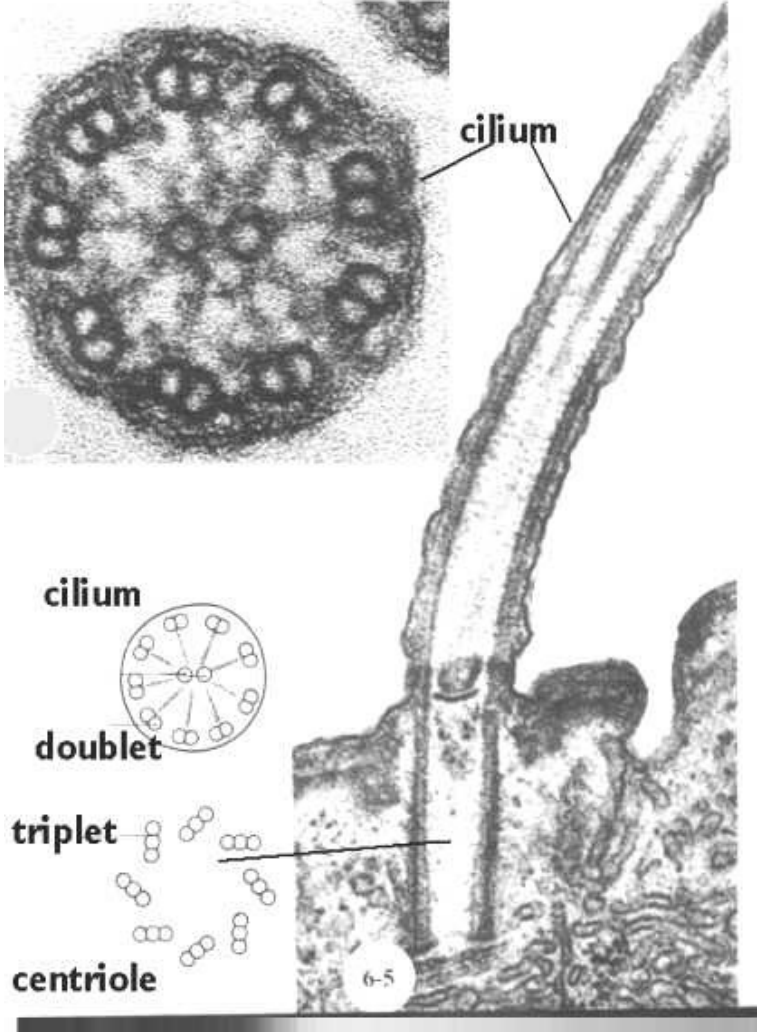


$$\sigma = F \times y/I$$

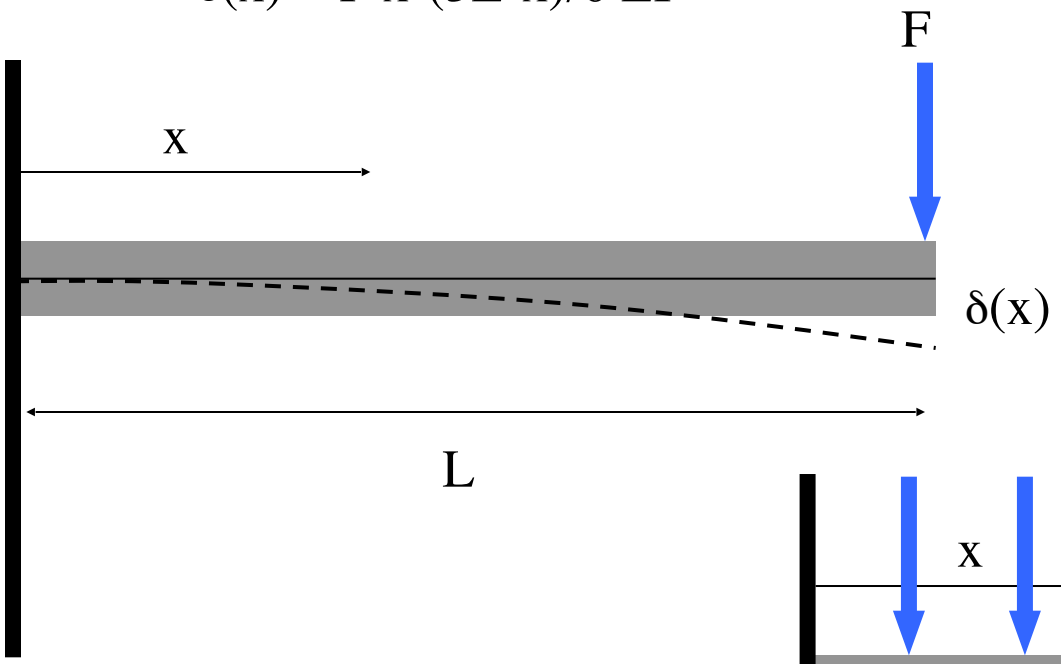
$$I = \pi R^4/4$$



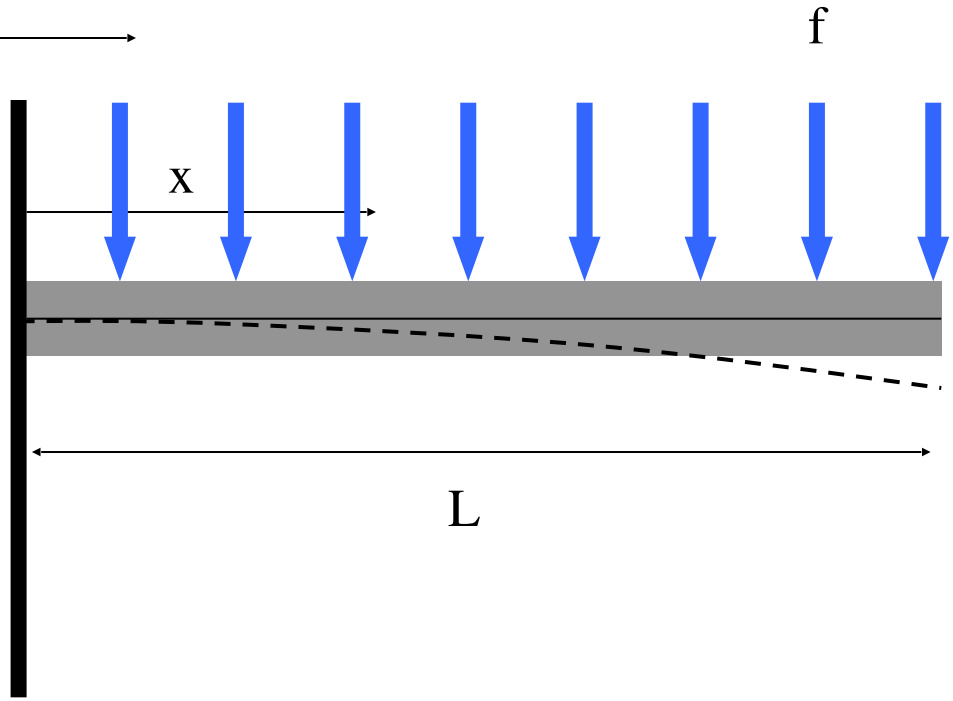
How would you estimate I for a cilium from a micrograph? Does the central doublet make a big difference?



$\delta(x) = F\, x^2(3L-x)/6\, EI$



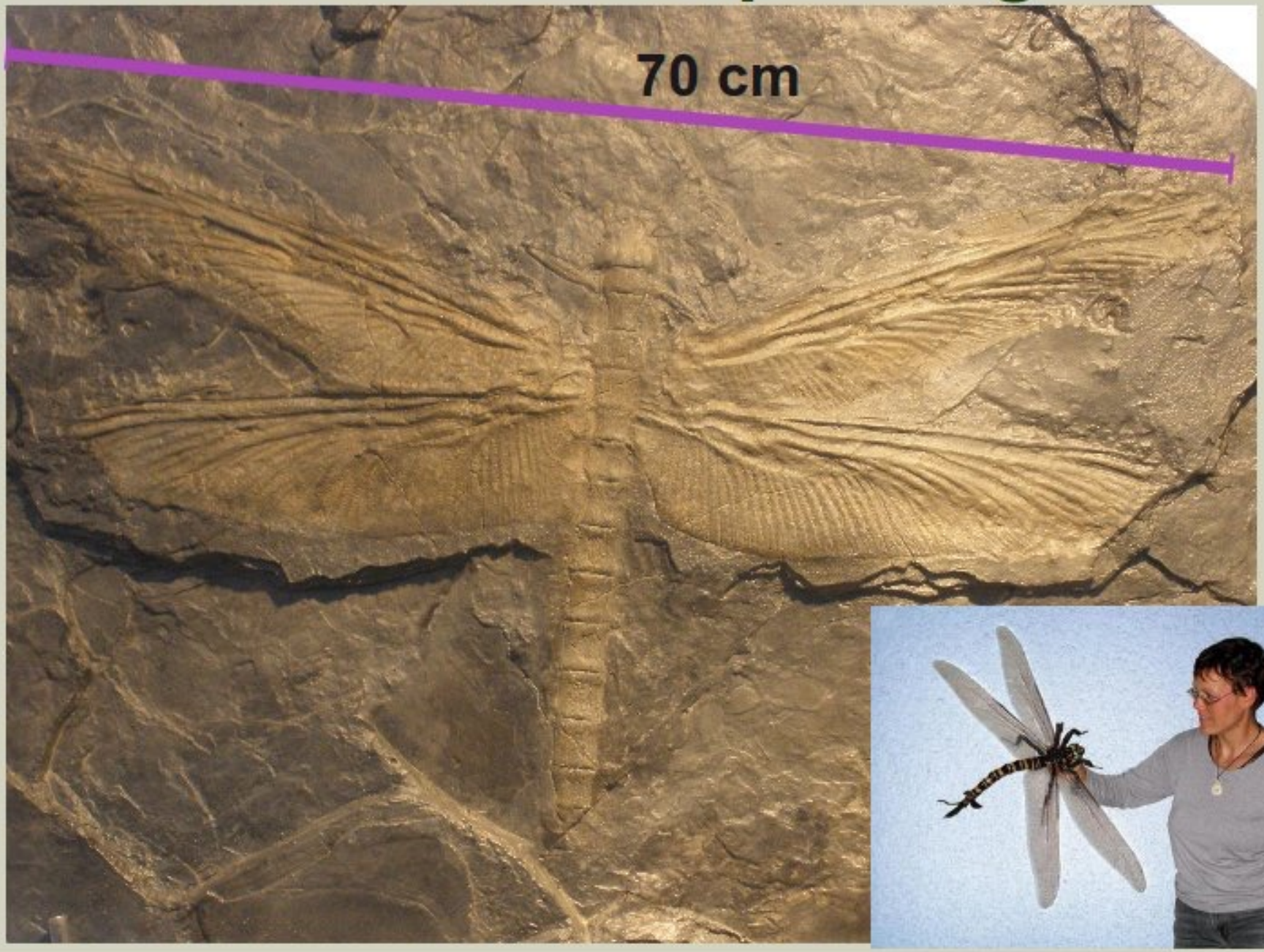
$$\delta_{tip} = \frac{FL^3}{3EI}$$



$\delta(x) = f\, x^2(x^2 + 6L^2 - 4Lx)/24\, EI$

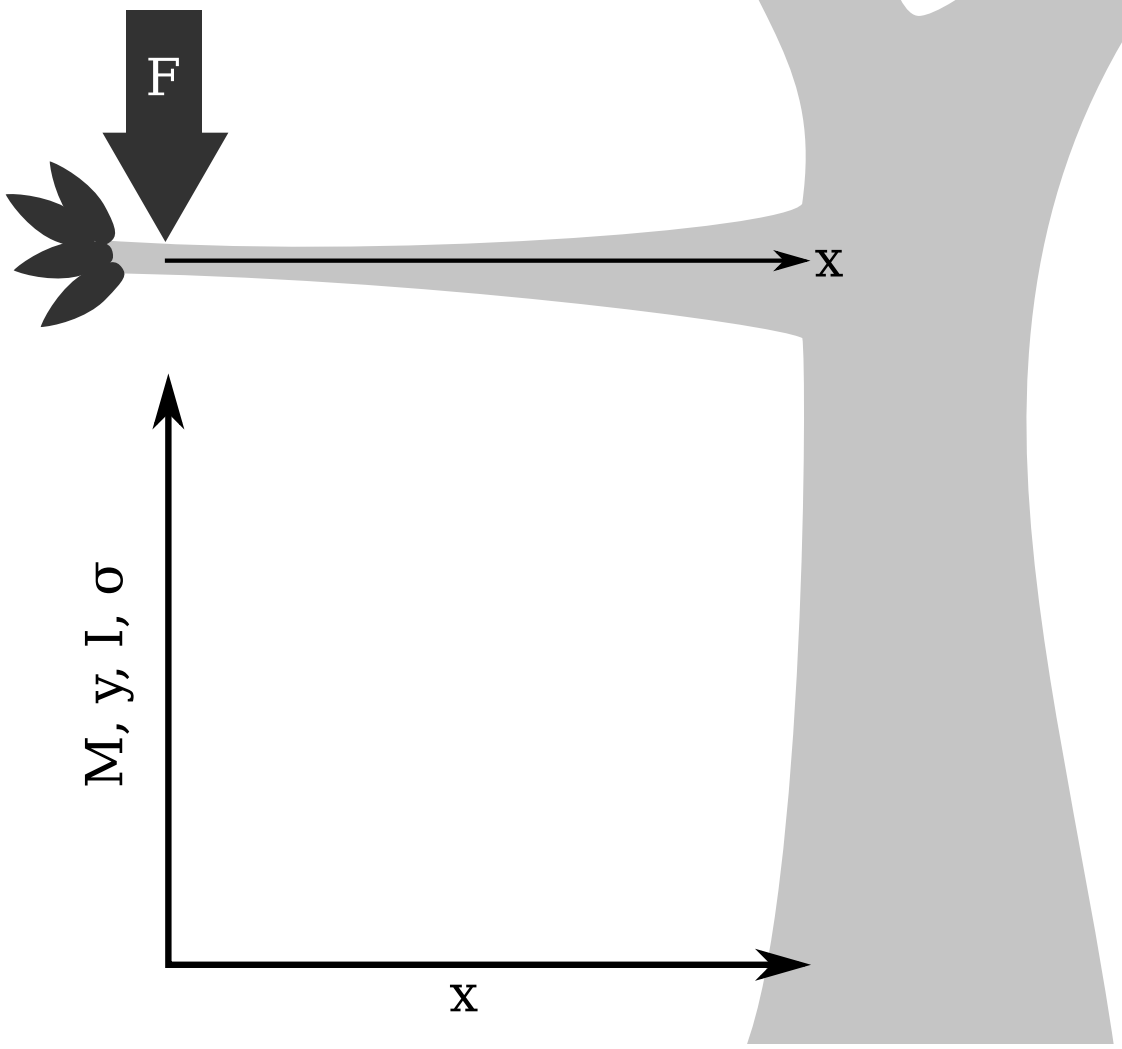
http://www.aps.anl.gov/asd/me/Calculators/ElasticBeam2.html

Carboniferous Arthropod Gigantism



Stress distributions in biological beams: tree branches. Where will this break?

$$\sigma = F \times y / I$$



$$0 \leq x \leq 2.0 \text{ m}$$

$$F = 1000\text{N}$$

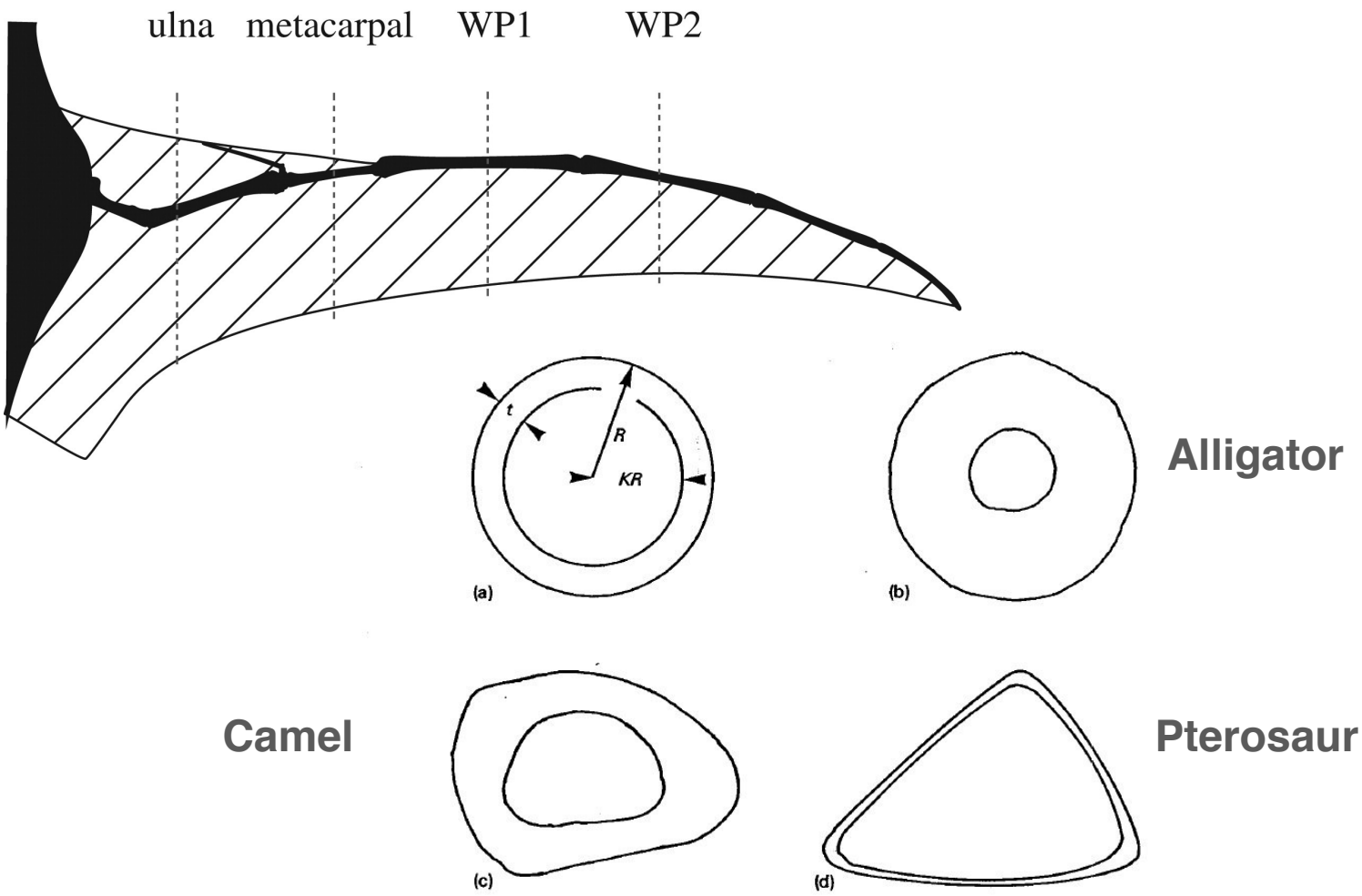
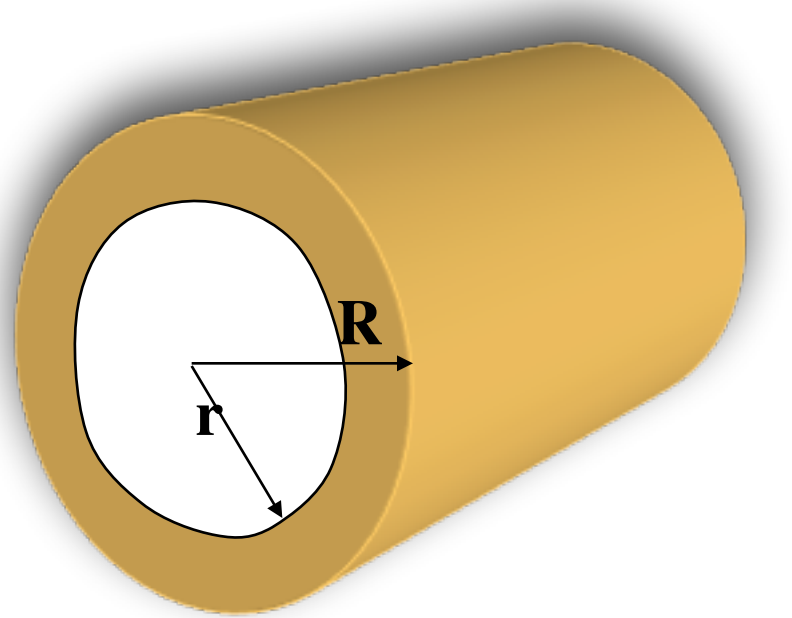


FIG. 1. (a) Conventions for R, t and K used in text. (b), (c), (d) Sketches of sections of bones of different values of K and R/t. (b) $K = 0{\cdot}35$, $R/t = 1{\cdot}5$; alligator femur. (c) $K = 0{\cdot}57$, $R/t = 2{\cdot}4$; camel tibia. (d) $K = 0{\cdot}91$, $R/t = 11$; *Pteranodon* first phalanx.

You've decided to trade in your endoskeleton for a sleek shiny exoskeleton. Assuming your exoskeleton is bone how thick would it have to be to surround your biceps for the same EI?



$$k = r/R$$

Bone	Hare	Fox	Lion	Camel	Buffalo	Swan
femur	0.57	0.63	0.56	0.62	0.54	0.60
humerus	0.55	0.59	0.57	0.66	0.51	0.92