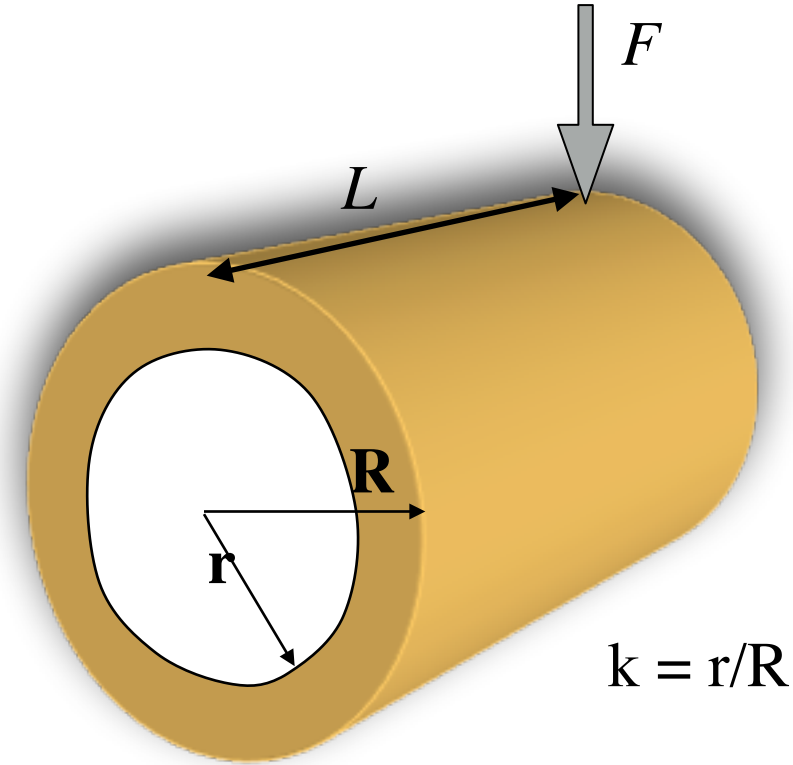
Biology 427 Biomechanics

Lecture 11. Vibrations

- Recap beams and EI — some structures are really hard to analyze.
- Examples of vibration problems in biomechanics: resonant frequencies, energy storage
- The mechanics of vibrating systems: free vibrations, resonance, and damping
- Forced vibrations and modes of excitation
- Mathematica Demonstration

$\sigma = F \times y/I \qquad \sigma_{max} = F\,L\,R/I$

A pattern of constant k suggests a common design constraint: What biomechanical factors might come to play?



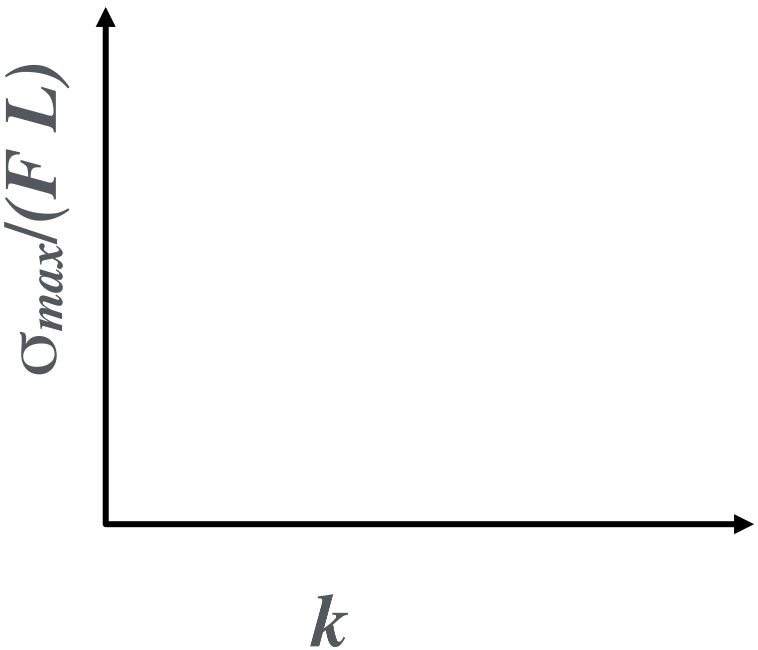
Bone	Hare	Fox	Lion	Camel	Buffalo	Swan
femur	0.57	0.63	0.56	0.62	0.54	0.60
humerus	0.55	0.59	0.57	0.66	0.51	0.92

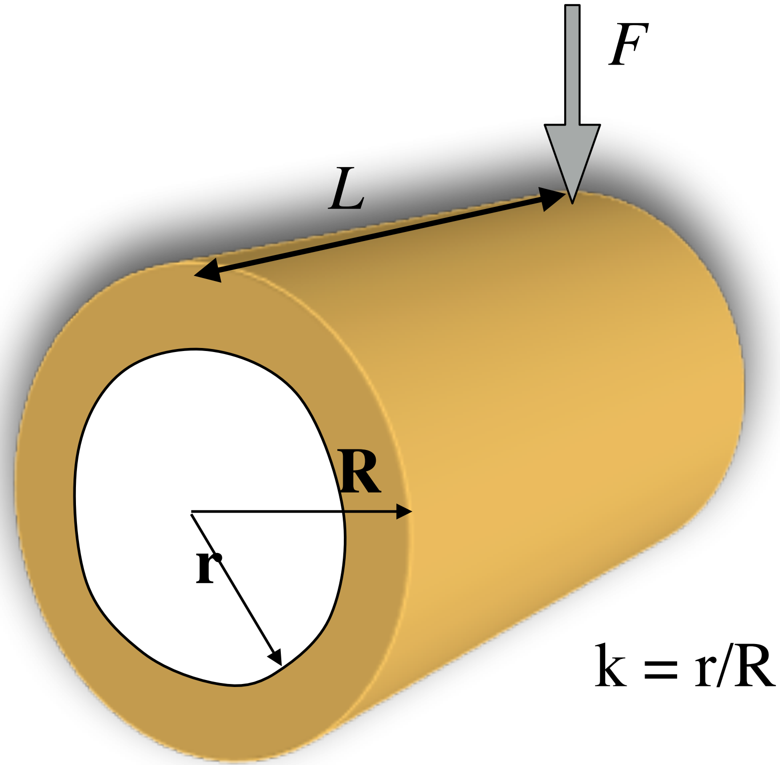
$$\sigma_{max} = F\,L\,R/I$$

$$I = \pi R^4/4 - \pi r^4/4$$

$$r = k\,R \quad (k < 1)\ (\text{thin: } k \to 1)$$

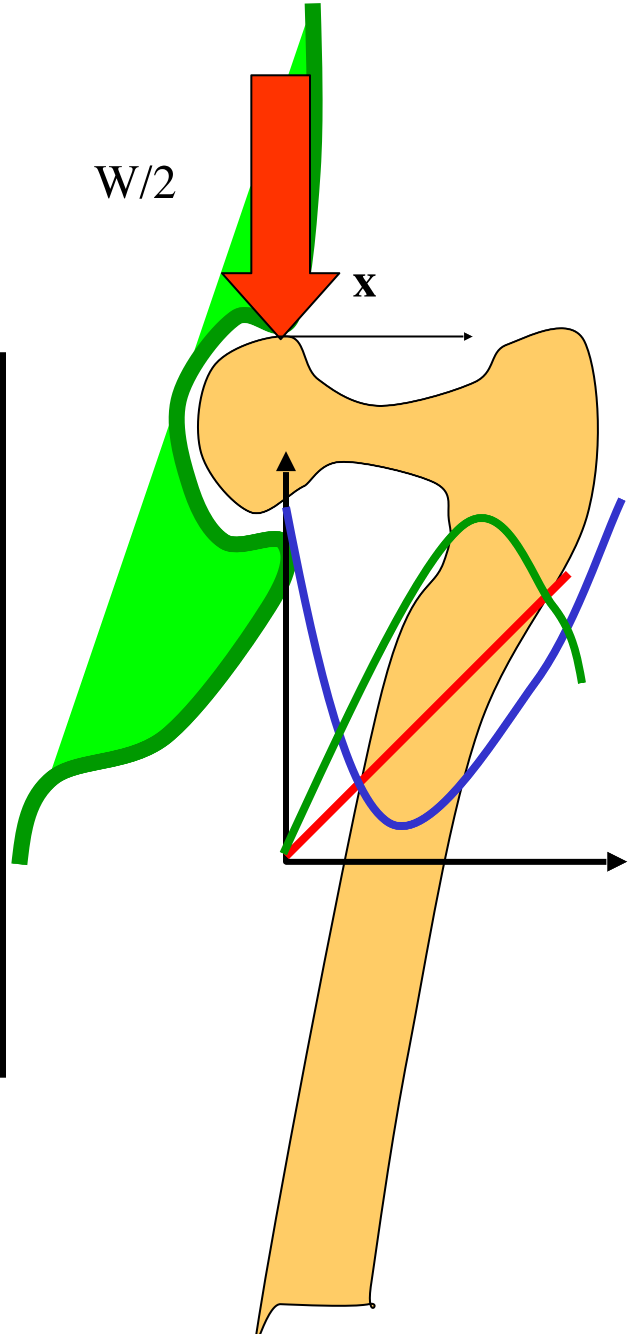
$$I = \pi R^4 (1 - k^4)/4$$





Where do you think the tensile stress is greatest?

Where is the most likely zone for failure?

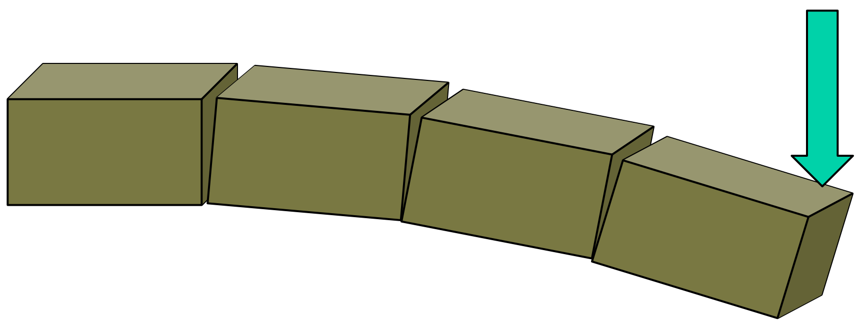


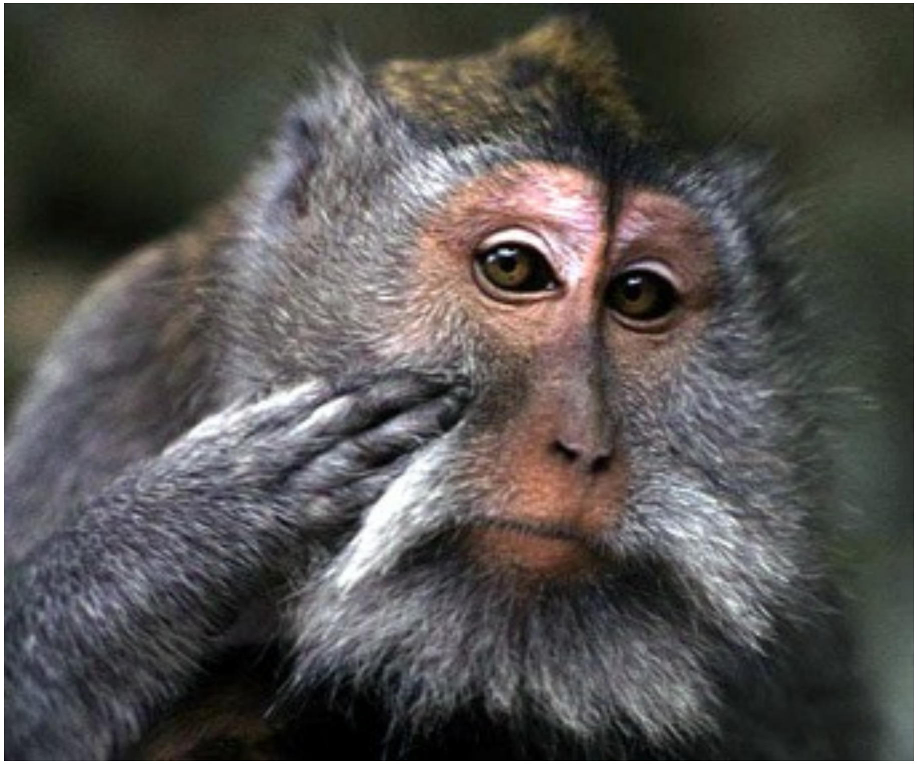
$$\sigma = F \times y/I$$

$$I = \pi R^4/4$$

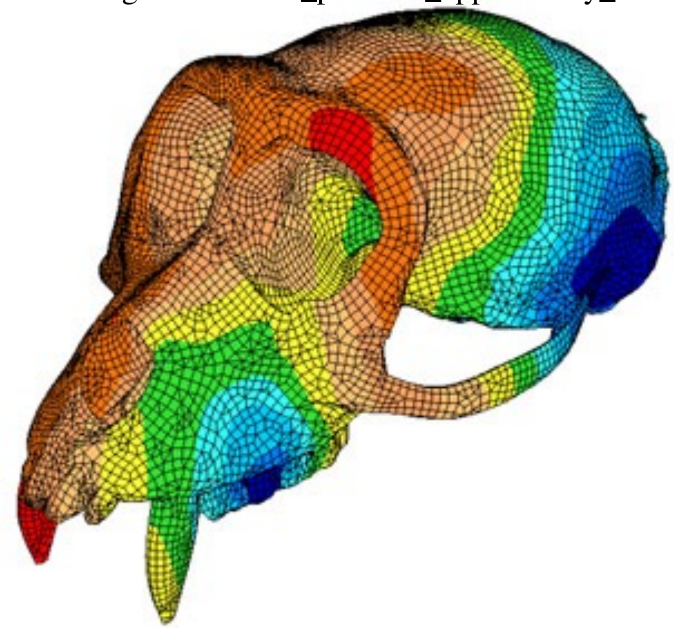
How do you handle even more complicated shapes?
Finite element methods….

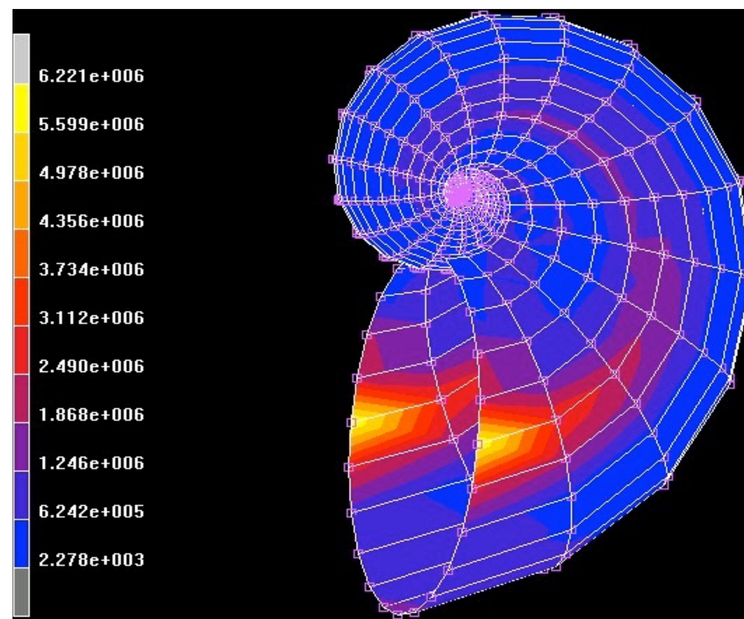
$$\delta_2 = \frac{FL^3}{3EI}$$



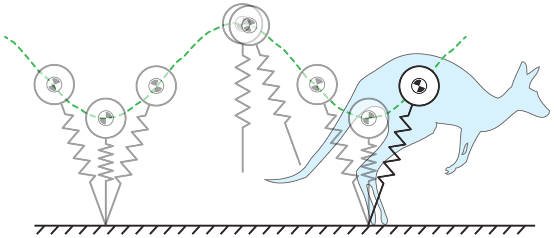


Monkey Skull
Displacement Magnitude
http://www.algor.com/news_pub/cust_app/monkey_skull/

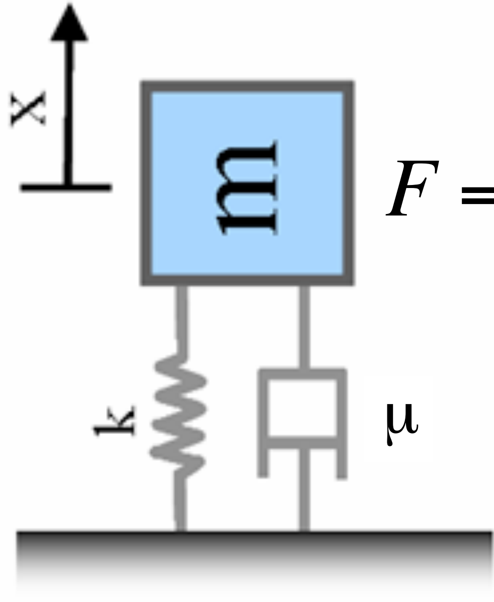




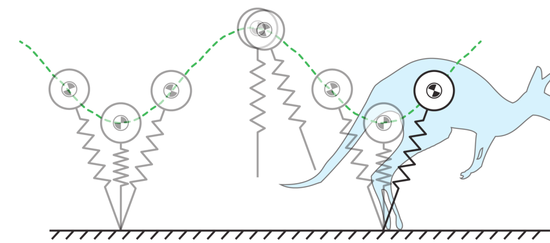
Biological examples of vibrating systems



What are the quantitative tools we have ?



$$F = ma = m\frac{d^2x}{dt^2}$$

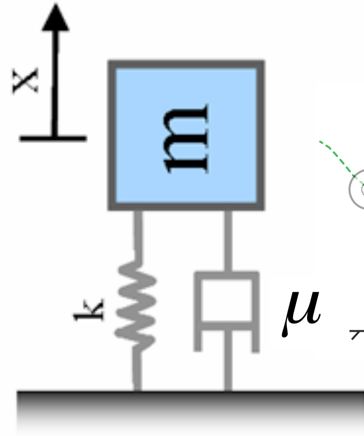


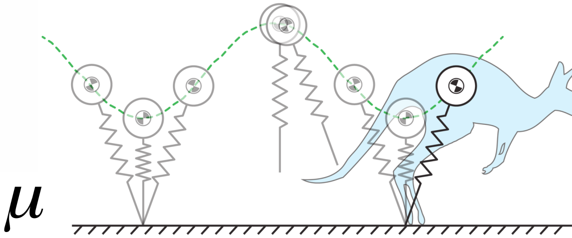
$$F = kx \qquad F = \mu\frac{dx}{dt}$$

Newton

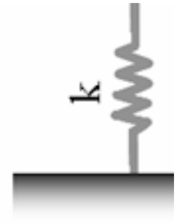
Hooke

If you know x, can you say what F is in each case?



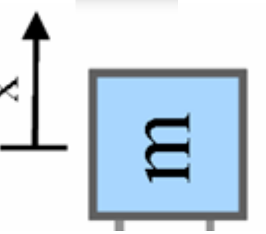


$$x = \sin(2\pi f t)$$



$$F = kx$$

$$F = \mu \frac{dx}{dt}$$



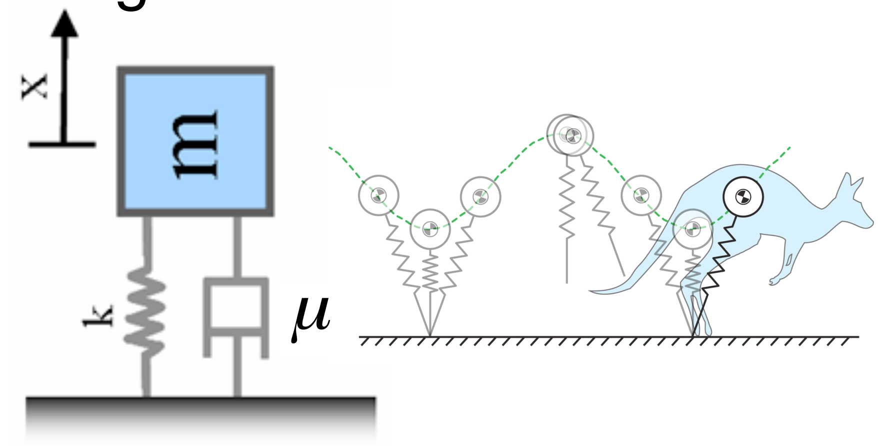
$$F = ma = m \frac{d^2 x}{dt^2}$$

If you know $x(t)$, can you say what $F(t)$ is in each case?

If you know x, what is F for our Kangaroo model?

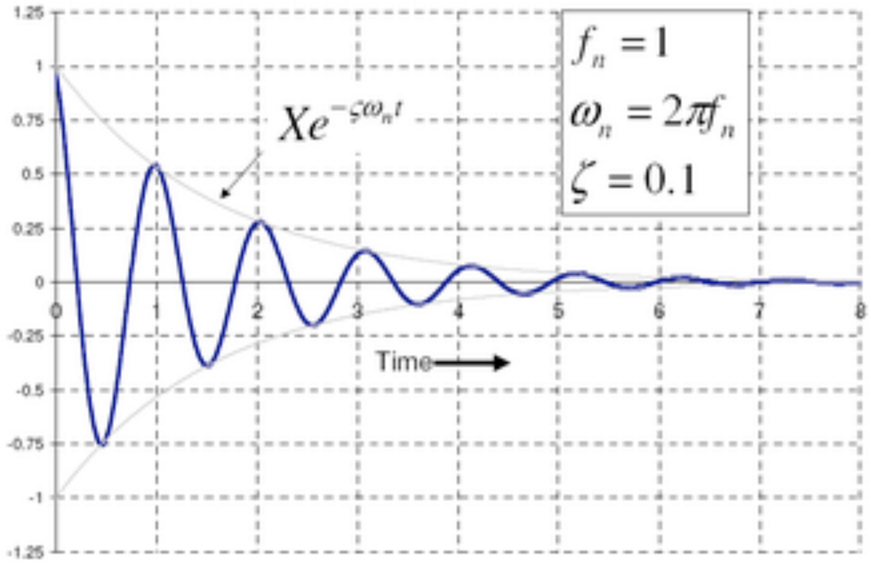
$$x = \sin(2\pi f t) \quad = \sin(\omega t)$$

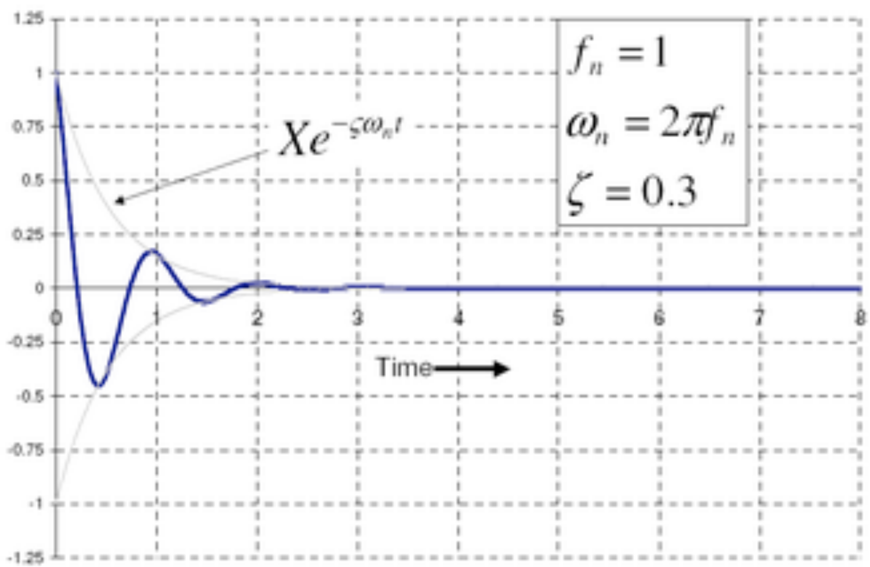
$$F = kx + \mu \frac{dx}{dt} + m \frac{d^2 x}{dt^2}$$



Can the kangaroo (or human or fly) operate at frequencies that minimize the energy?

How can I find the frequency that minimizes energy? (this should resonate with you) A demonstration in Mathematica.





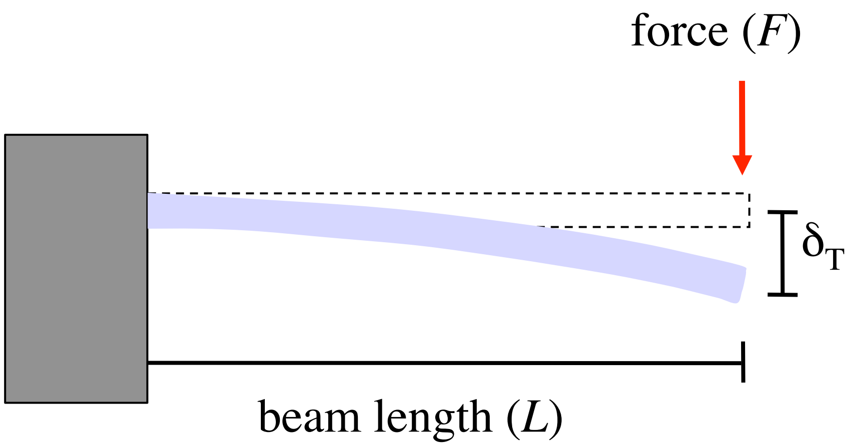
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

Free vibration: simple ways to quantify the stiffness, damping, and mass of a system

$$EI = \frac{FL^3}{3\delta}$$

$$\omega = 3.5\sqrt{\frac{EI}{\rho A L^4}}$$



How does the motion (x) change as we input a force near resonance?

A sample demonstration in Mathematica.