Biology 427 Biomechanics
Lecture 16. Basic fluid dynamics II: continuity and Bernoulli's principle.

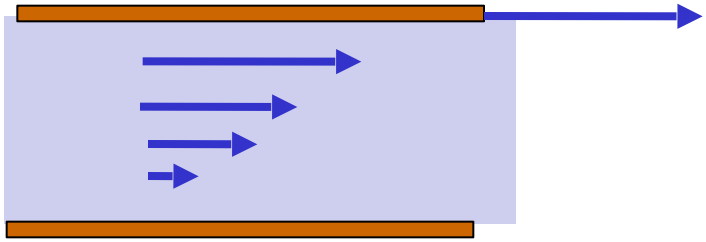
- Recap definition of a fluid and Newton's law of viscosity and a coevolutionary story.
- Conservation of mass and continuity
- An application of continuity
- Conservation of energy and Bernoulli's principle
- Applications of Bernoulli's principle

2 rules govern flow

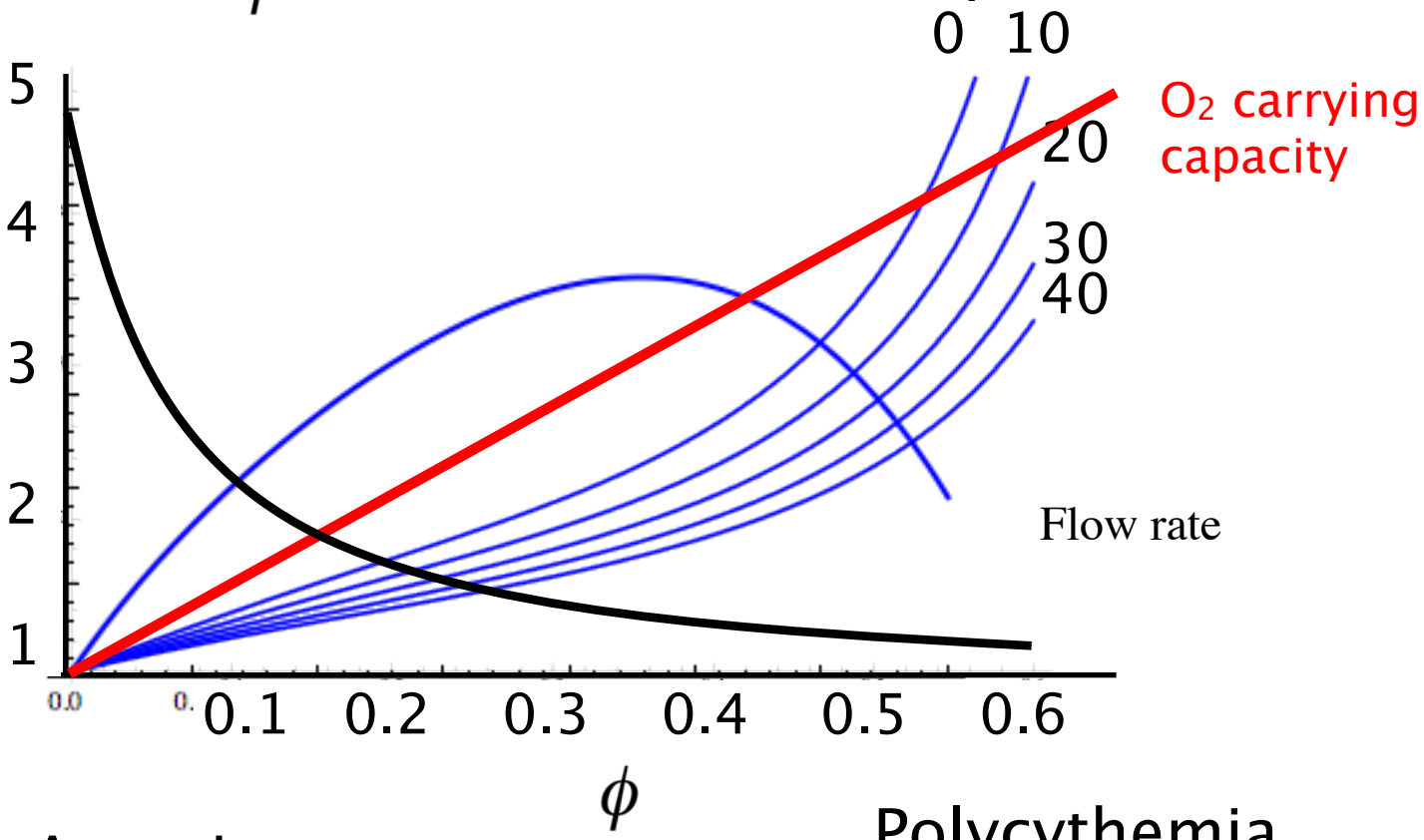
- Newton's Law of viscosity

$$\tau = \mu \; du/dy = \nu \; d(\rho \, u)/dy$$

- No slip condition (fluid adheres to solid surfaces)



Blood viscosity depends non-linearly on red blood cell concentration (ϕ hematocrit) and Temperature



Anemia

genetic, low iron, malaria

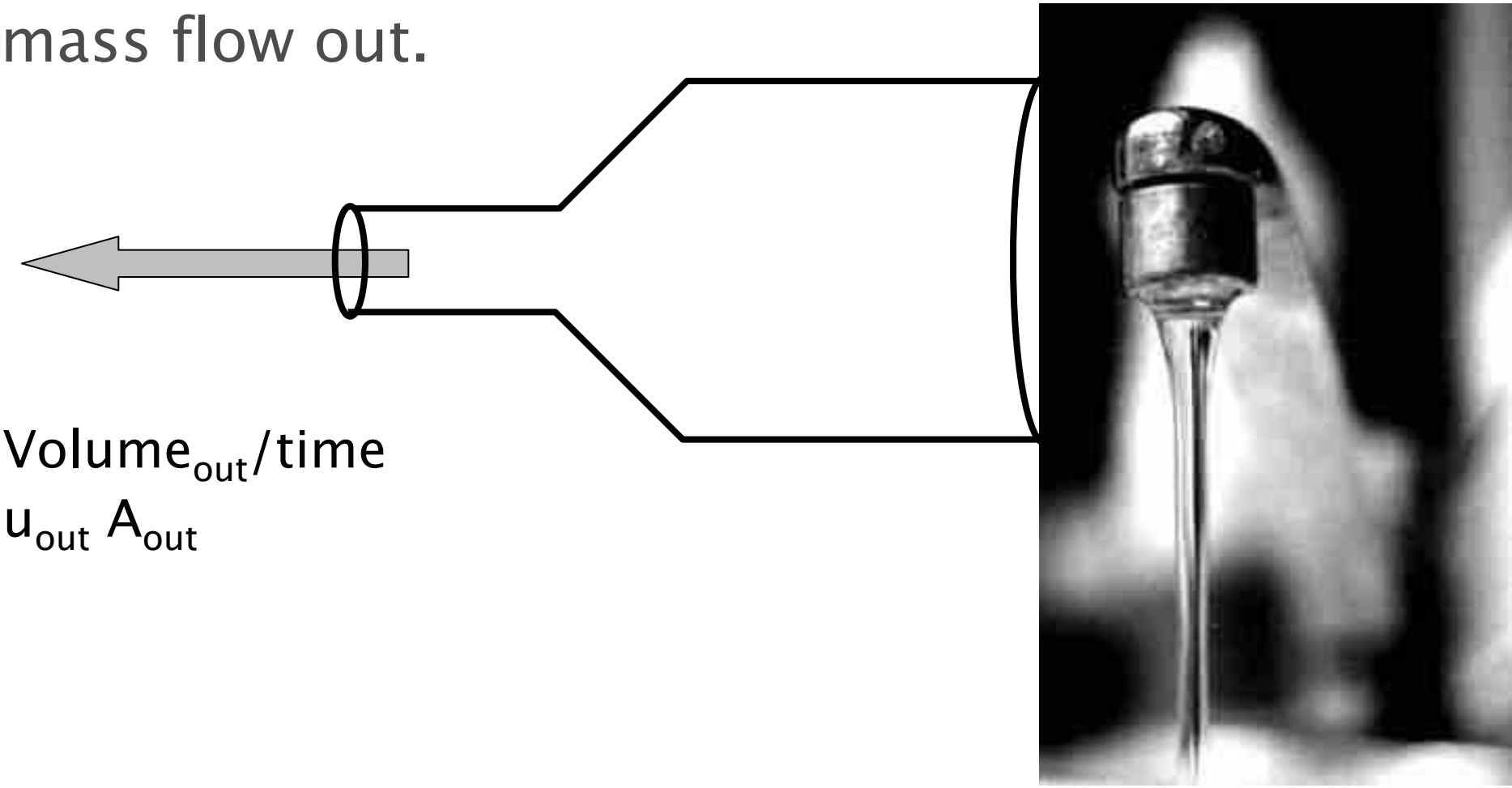
Polycythemia

genetic, EPO, blood doping, dehydration

2 more rules govern flow-- and applications of those rules to biology:

- Conservation of mass -- the Principle of Continuity
- Conservation of energy -- Bernoulli's Principle

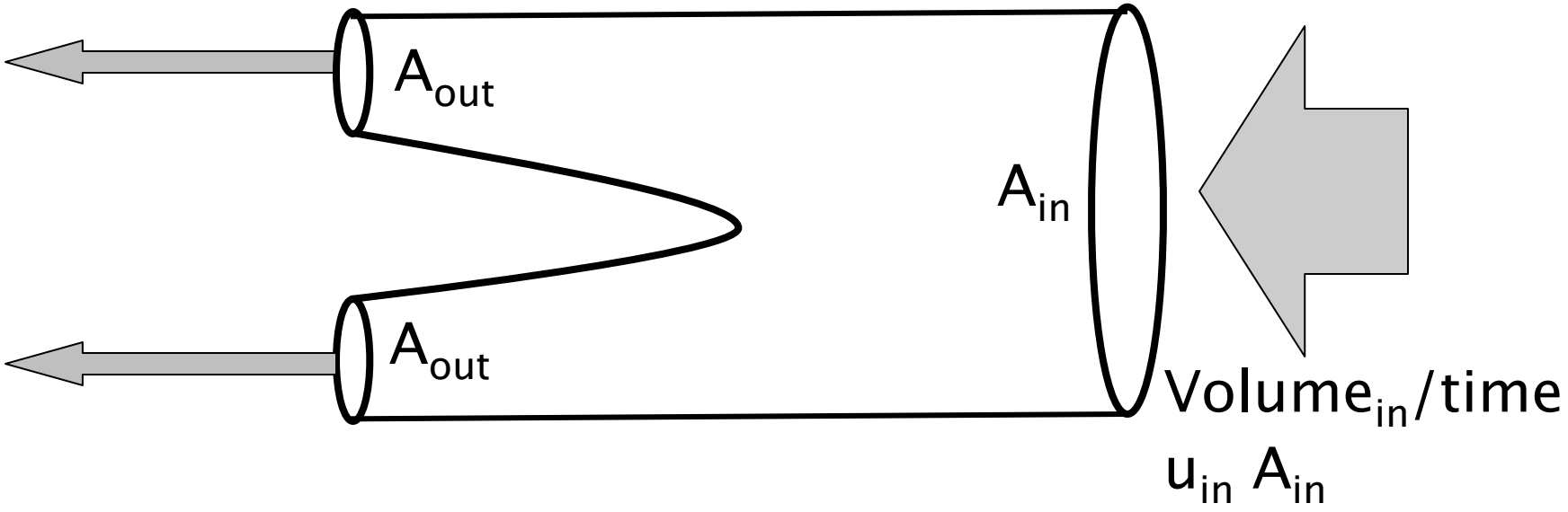
Conservation of mass -- the “principle of continuity”: rate of mass flow in must equal rate of mass flow out.



$Volume_{out}$/time

u_{out} A_{out}

Conservation of mass -- the "principle of continuity": rate of mass flow in must equal rate of mass flow out.



$$\Sigma u_{in}\, A_{in} = \Sigma\, u_{out}\, A_{out}$$

How many capillaries do we have?

Aorta

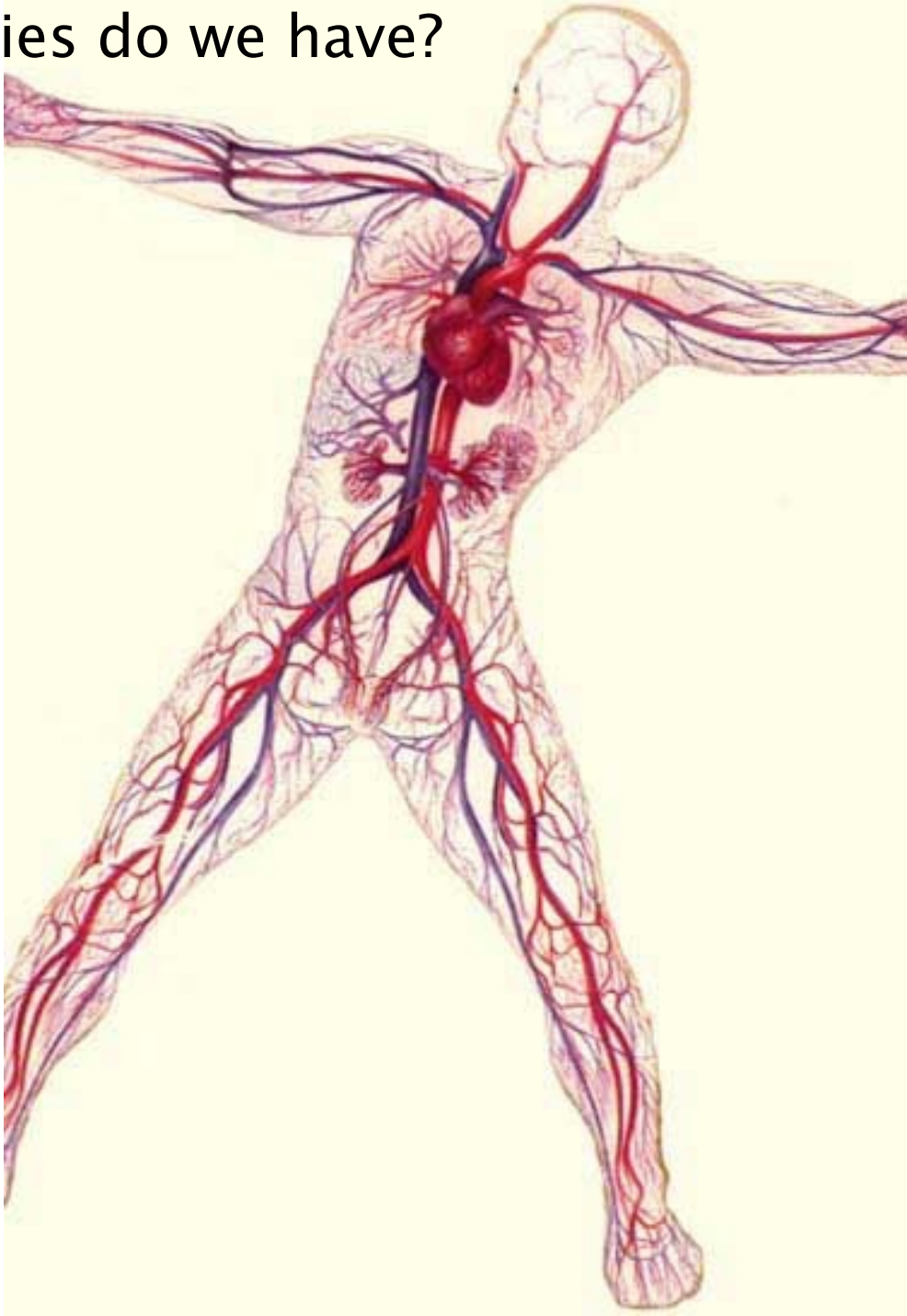
Area 1 cm^2

Blood velocity 10 cm/s

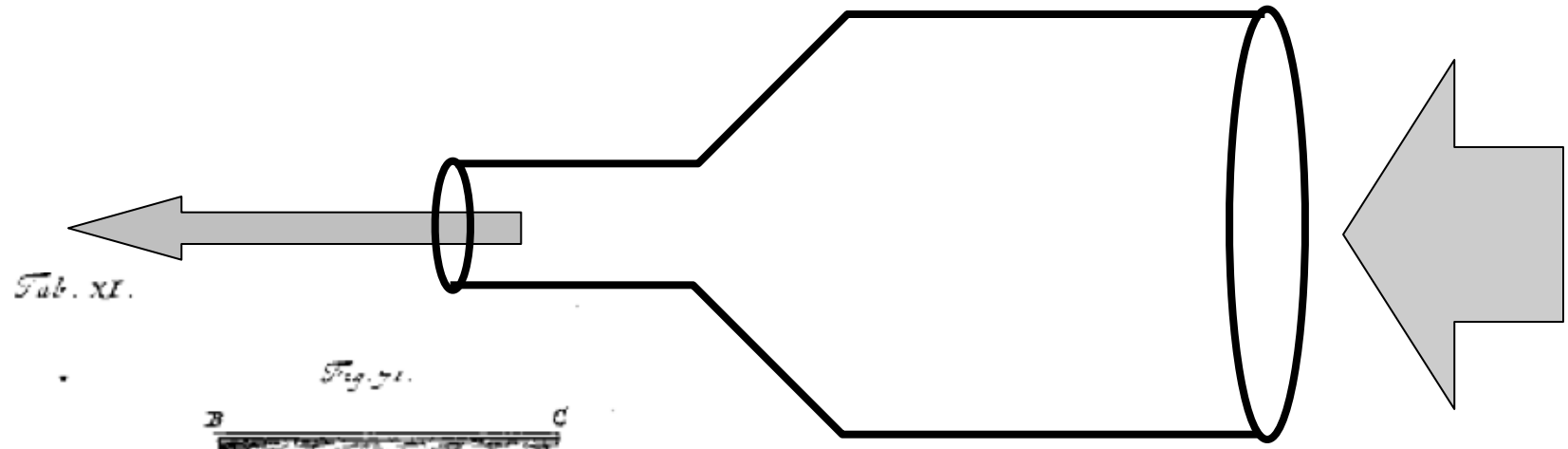
Capillary

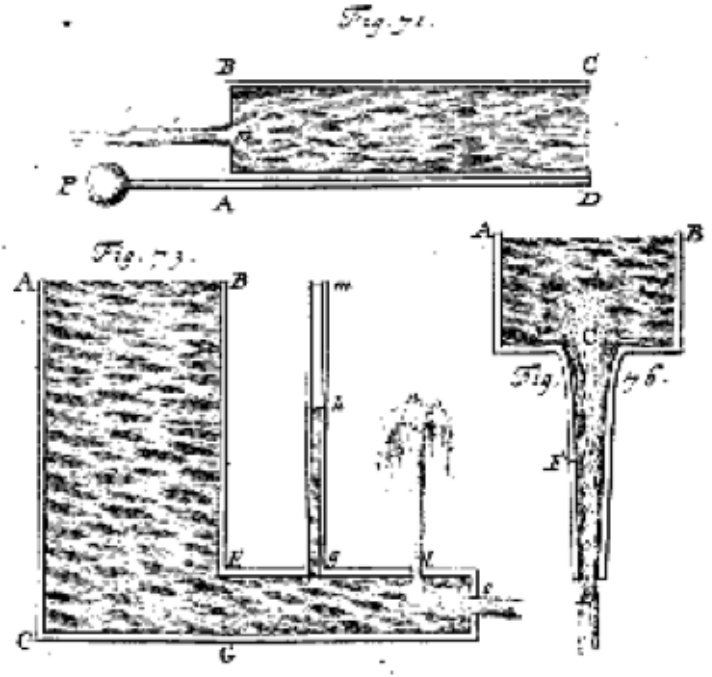
Area 2 10^{-7} cm^2

Blood velocity 0.1 cm/s



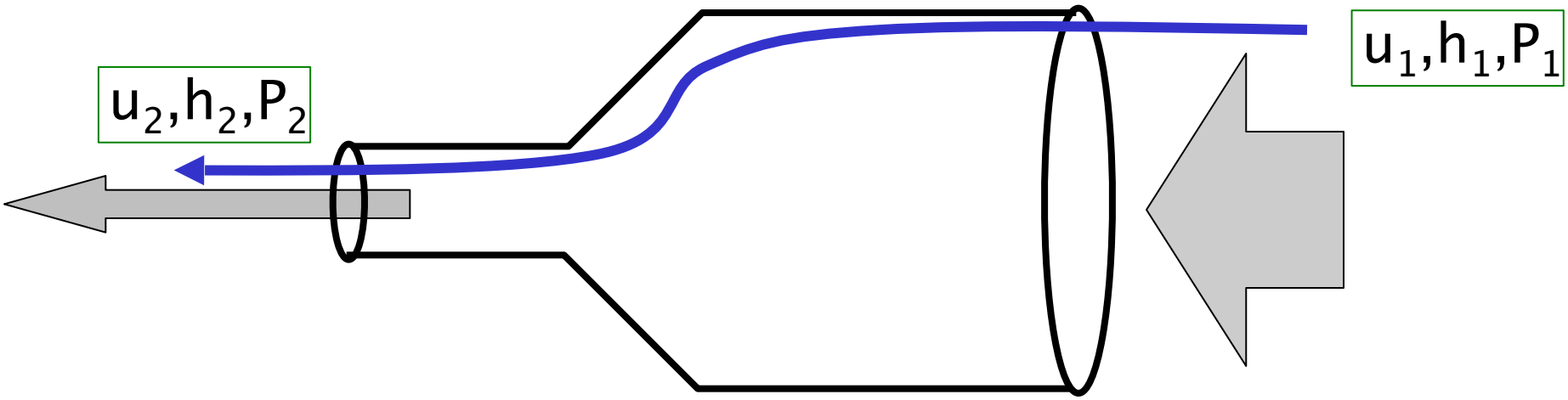
Conservation of Energy: what is the relationship between fluid motion and pressure?





Daniel Bernoulli
1738

Conservation of Energy: what is the relationship between fluid motion and pressure?



3 assumptions:

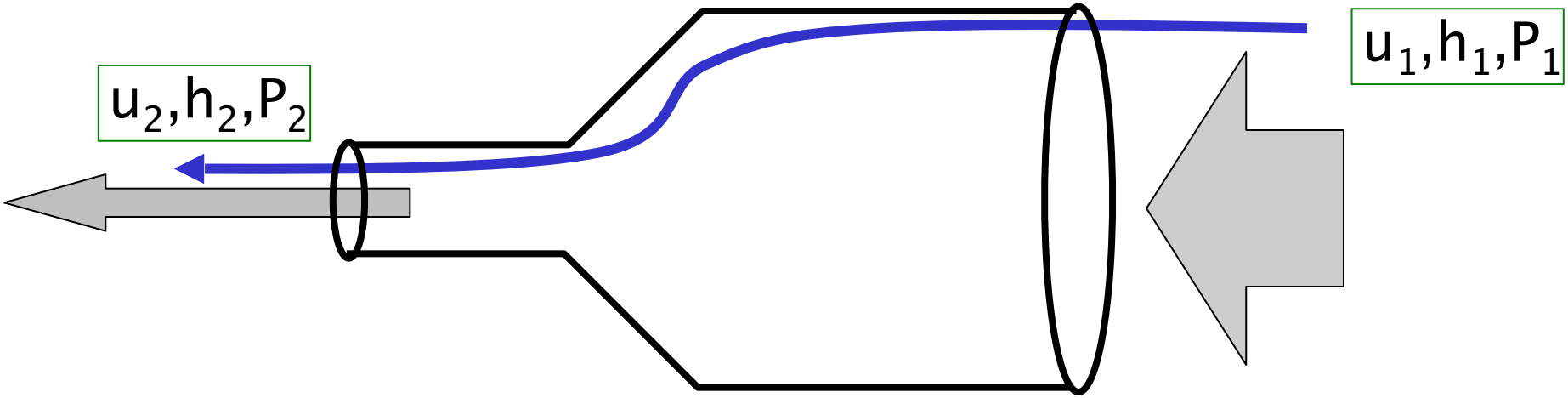
flow is steady (no acceleration at a point in space,

$du/dt = 0$

fluid is incompressible (ρ = constant)

flow is "inviscid" ($\mu = 0$)

Conservation of Energy: what is the relationship between fluid motion and pressure?



Along a stream line we consider three forms of energy

Potential energy (PE = m g h)

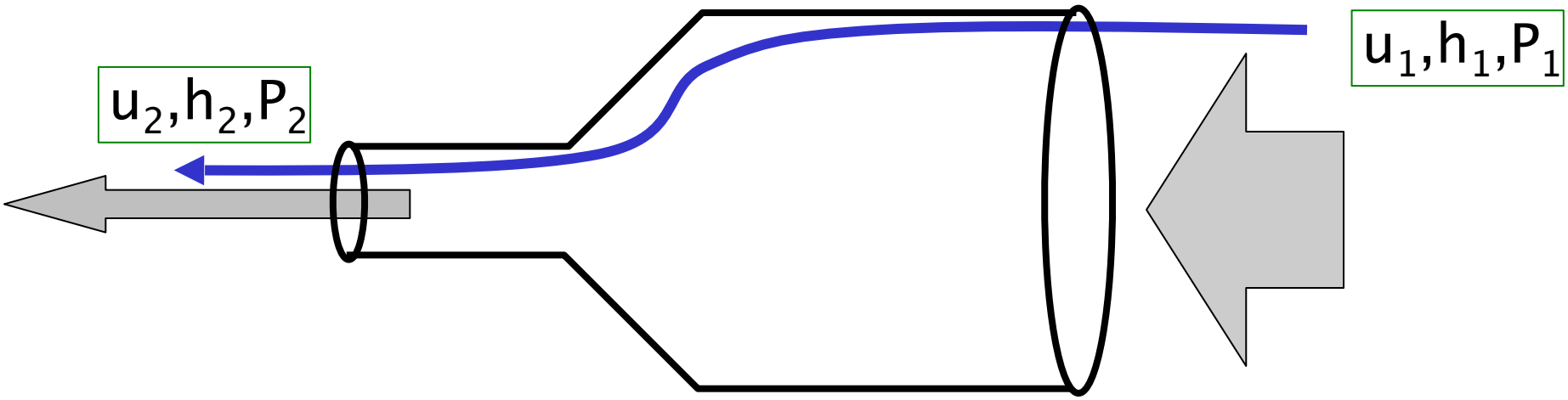
Kinetic energy (KE = m $u^2/2$)

Mechanical work (W = F d =

P Area d = P V = P m/ ρ)

Along a streamline PE + KE + W = constant

Conservation of Energy: what is the relationship between fluid motion and pressure?



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Potential energy (PE = g h)

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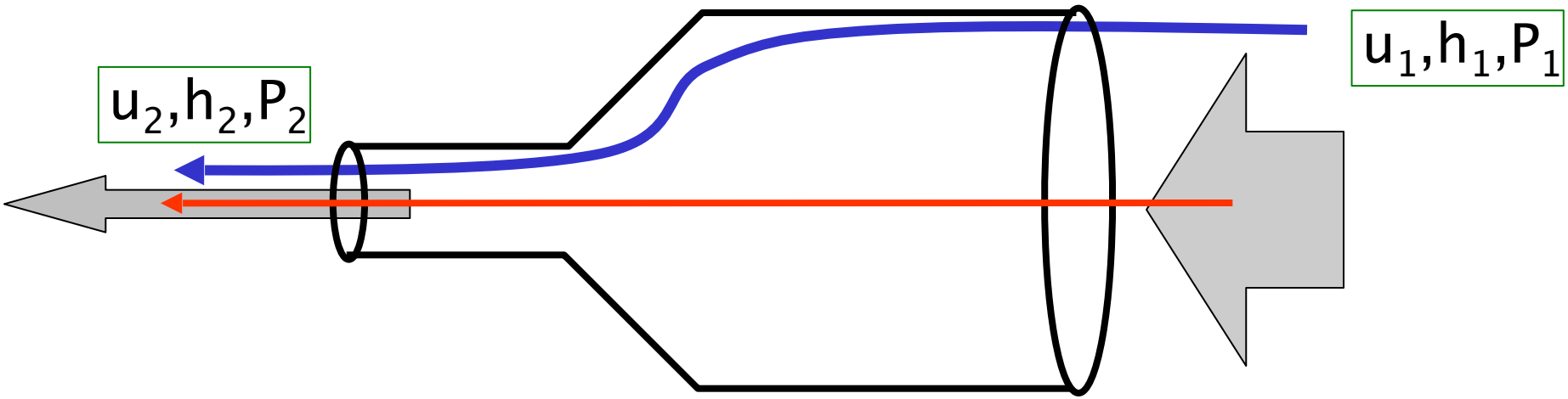
Mechanical work (W = F d = P Area d = P V = P / ρ)

Along a streamline (PE + KE + W)/m = constant

Conservation of Energy: what is the relationship between fluid motion and pressure? $(P_2 - P_1)/\rho + (u_2^2 - u_1^2)/2 = 0$



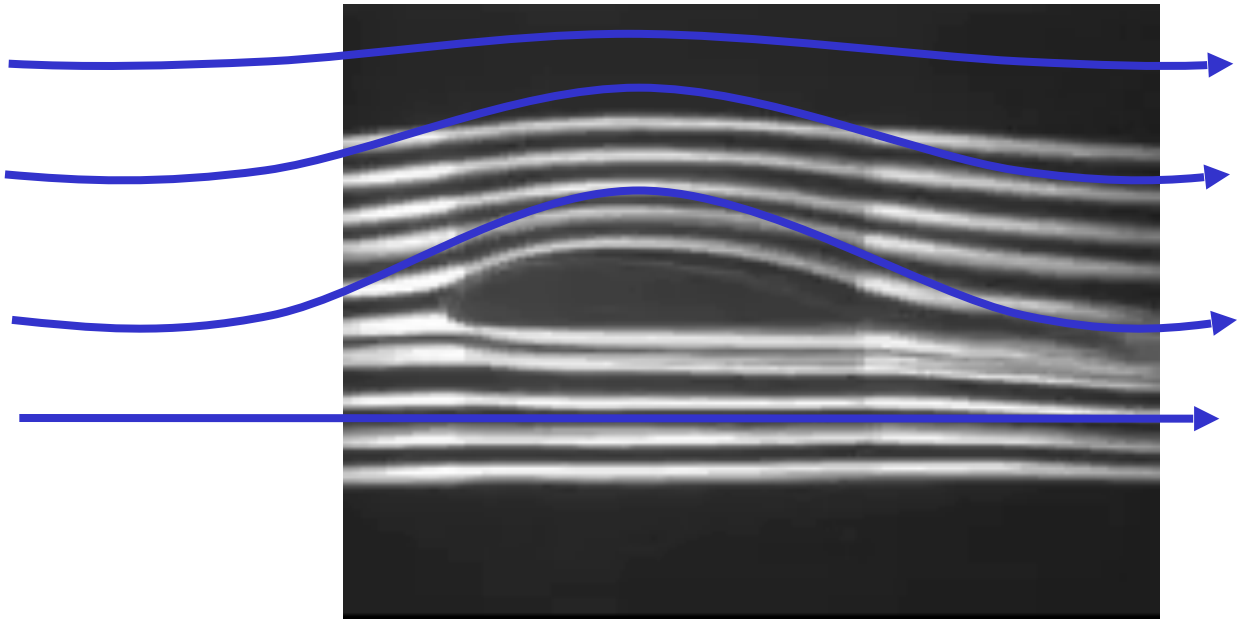
$$P_2/\rho + gh_2 + u_2^2/2 = P_1/\rho + gh_1 + u_1^2/2$$

$$(P_2 - P_1)/\rho + g(h_2 - h_1) + (u_2^2 - u_1^2)/2 = 0$$

Along a streamline $(PE + KE + W)/m = \text{constant}$

Conservation of Energy: what is the relationship between fluid motion and pressure?

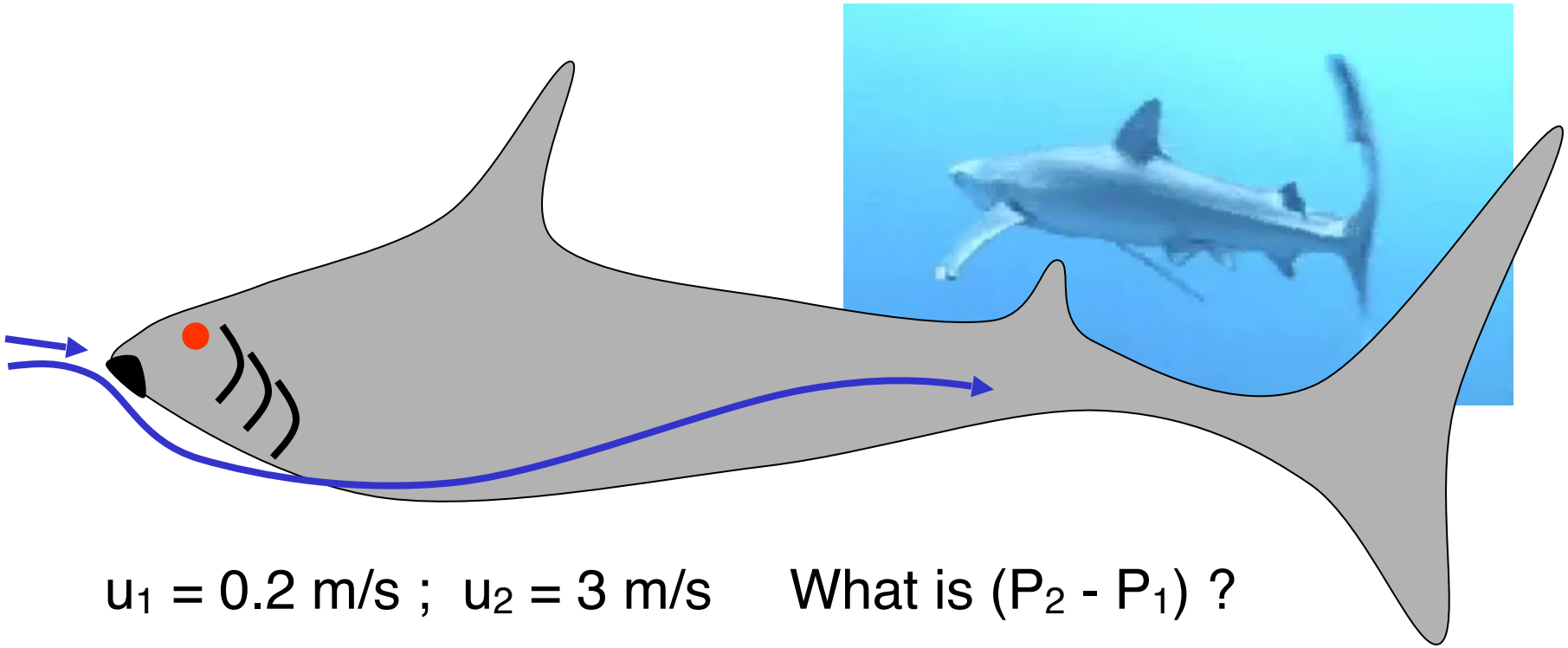
$$(P_2 - P_{1)} / \rho + (u_2^2 - u_1^2)/2 = 0$$



Why is fluid faster over the top?

Conservation of Energy: what is the relationship between fluid motion and pressure?

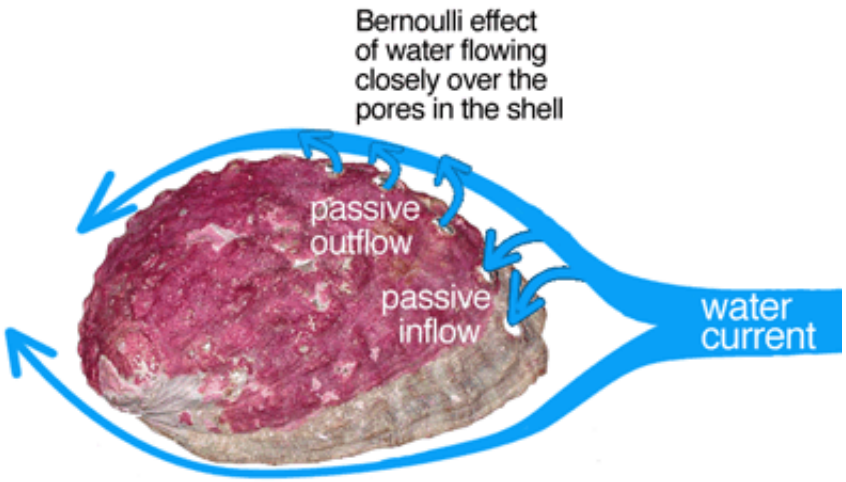
$$(P_2 - P_1)/\rho + (u_2^2 - u_1^2)/2 = 0$$

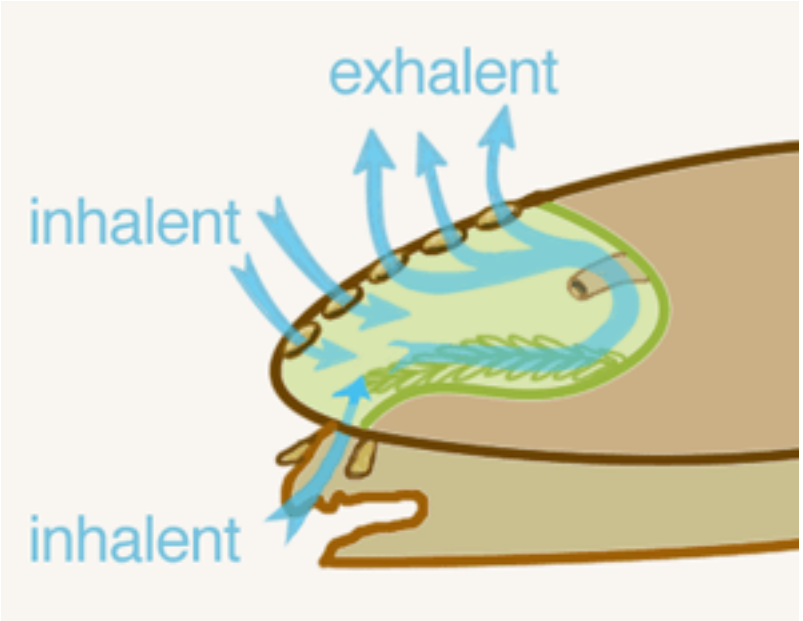


$u_1 = 0.2$ m/s ; $u_2 = 3$ m/s What is $(P_2 - P_1)$?

Conservation of Energy: what is the relationship between fluid motion and pressure?

$$(P_2 - P_{1)} / \rho + (u_2^2 - u_1^2)/2 = 0$$





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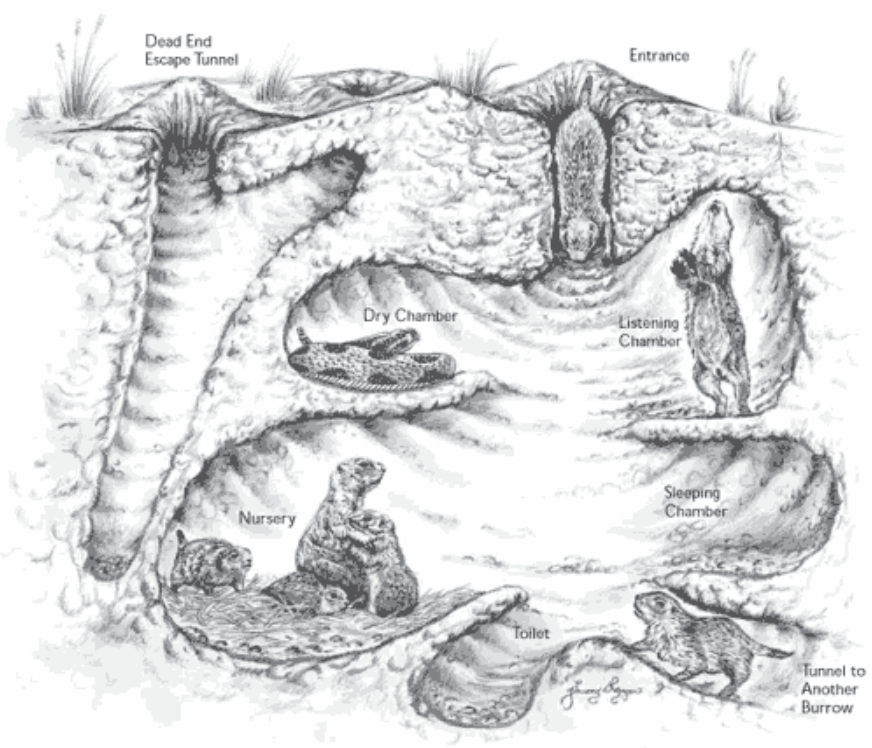

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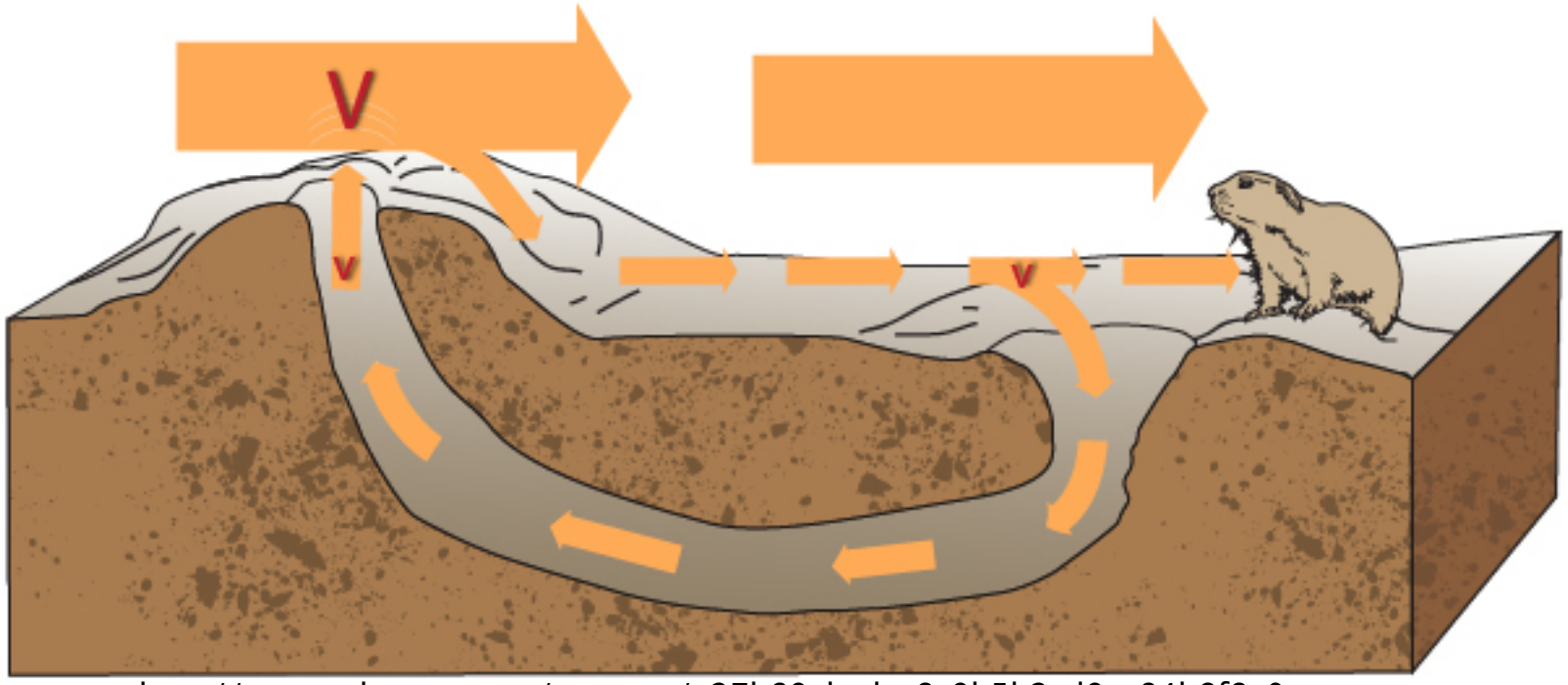
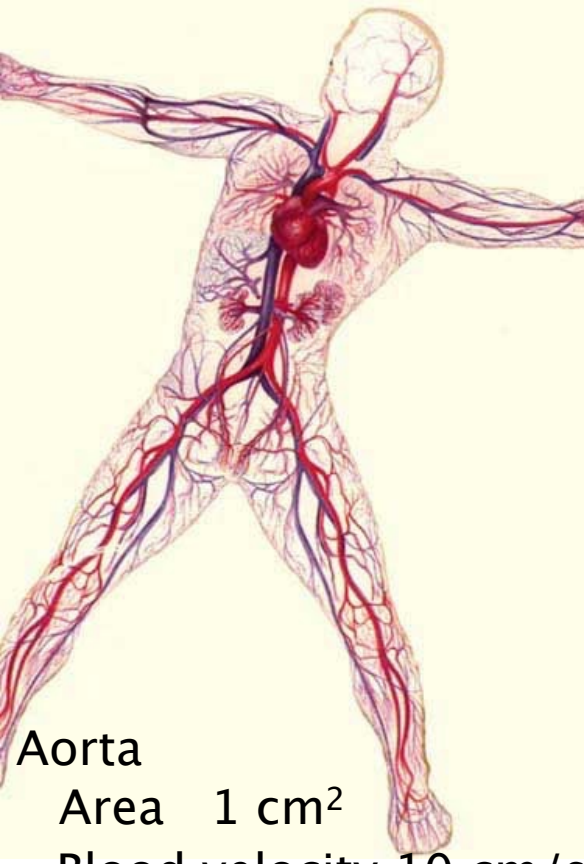
Is the direction drawn here correct?



http://www.asknature.org/strategy/e27b89ebcdec8c9b5b2cd9ac84b8f8a0

Worksheet



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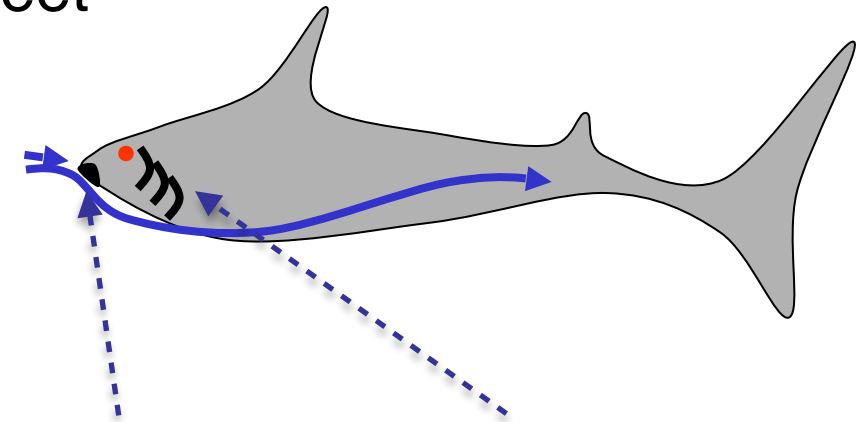

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What is $(P_2 - P_1)$?