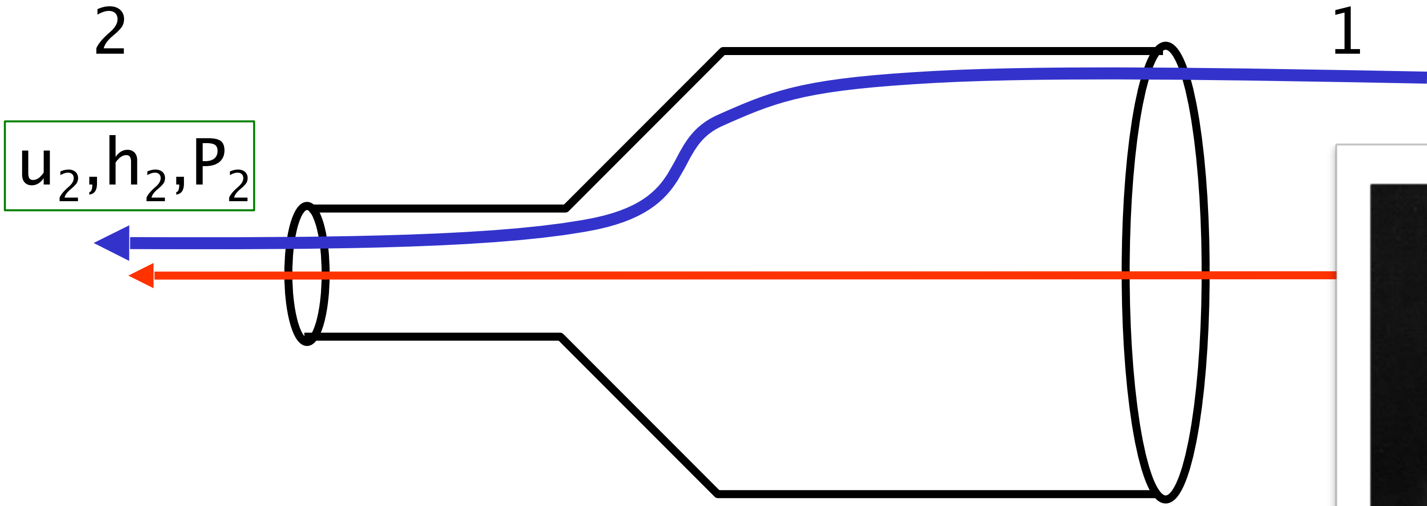
Biology 427 Biomechanics
Lecture 17. Drag and the Reynolds number.

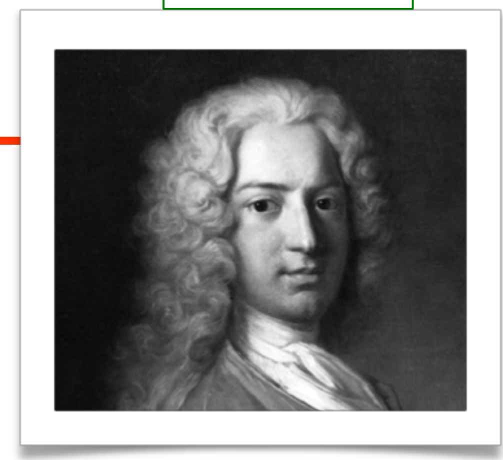
- Recap conservation of energy and mass
- D'Alembert's Paradox and the missing energy
- A wake is a separate issue
- The Reynolds number measures the relevance of viscous and inertial stresses

Conservation of mass (continuity) and energy (Bernoulli)

$$\Sigma\, u_{in} A_{in} = \Sigma\, u_{out} A_{out}$$

$$(P_2 - P_1)/\rho + (u_2^2 - u_1^2)/2 = 0$$



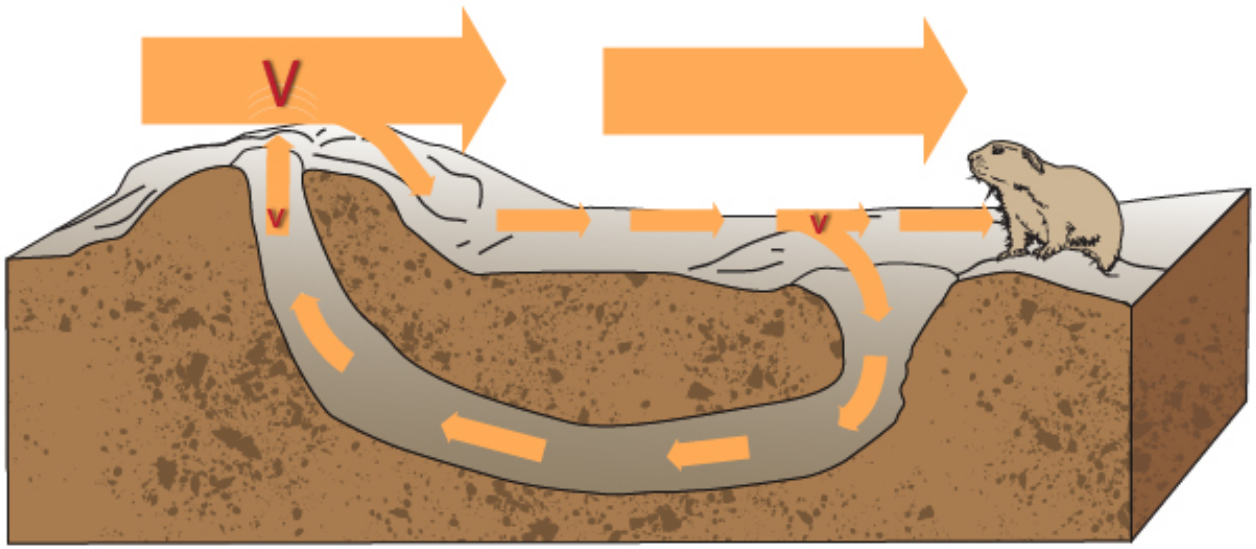


Daniel Bernoulli
1738

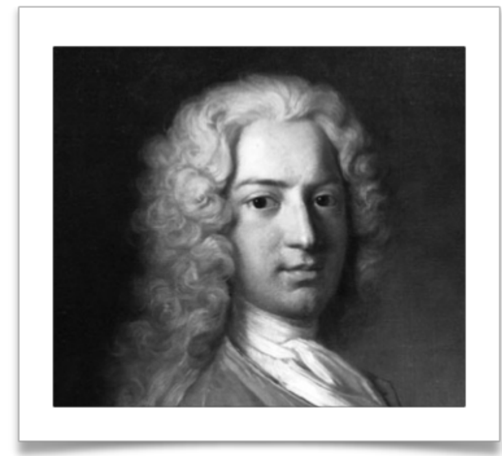
Conservation of Energy: what is the relationship between fluid motion and pressure?

$$(P_2 - P_{1)} /\rho + (u_2^2 - u_1^2)/2 = 0$$

Is the direction drawn here correct?

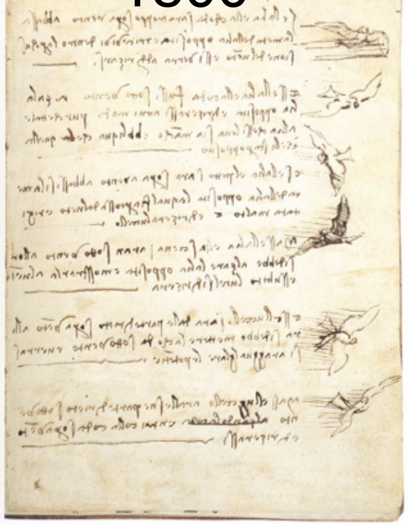


http://www.asknature.org/strategy/e27b89ebcdec8c9b5b2cd9ac84b8f8a0



Daniel Bernoulli
1738

Leonardo da Vinci
~1500



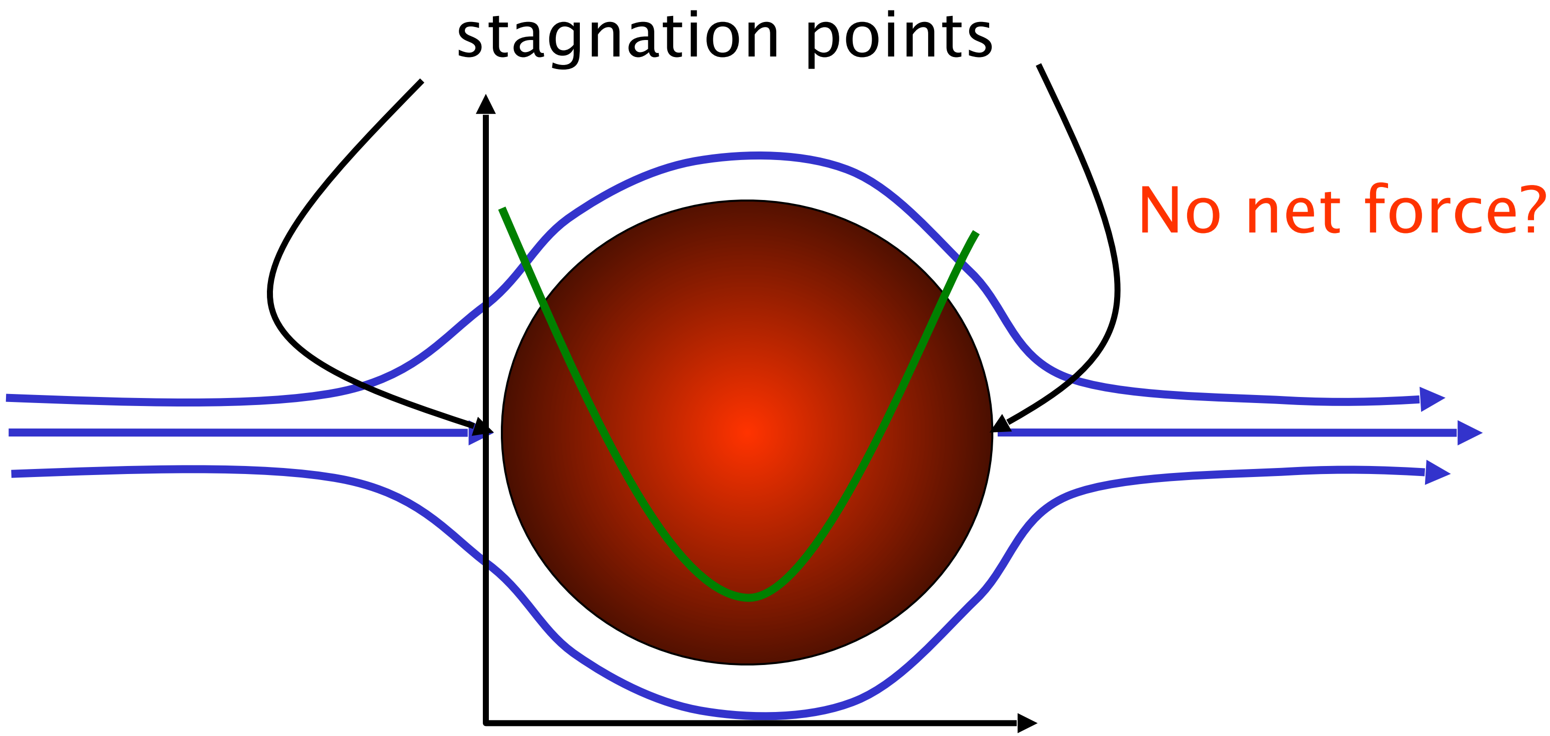
"what quality of air surround birds in flight? The air surrounding the bird is above thinner than the usual thinness of the other air, as below it is thicker than the same... in proportion to the velocity of the bird in its motion forward ..."

Sul volo degli Uccelli (On the flight of birds)
Leonardo da Vinci ~1500

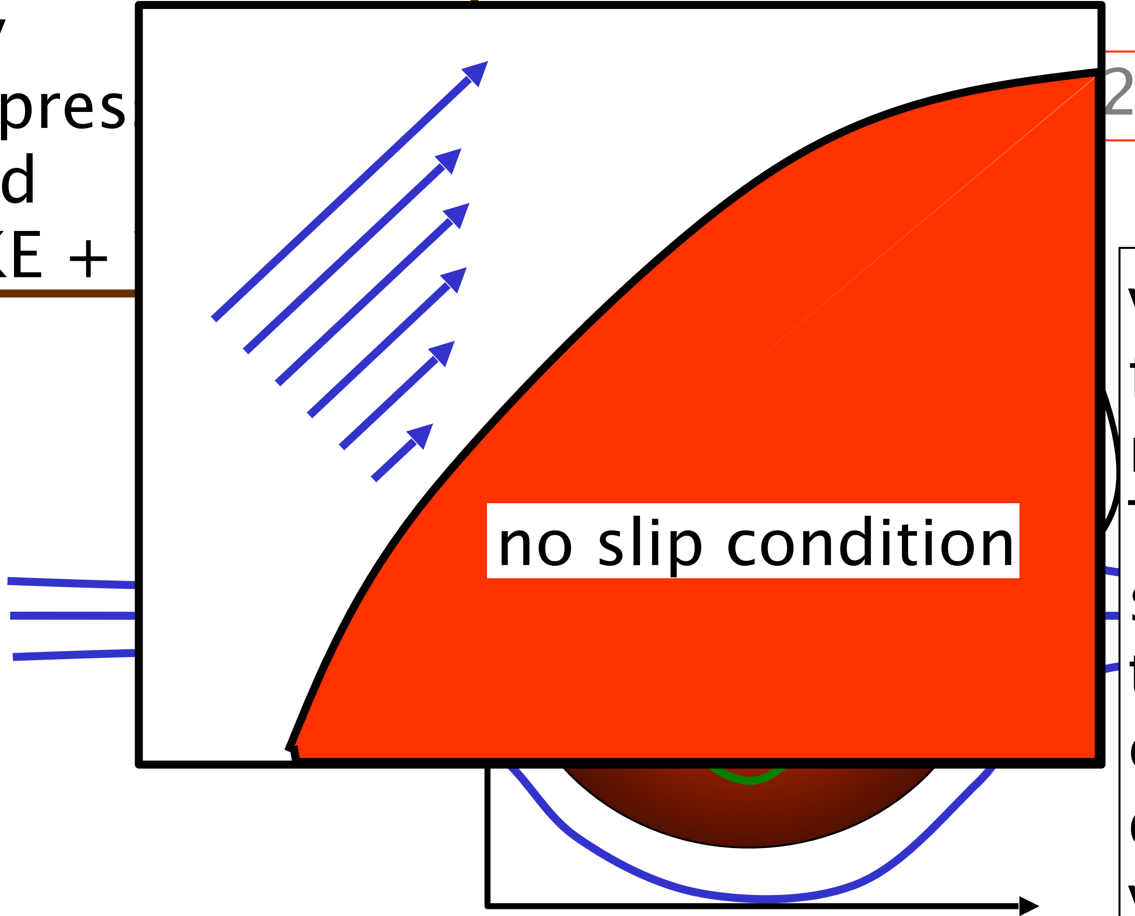
Daniel Bernoulli
1738

D'Alembert's Paradox

u: high P:low $(P_2 - P_1)/\rho = (u_1^2 - u_2^2)/2$ u = 0, P = high



assumptions
steady
incompres
inviscid
PE + KE +



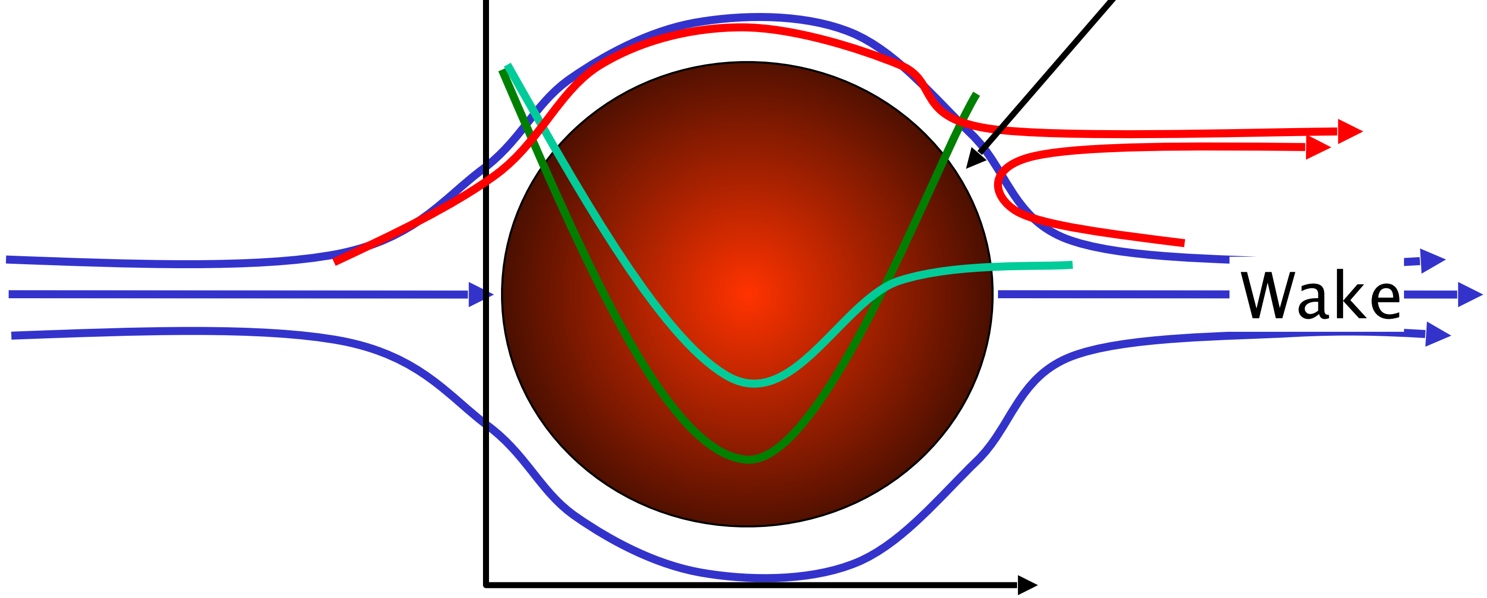
2 u = 0, P = high

viscosity robs fluid of its momentum. There is a shear stress exerted on the sphere and energy is dissipated by viscosity

$$(P_2 - P_1) / \rho = (u_1^2 - u_2^2)/2$$

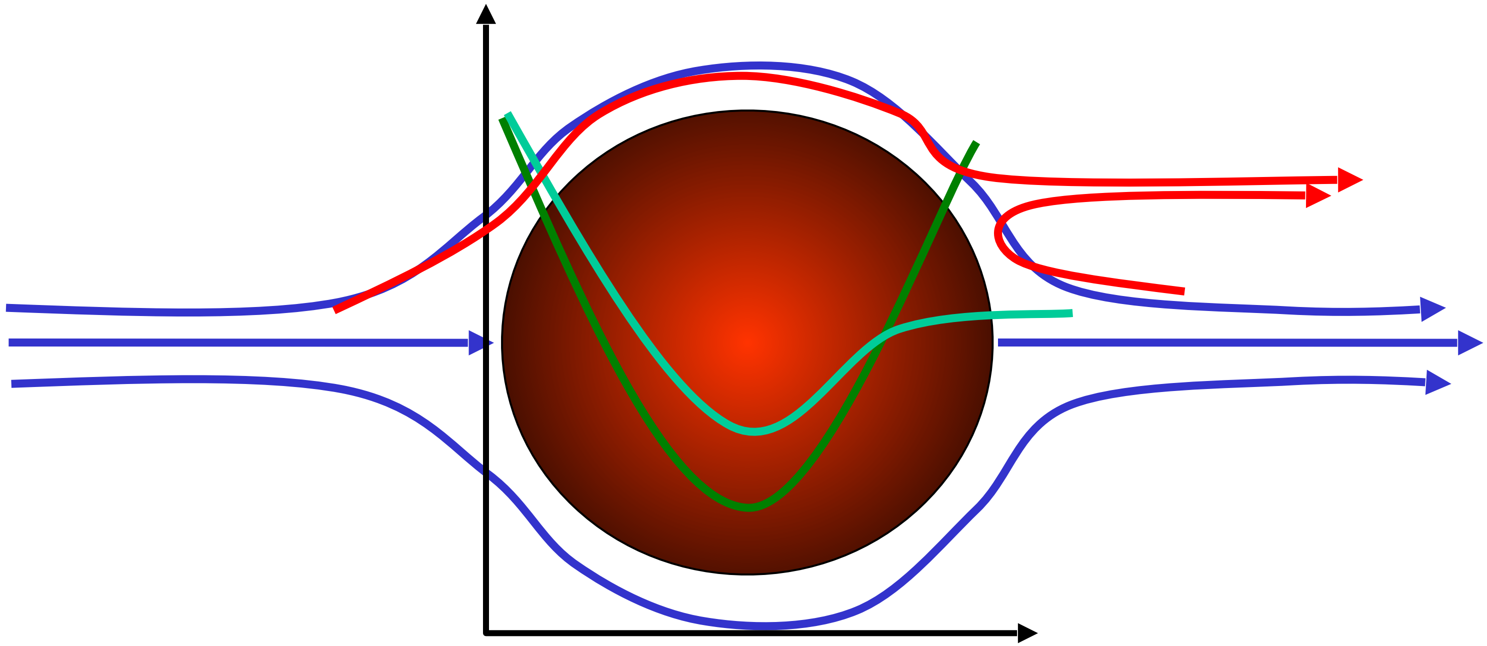
because of viscosity, velocity cannot increase as much as in the inviscid case.

new stagnation point where the flow separates

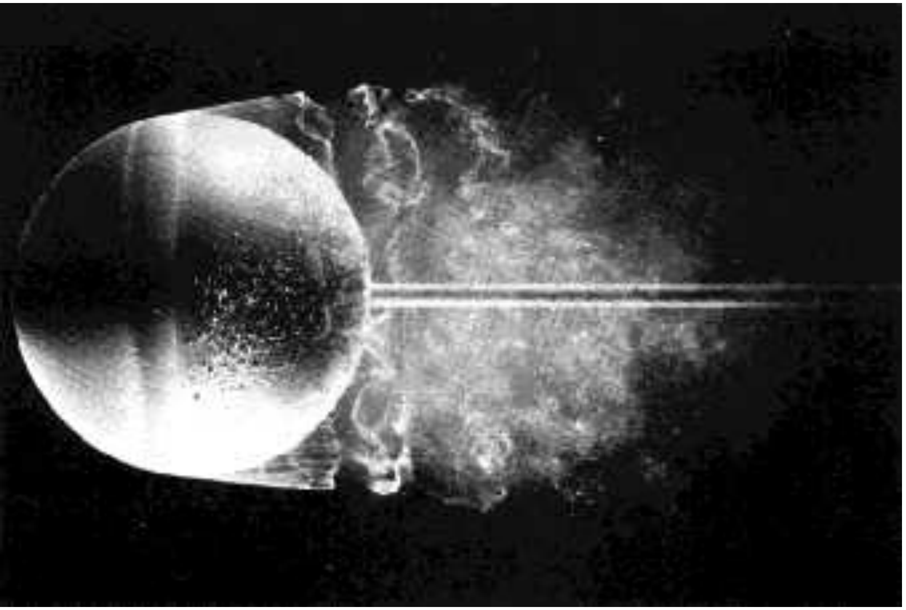
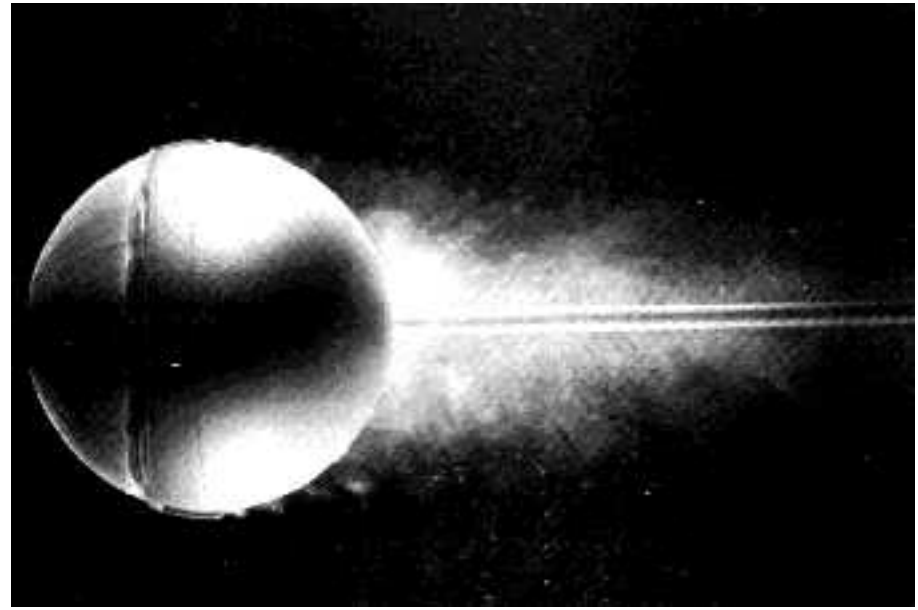


$(P_2 - P_1)/\rho = (u_1^2 - u_2^2)/2$

There are two mechanisms leading to force:
shear stress : friction drag
the fore-aft asymmetry in pressure : pressure drag

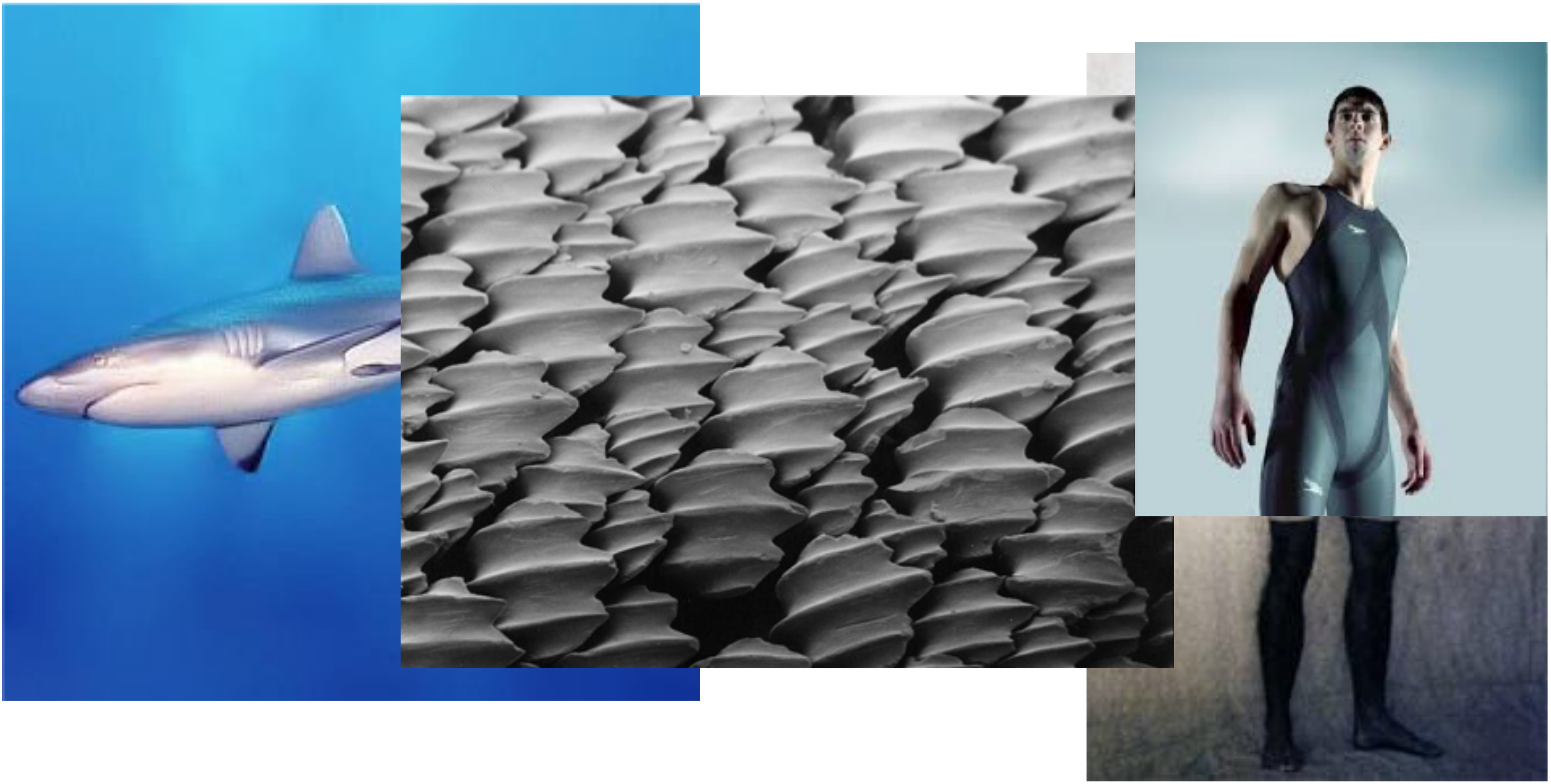


The total drag is a composite of these and depends on size and shape of the body

MOVING

The total drag is a composite of pressure and shear stresses



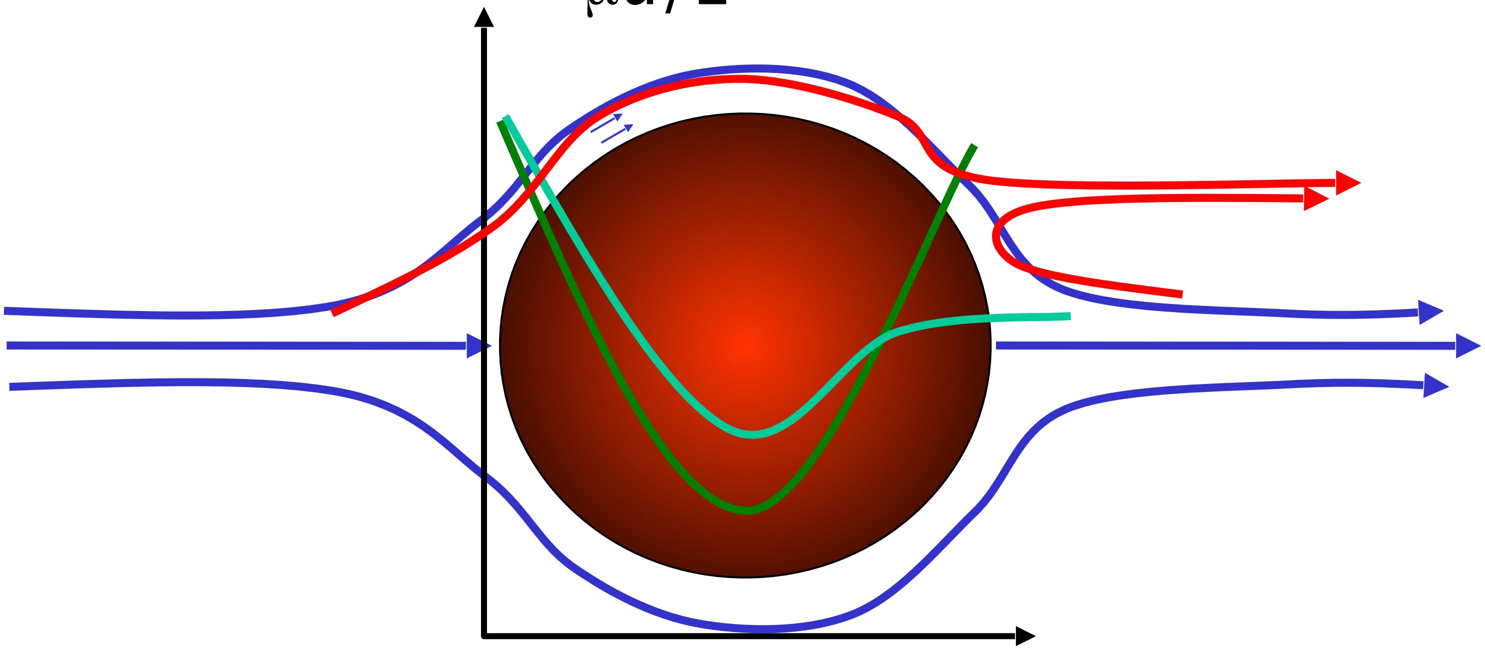
The total drag is a composite of pressure and shear stresses

$$(P_2 - P_1)/\rho = (u_1^2 - u_2^2)/2$$

Pressure stress: $P \sim \rho u^2$

Shear stress: $\tau \sim \mu u/L$

$$\frac{P \sim \rho u^2}{\tau \sim \mu u/L} = Re = \frac{\rho u L}{\mu}$$



What is your Reynolds number ?

Water: $\rho = 1000 \text{kg m}^{-3}$; $\nu = 1 \times 10^{-6}$
Air : $\rho = 1 \text{ kg m}^{-3}$; $\nu = 15 \times 10^{-6}$

$$\mathrm{Re} = \frac{\rho UL}{\mu} = \frac{UL}{\nu}$$

Bacterium $\sim 10^{-5}$
Spermatozoa $\sim 10^{-4}$
Ciliate $\sim 10^{-1}$
Smallest Fish ~ 1
Drosphila flight$\sim 1 \times 10^{2}$
Manduca flight$\sim 1 \times 10^{3}$
Birds flying 1×10^{4}
Typical pitch in Major League Baseball $\sim 2 \times 10^{5}$
Person swimming $\sim 4 \times 10^{6}$
Fastest Fish $\sim 10^{6}$
Blue Whale $\sim 3 \times 10^{8}$
A large ship (RMS Queen Elizabeth 2) $\sim 5 \times 10^{9}$

What is your Reynolds number ?

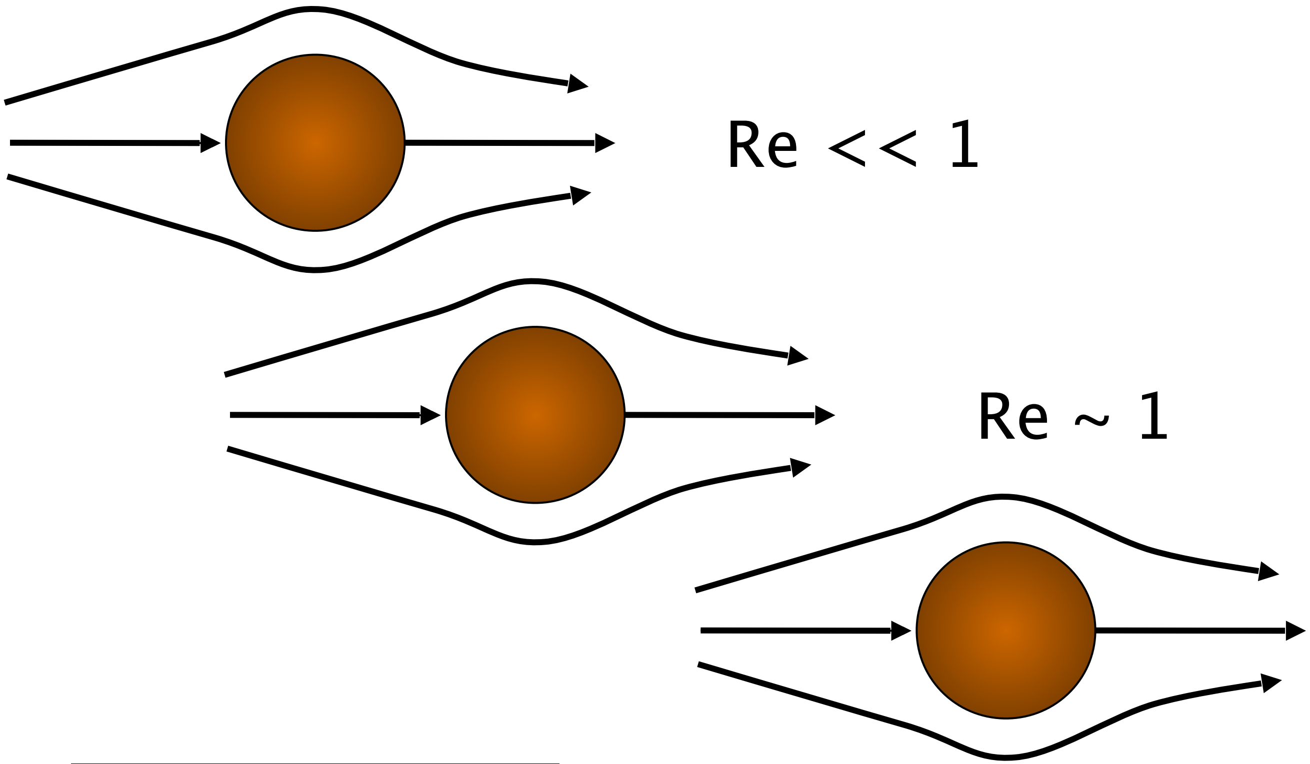
Water: ρ = 1000kg m^{-3}; ν =1x10^{-6}
Air : ρ = 1 kg m^{-3}; ν = 15x10^{-6}

$$\mathrm{Re} = \frac{\rho UL}{\mu} = \frac{UL}{\nu}$$

What would the viscosity of the medium have to be for you to feel the flows like a bacterium does?

How far would a bacterium glide if it turned off its flagellum?

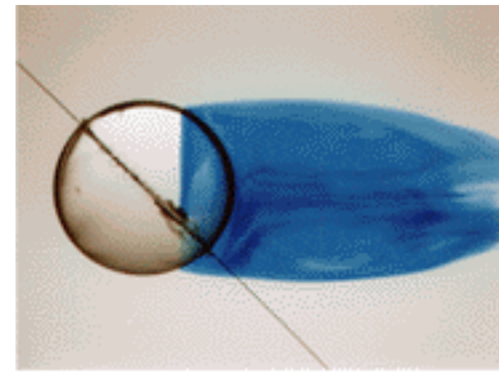
The Reynolds number measures the relative importance of intertial and viscous stresses in determining the flow. Conservation of Re implies identical flow patterns

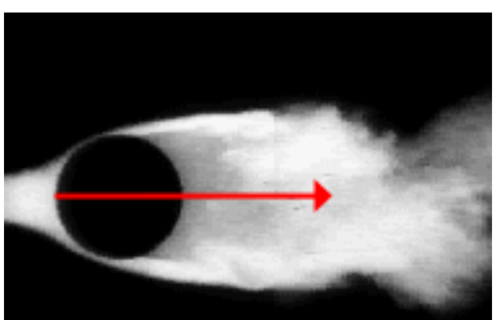


Re << 1

Re ~ 1

Re >> 1





$$Re = \frac{\rho u L}{\mu}$$