Zoology 427 Biomechanics
Lecture 18. Shape and drag

- A comment about poster projects
- Recap drag and the Reynolds number
- Drag and its coefficient
- How size, shape, and the Reynolds number determine drag
- Streamlined bodies and swimming energetics
- When does drag matter?

Because of viscosity, velocity cannot increase as much as in the inviscid case.

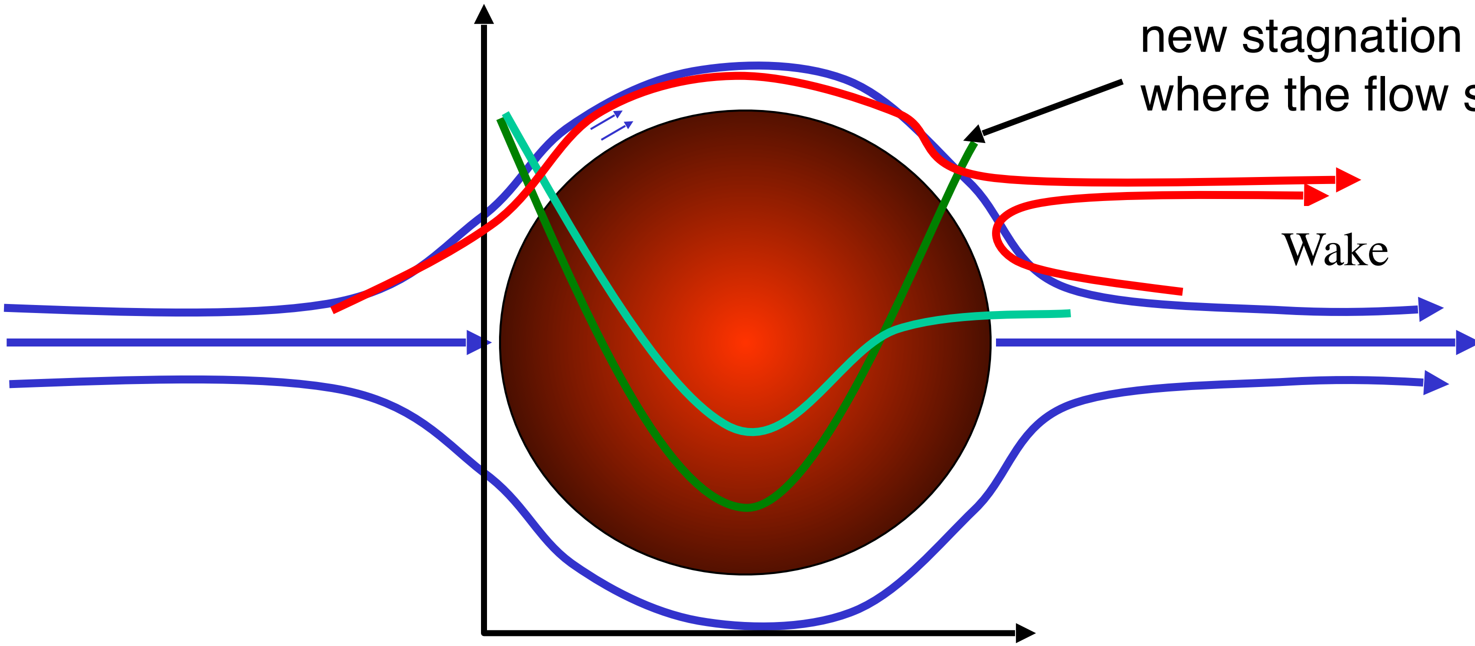
PE + KE + W + Ediss = const

$(P_2 - P_1)/\rho = (u_1^2 - u_2^2)/2$

Pressure stress: $P \sim \rho u^2$

Shear stress: $\tau \sim \mu \, du/dr$

$$Re = \frac{\rho u L}{\mu}$$



Drag *force* arises from pressure and frictional *stresses*

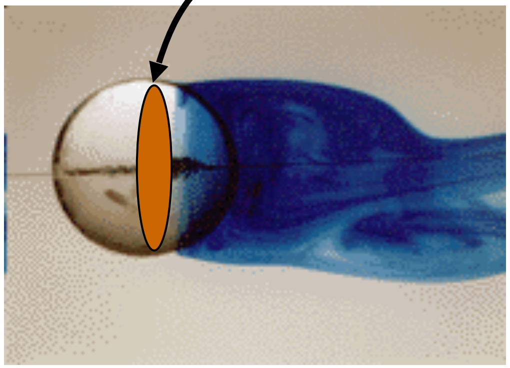
$$\mathbf{D} \sim \rho u^2/2$$

$$D = C_d \rho S u^2/2$$

drag coefficient

projected surface area

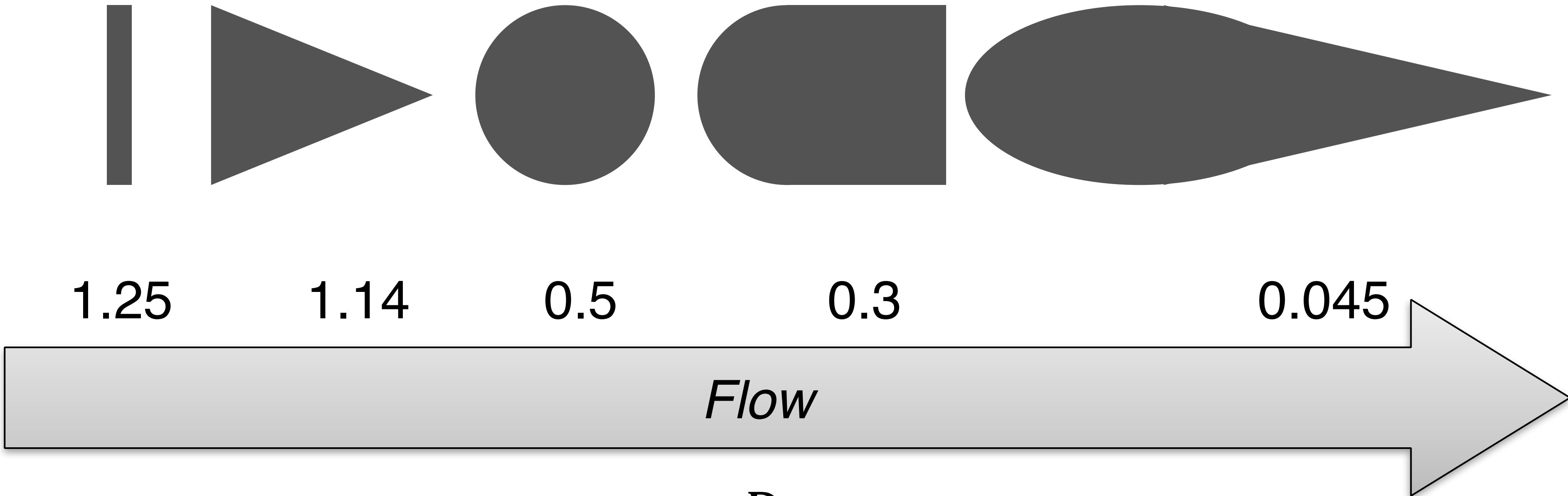
shape
Reynolds number



size

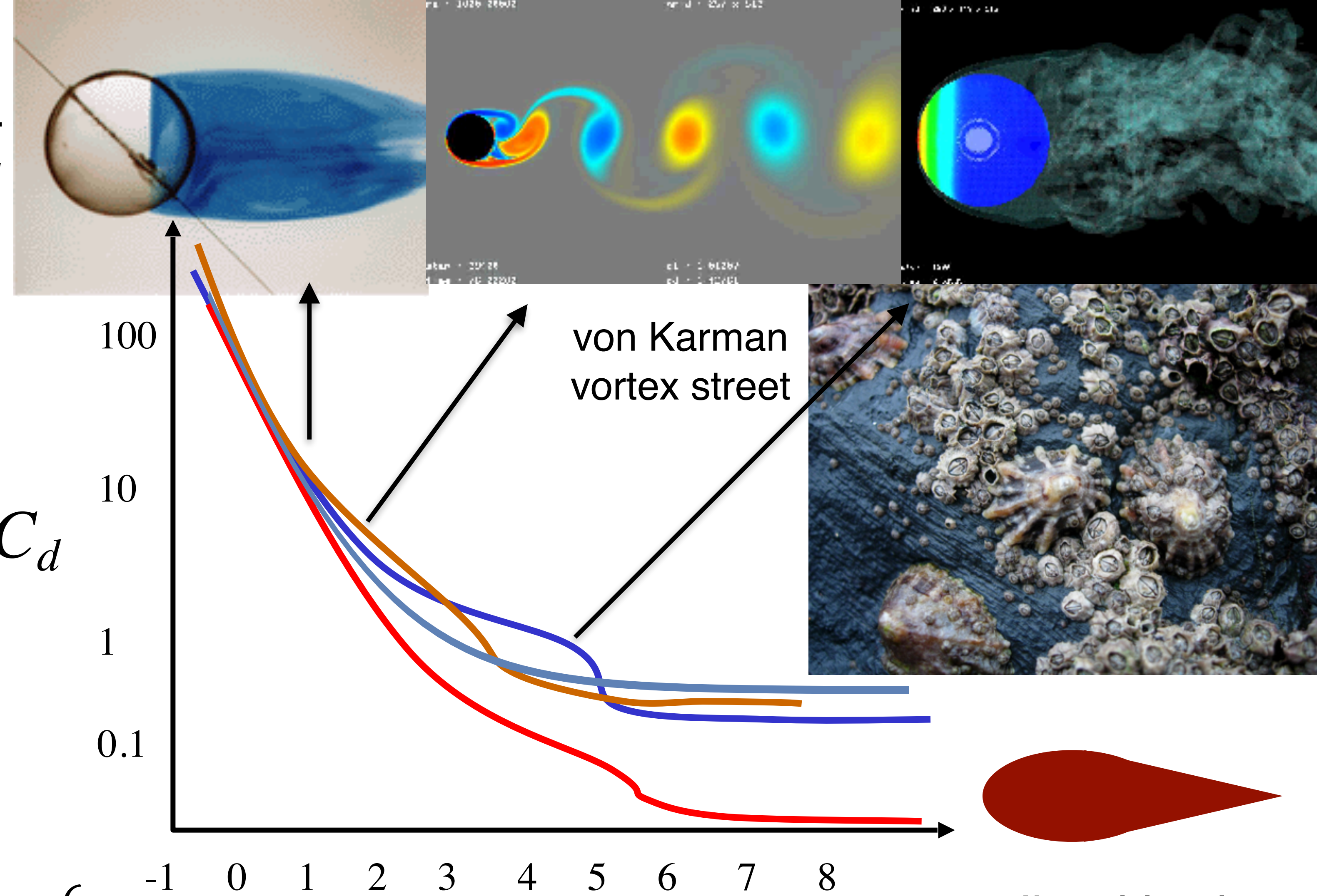
$$C_d = \frac{D}{\frac{1}{2}\rho U^2 S}$$

Shape and Drag



$$C_d = \frac{D}{\frac{1}{2}\rho U^2 S}$$

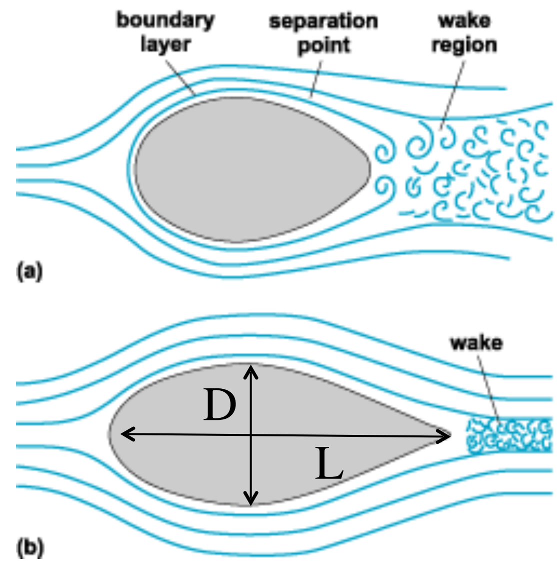
$$C_d = \frac{D}{\frac{1}{2}\rho U^2 S}$$



$$C_d = \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} + 0.4$$

Streamlining – how much?

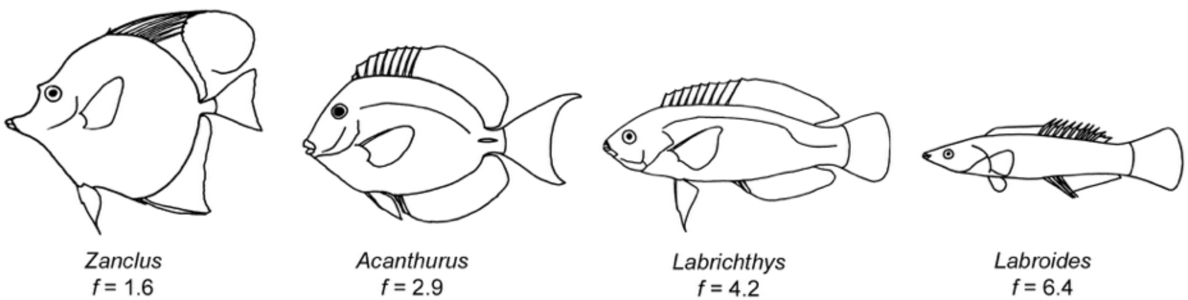
manage your wake and the adverse pressure gradient



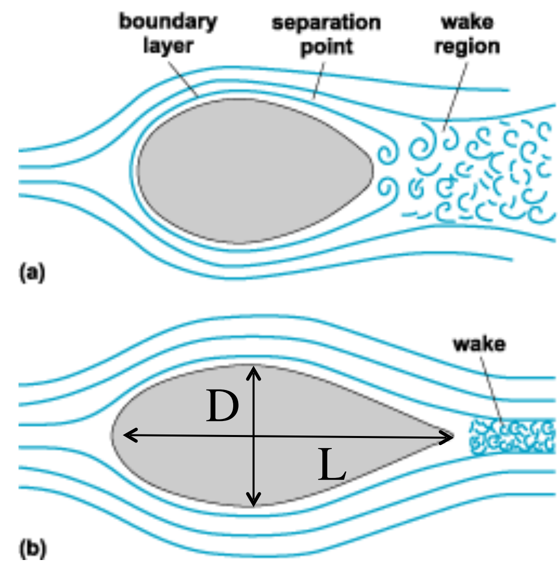
Fineness ratio = L/D

Streamlining – how much?

Challenge: you are given a fixed volume of fish and must arrange it to minimize the total drag. The volume can be distributed with any fineness ratio. Generate a hypothetical plot for the drag as a function of fineness ratio.

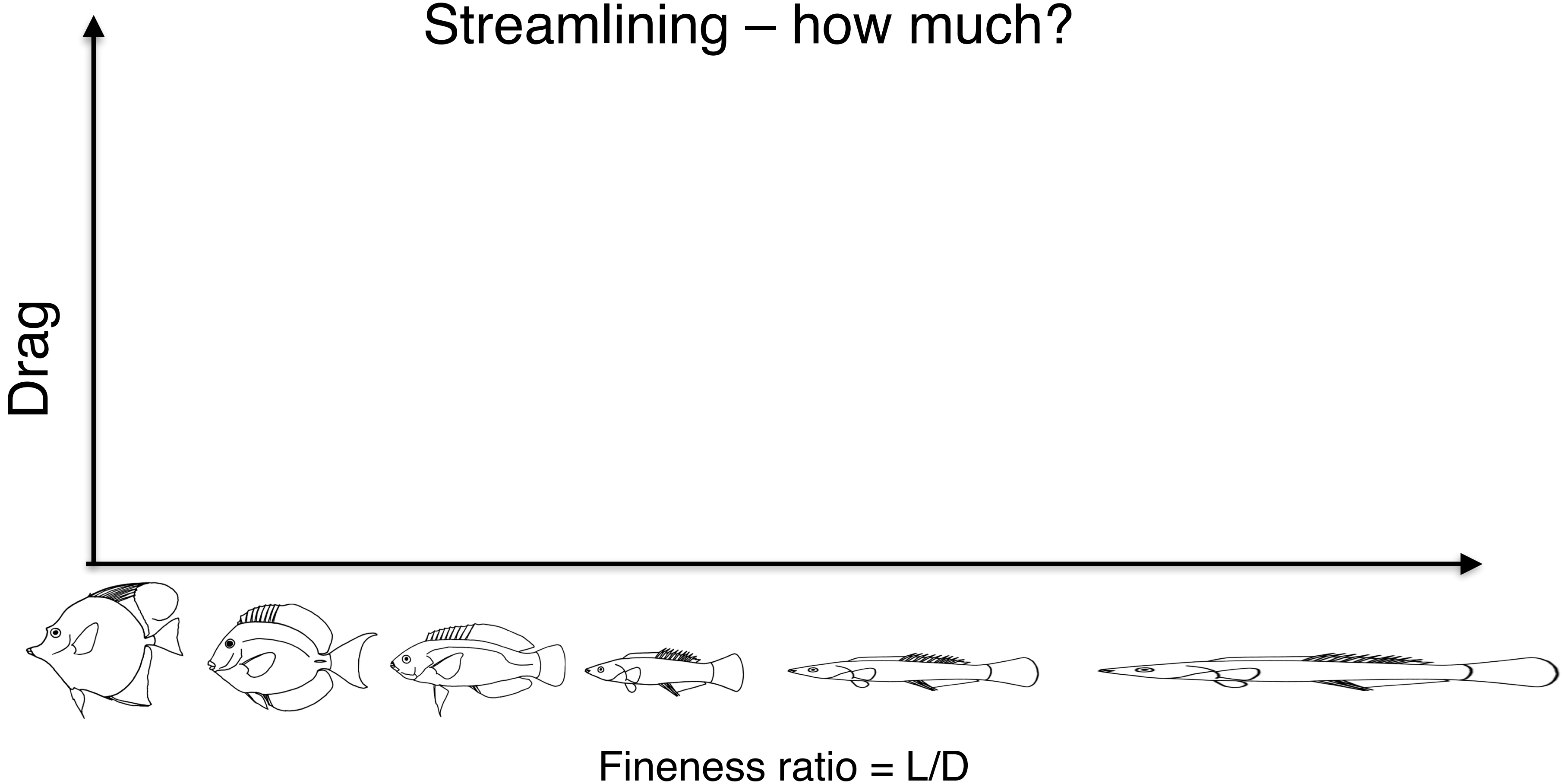


manage your wake and the adverse pressure gradient



Fineness ratio = L/D

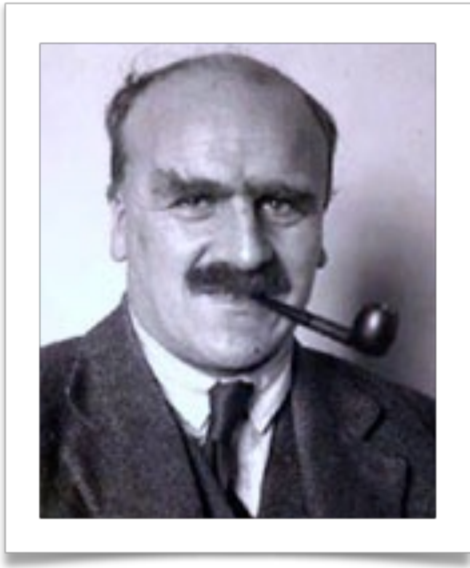
Streamlining – how much?



“You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes”. JBS Haldane 1926. On Being the Right Size.

$D = 0.5\ C_d\ \rho\ S\ u^2$

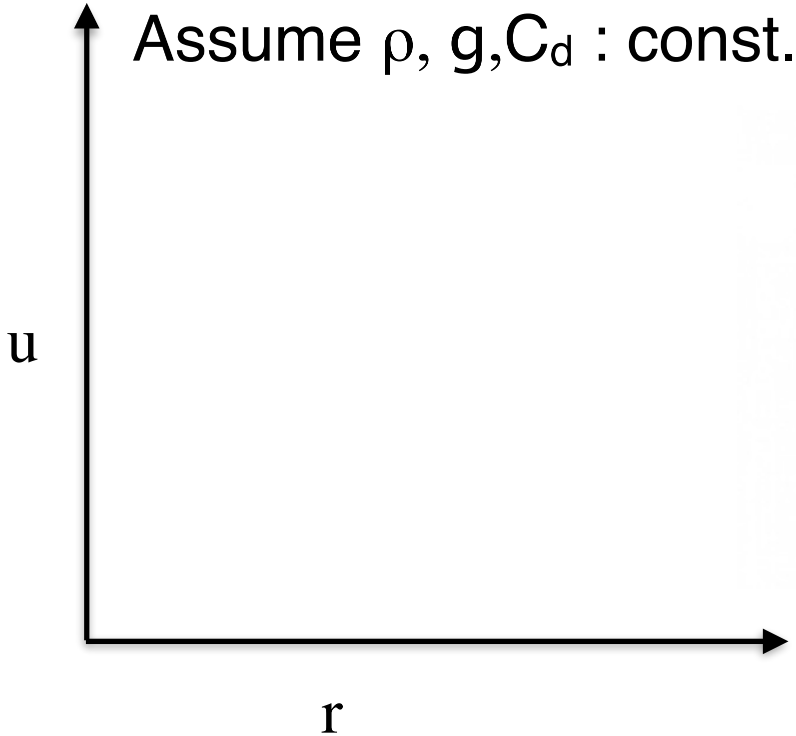
$m\ g$



$$m\ g = 0.5\ C_d\ \rho\ S\ u^2$$

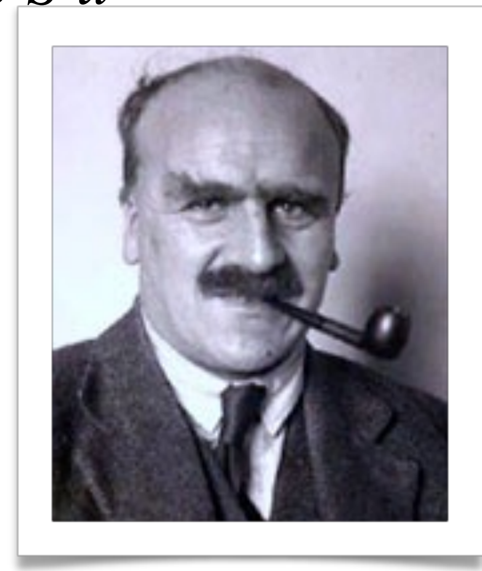
$$m = \rho\, V = \rho\, \frac{4}{3}\pi r^3 \qquad S = \pi r^2$$

Assume ρ, g, C_d : const.



$$D = 0.5\, C_d\, \rho\, S\, u^2$$

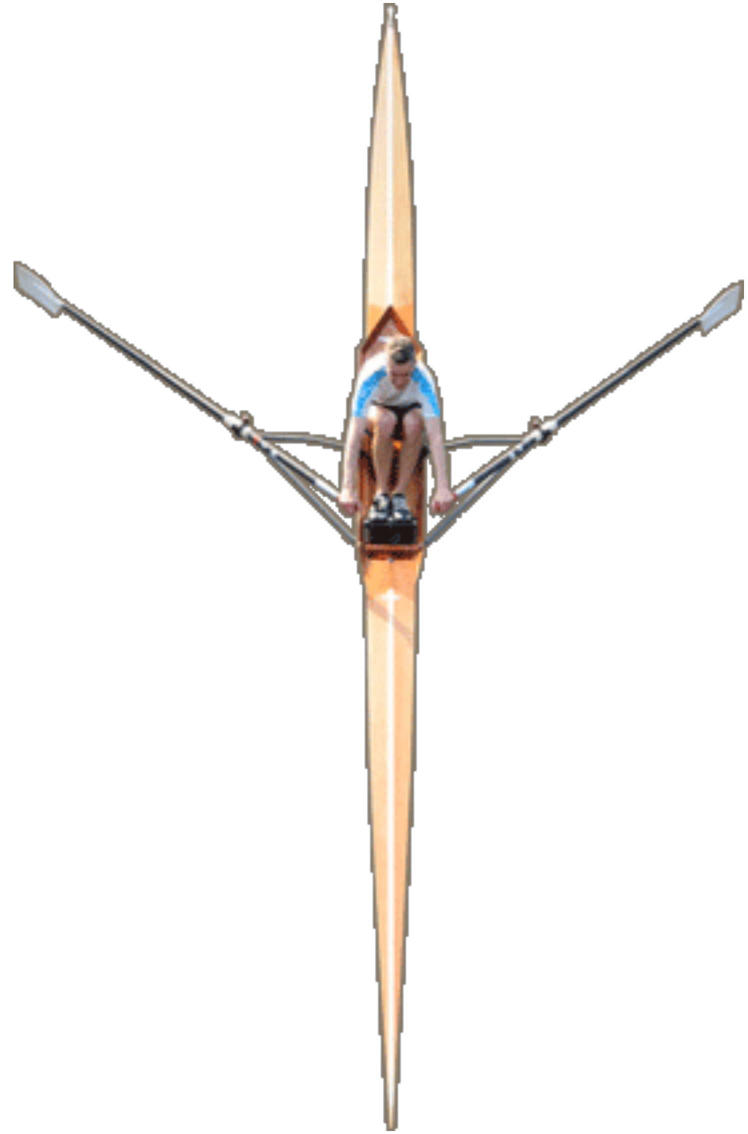
$m\, g$



$$m\, g = 0.5\, C_d\, \rho\, S\, u^2$$

$$m = \rho\, V = \rho\, \frac{4}{3}\pi r^3 \qquad S = \pi r^2$$

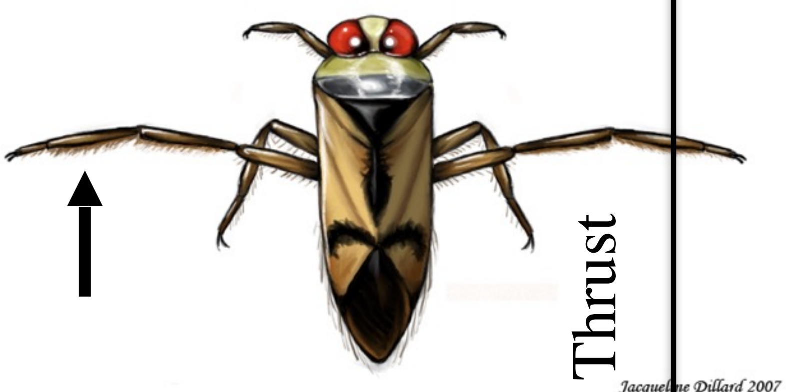
Drag is a propulsive force

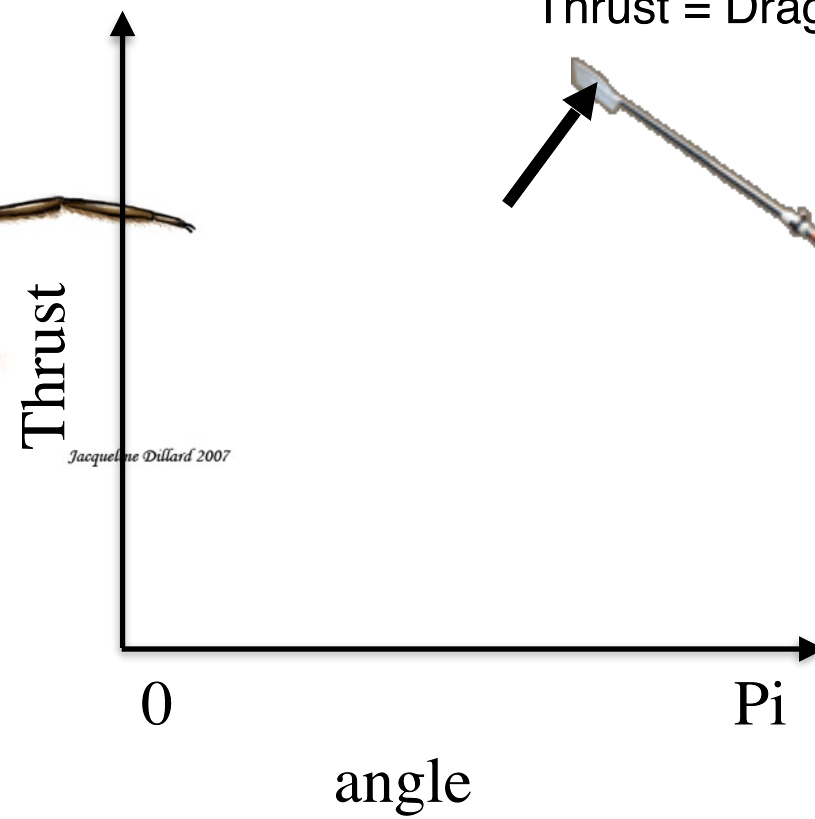


Drag is a propulsive force
Drag on oar generates thrust

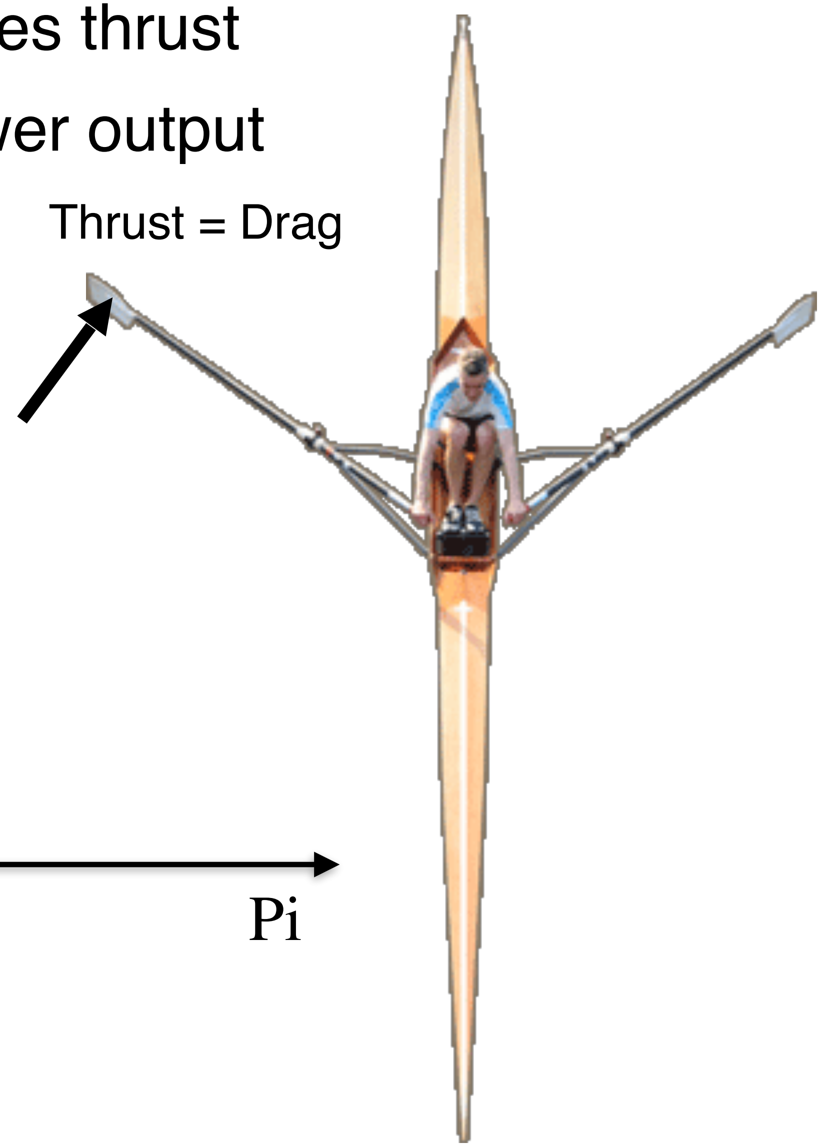
Drag * Velocity = Power output

Thrust = Drag

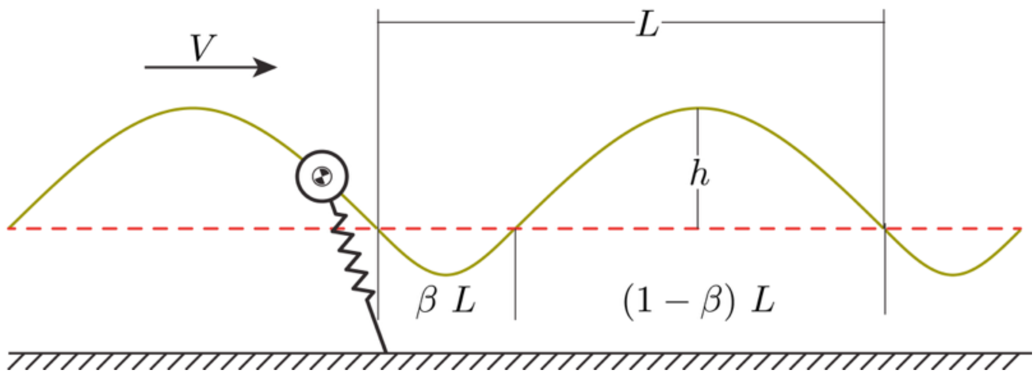




Thrust = Drag



When does drag matter?



$$T = \frac{L}{V}$$

$$T_{\text{stance}} = \frac{\beta\ L}{V} \qquad T_{\text{air}} = \frac{(1-\beta)\ L}{V}$$

$$h = \frac{1}{2} g (\frac{T_{\text{air}}}{2})^2 = (1-\beta)^2 \frac{m\ g\ L^2}{8\ V^2}$$

Energy

$$W = PE + KE$$

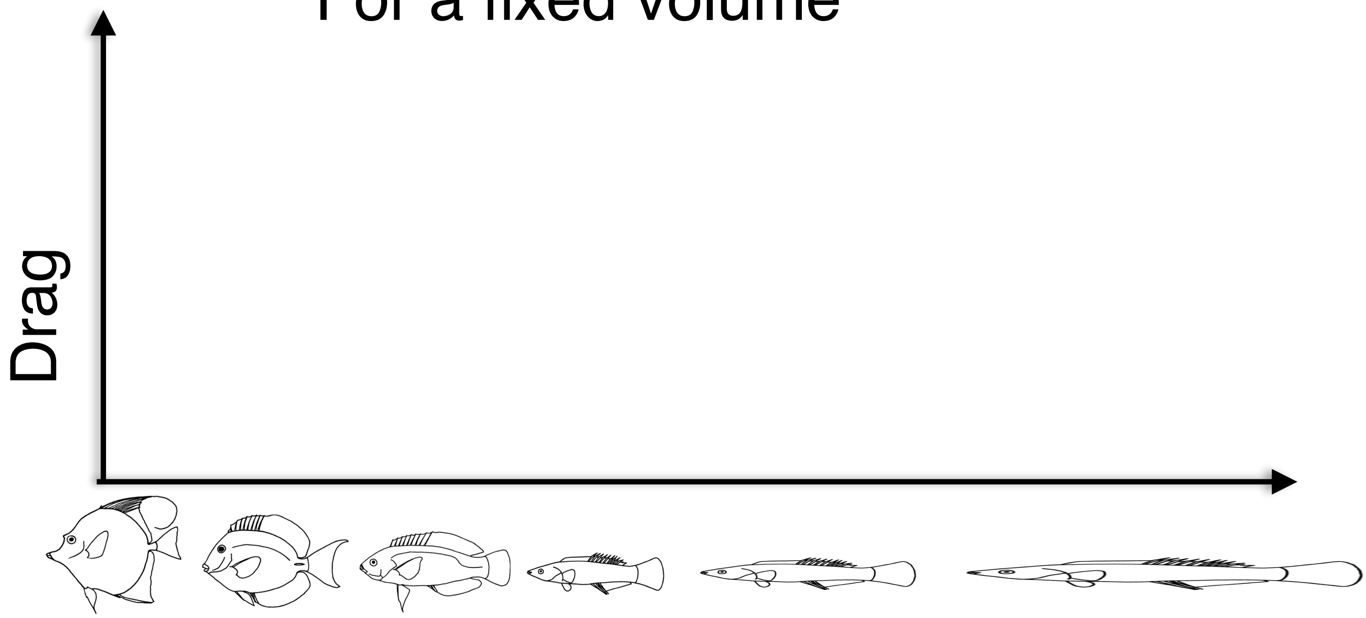
$$PE = m\ g\ h = (1-\beta)^2 \frac{m\ g^2\ L^2}{8\ V^2}$$

$$KE = \frac{1}{2} m\ V^2$$

What is the average power per stride if we consider drag?

$$(1-\beta)^2 \frac{mg^2 L}{8V} + \frac{mV^3}{2L} + \frac{1}{2}\rho S C_d V^3$$

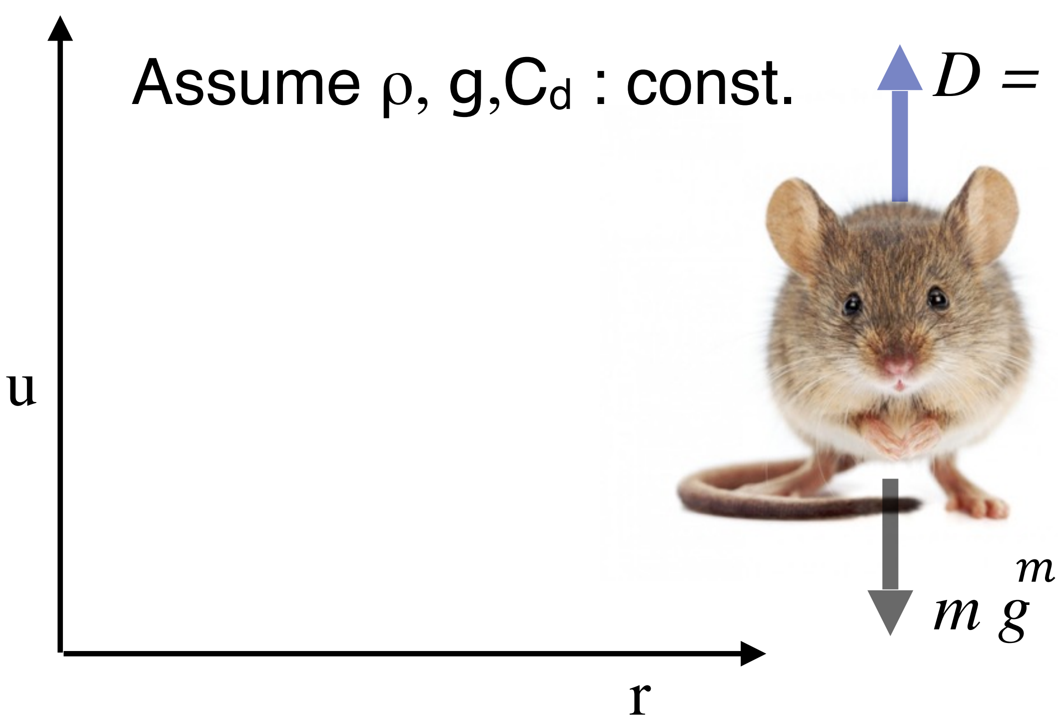
For a fixed volume



Assume ρ, g, C_d : const.

$$D = 0.5\, C_d\, \rho\, S\, u^2$$

$$S = \pi r^2$$



$$m\, g$$

$$m = \rho\, V = \rho\, \frac{4}{3}\pi r^3$$