# Zoology 427 Biomechanics
## Lecture 22. Internal biofluiddynamics: the world is your vessel....

- Recap external flows
- Basic rules for internal flows ( biological plumbing)
  velocity profiles in tubes
- Volumetric flow rate and the Hagen-Poiseuille law:

$$Q = \pi R^4 \Delta P/8\mu L$$

- Case study: asthmatic constrictions
- Case study: atherosclerosis

# Where you've been

Newton's law of viscosity

Conservation of mass

Conservation of energy, Bernoulli

Drag

Lift

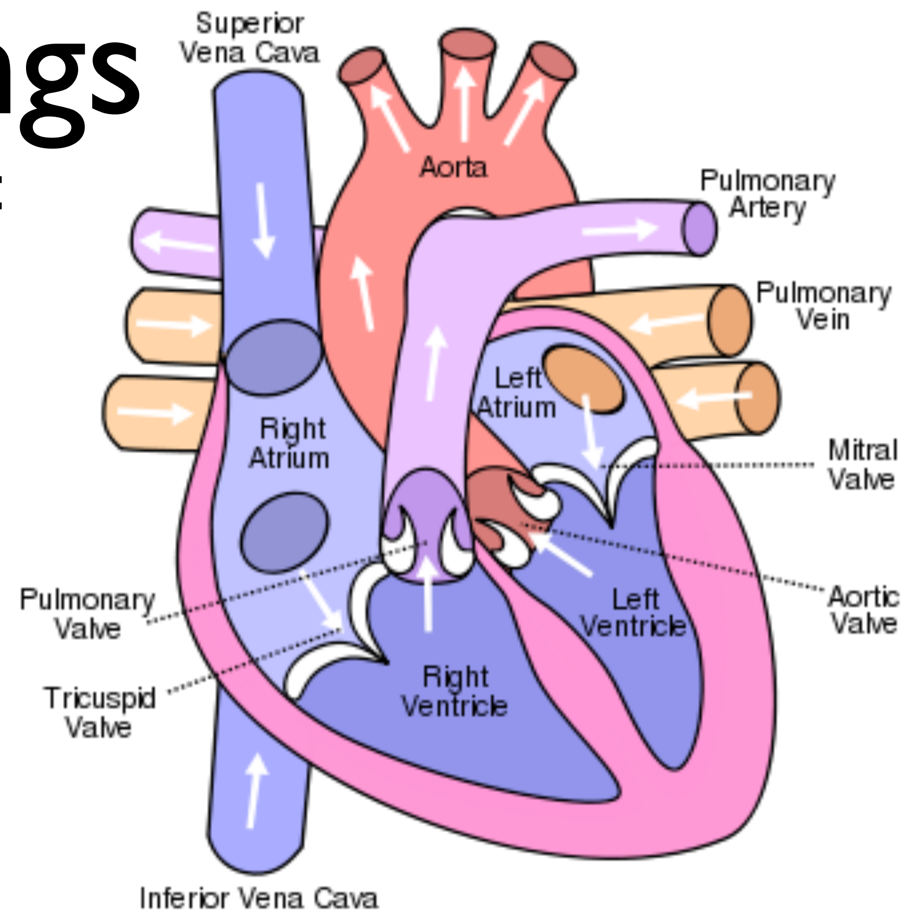
Flight

## Lift propels

# Flow inside things

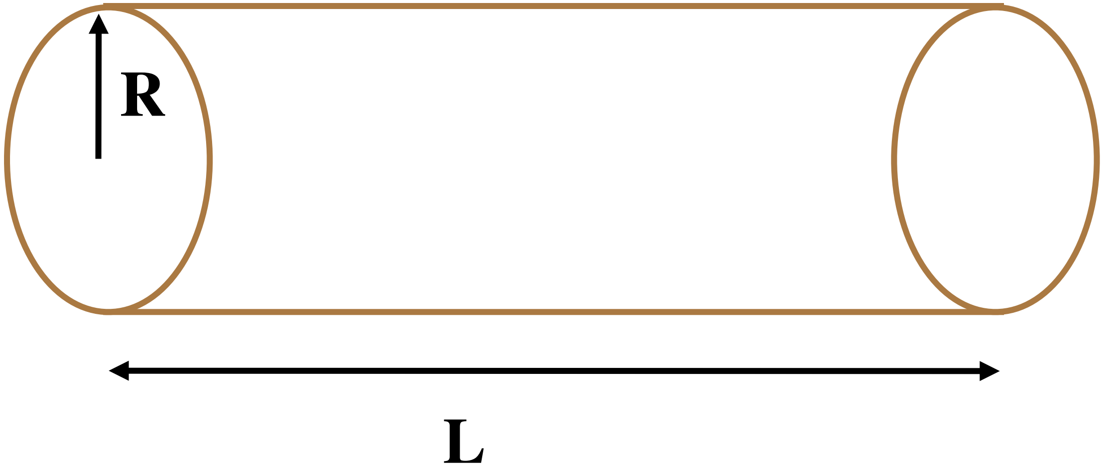
Internal biological fluid dynamics:

Blood flow
Respiratory flow
Flow of synovial joints
Flow in burrows
Any other examples?



# Rigid circular vessel of constant radius R and length L

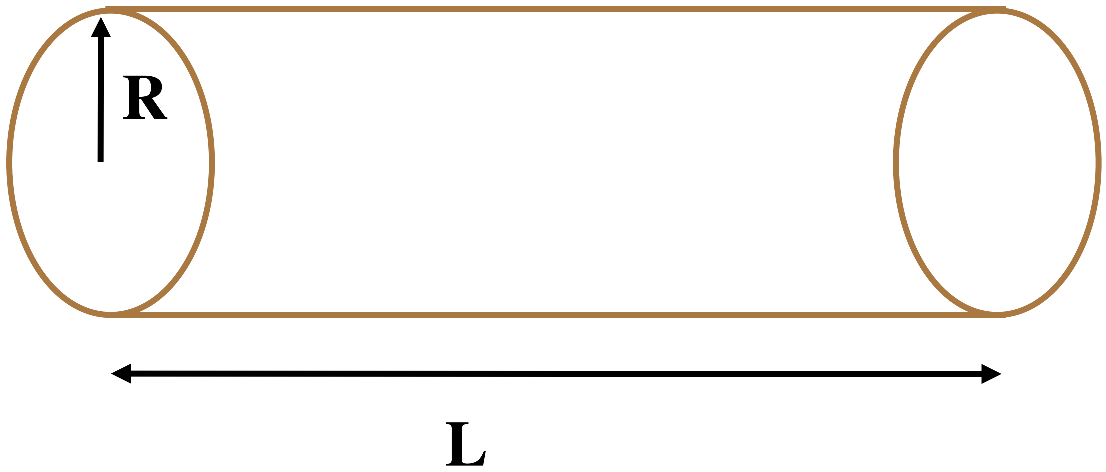
Where is the flow maximum? What is the velocity profile?




| | |
|---|---|
| Steady flow: | $u(r) = \text{const}$, $du/dt = 0$ |
| Pressure gradient | $\Delta P/L$ $(dP/dx)$ |
| Newtonian flow : | $\tau = \mu\, du/dr$ $(0 \leq r \leq R)$ |
| No slip condition: | $u(r) = 0$ at $r = R$ |

Rigid circular vessel of constant radius R and length L

$Q = \pi R^4 \Delta P / 8\mu L$  Hagen-Poiseuille law

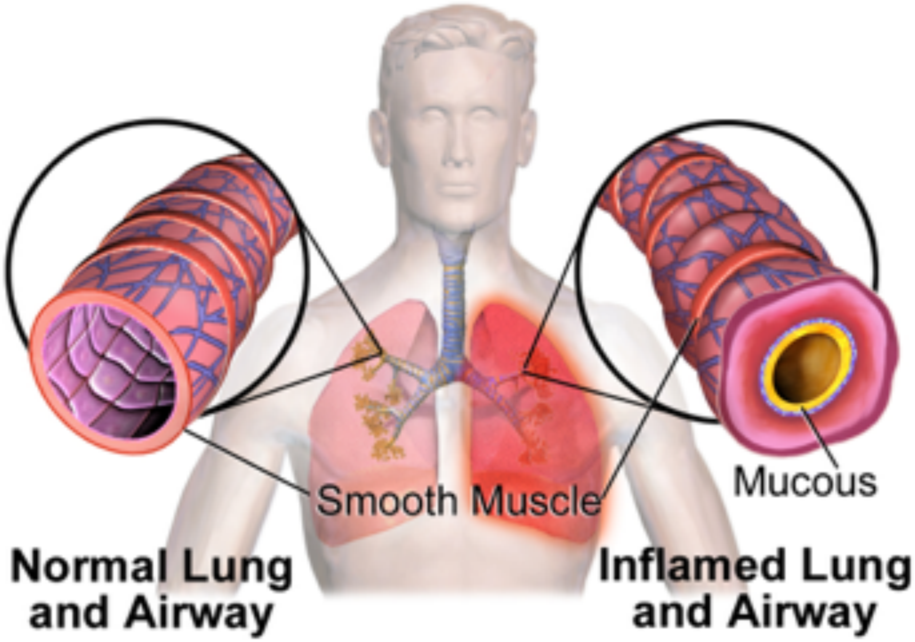



$u(r) = \Delta P\, R^2 (1 - (r/R)^2) / 4\, \mu\, L$

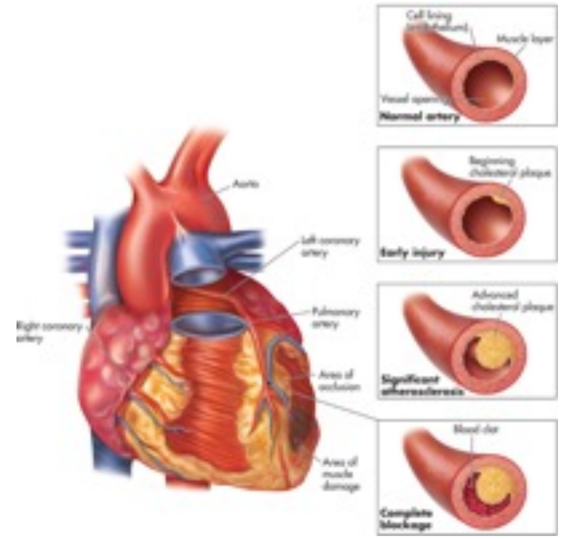
Volume flow rate (volume/time)

$Q = \iint u(r)\, r\, dr\, d\theta$

$Q = \pi R^4 \Delta P/8\mu L$ Hagen-Poiseuille law



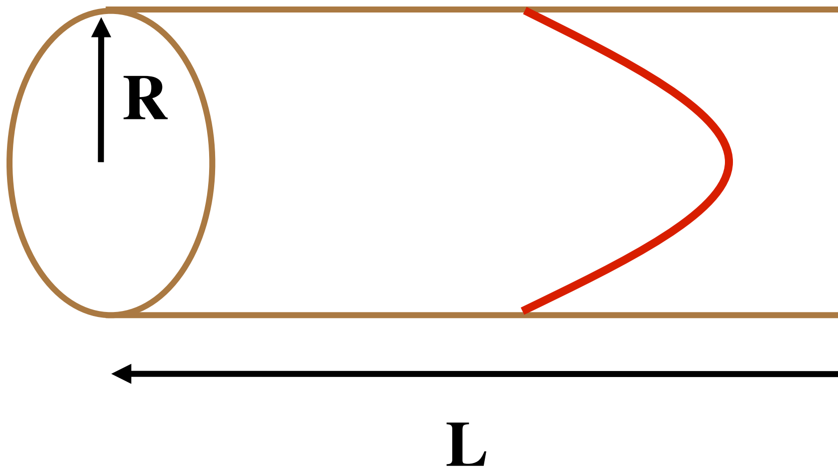

Asthma

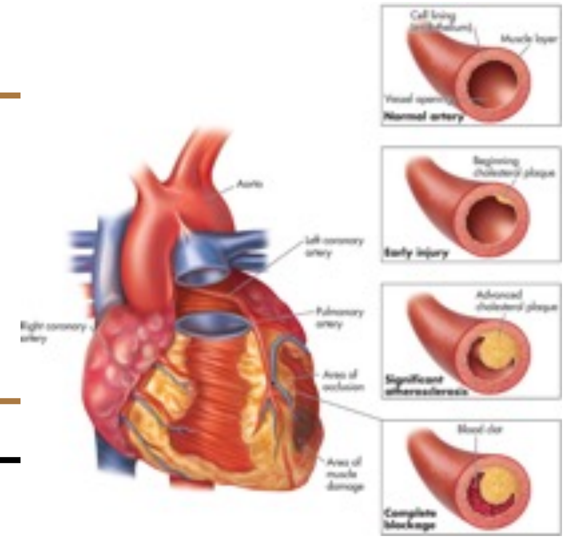


How much is the flow reduced (for a constant pressure) with a 25% change reduction in the internal radius?

Atherosclerosis

$Q = \pi R^4 \Delta P / 8\mu L$ Hagen-Poiseuille law



$$u(r) = \Delta P\,(R^2 - r^2)/4\,\mu\,L$$

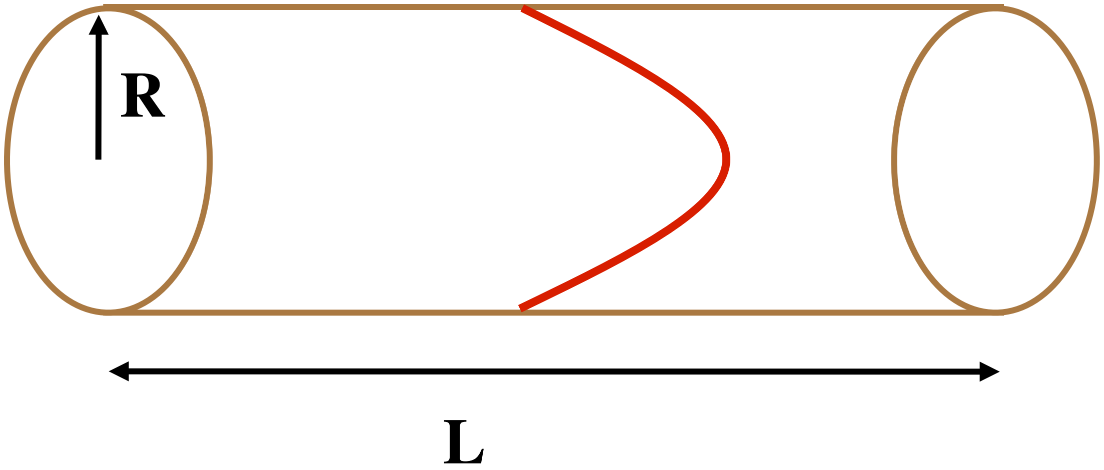
$$\tau = \mu\, du/dr = -\Delta P\, r/\,2\,L \qquad \tau = -\Delta P\, R/\,2\,L$$

$$\tau = -Q\, 2\mu/\pi R^3$$

Atherosclerosis

Rigid circular vessel of constant radius R and length L

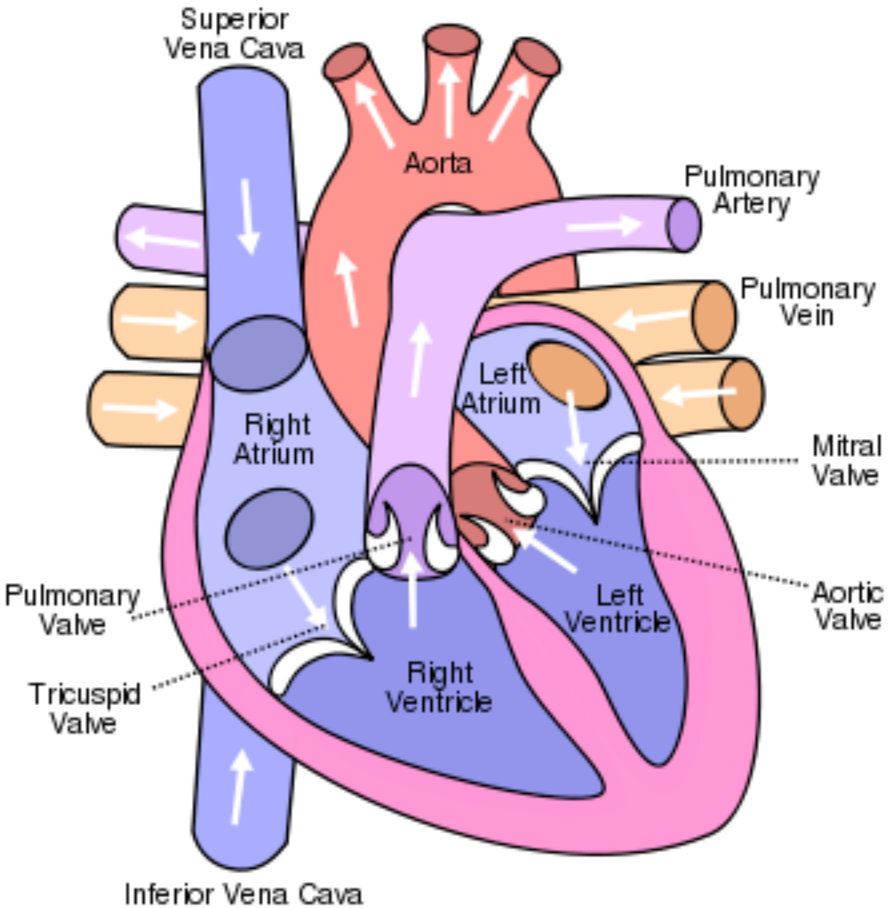
$Q = \pi R^4 \Delta P / 8\mu L$ Hagen-Poiseuille law




$u(r) = \Delta P R^2 (1 - (r/R)^2) / 4 \mu L$

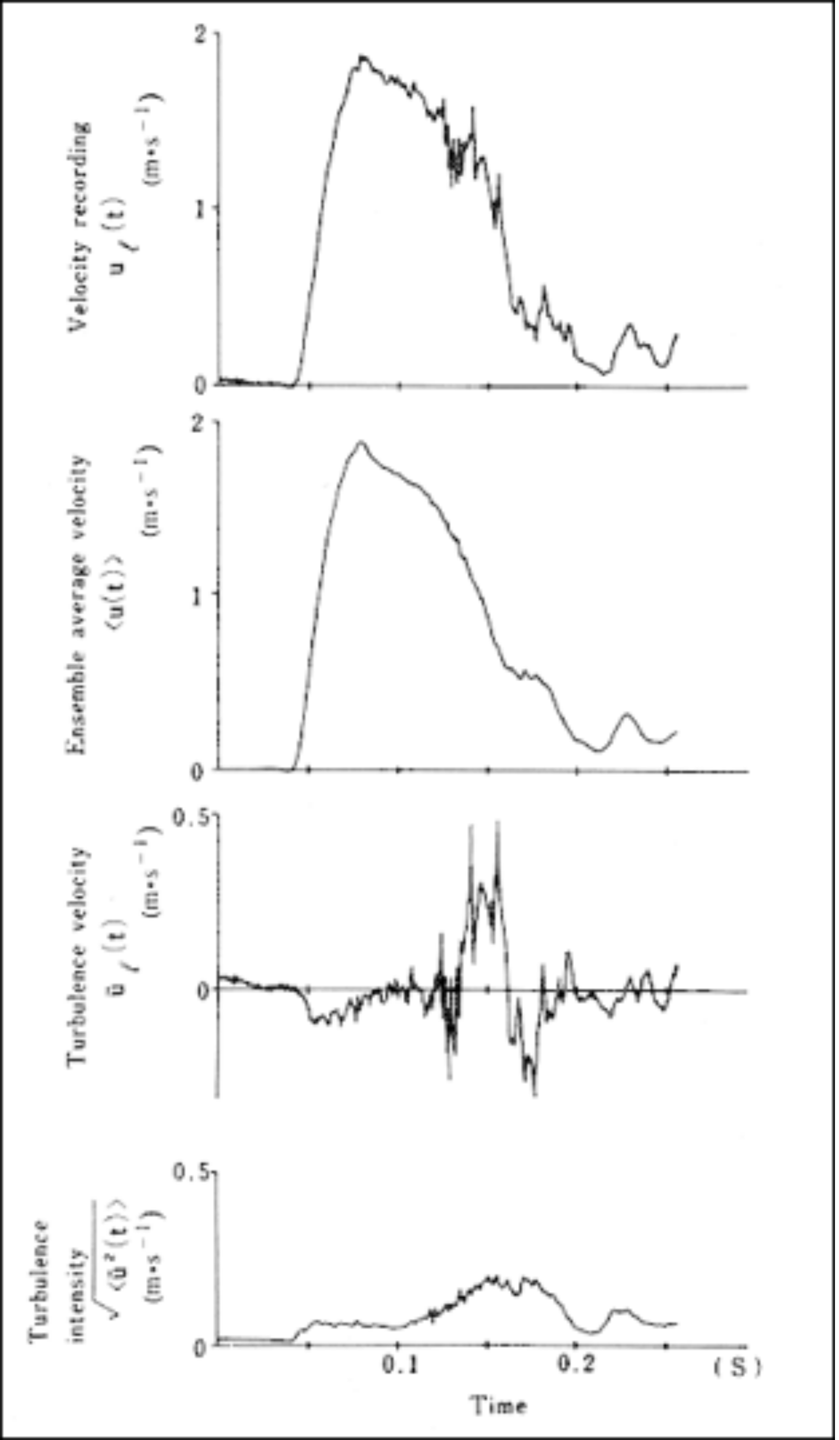
Pumping power = $\Delta P \, dV/dt = \Delta P \, Q$

Why might the pressure required to move blood through the aorta be much greater than a straight tube of equivalent length?




Equivalent lengths

1.0 straight tube
1.4 45 deg bend
1.4 T junction
2.5 90 deg. bend




Turbulence and unsteadiness

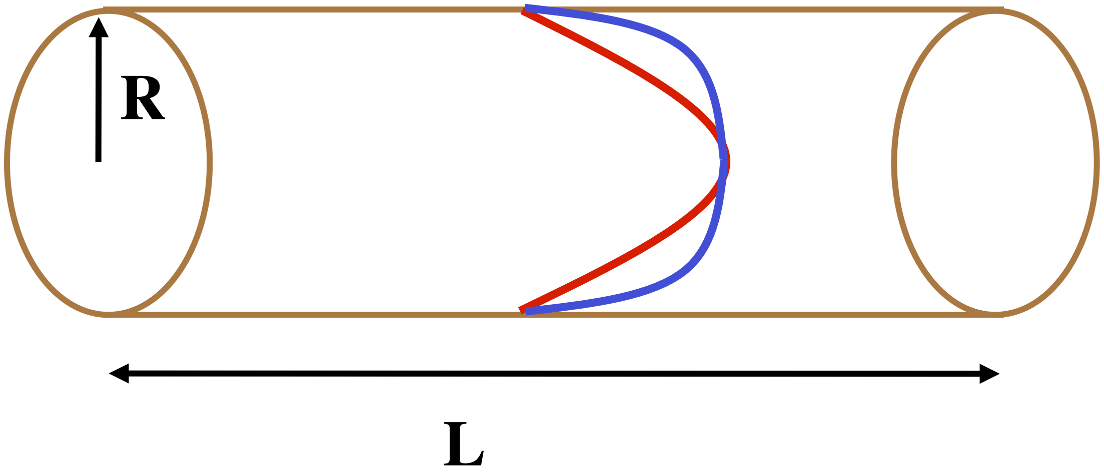
Womersely Number

$Wo = 2 \pi f D/U$

when large, acceleration effects are important

Pulse pressure is a function of accelerations and:
bends (a 180 bend acts like a 5 in the length of tube)

Rigid circular vessel of constant radius R




$Q = \pi R^4 \Delta P / 8\mu L$  Hagen-Poiseuille law

Valid until the flow is turbulent!

$Re = UD/\nu$

($Re > \sim 2000$)