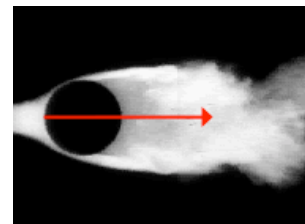
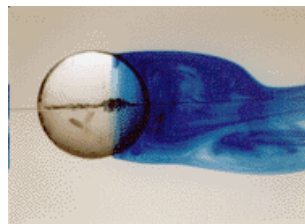
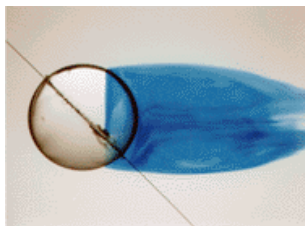
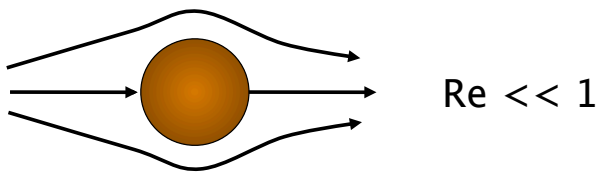


Zoology 427 Biomechanics 2011  
Lecture 19. Shape and drag

- Recap drag and the Reynolds number
- Drag and its coefficient
- How size, shape, and the Reynolds number determine drag
- Streamlined bodies and swimming energetics
- When does drag matter? A return to earlier biomechanical problems.

The Reynolds number measures the relative importance of inertial and viscous stresses in determining the flow. Conservation of Re implies identical flow patterns



$$\text{Pressure stress: } \frac{P \sim \rho u^2}{\text{Shear stress: } \tau \sim \mu \frac{u}{L}} = \text{Re} = \frac{\rho u L}{\mu}$$

Pressure stress:  $\frac{P \sim \rho u^2}{\tau \sim \mu \quad u/L} = \text{Re} = \frac{\rho u L}{\mu}$

Bacteria  $\sim 10^{-5}$

Spermatozoa  $\sim 10^{-4}$

Ciliate  $\sim 10^{-1}$

Smallest Fish  $\sim 1$

Blood flow in brain  $\sim 1 \times 10^2$

Blood flow in aorta  $\sim 1 \times 10^3$

Birds flying  $1 \times 10^4$

Typical pitch in Major League Baseball  $\sim 2 \times 10^5$

Person swimming  $\sim 4 \times 10^6$

Fastest Fish  $\sim 10^6$

Blue Whale  $\sim 3 \times 10^8$

A large ship (RMS Queen Elizabeth 2)  $\sim 5 \times 10^9$

Drag arises from pressure and frictional stresses

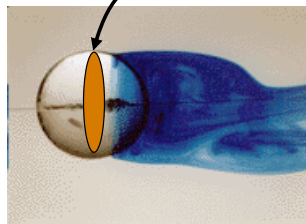
a force  $D \sim \rho S u^2/2$

$$D = C_d \rho S u^2/2$$

drag coefficient

projected surface area

shape



size

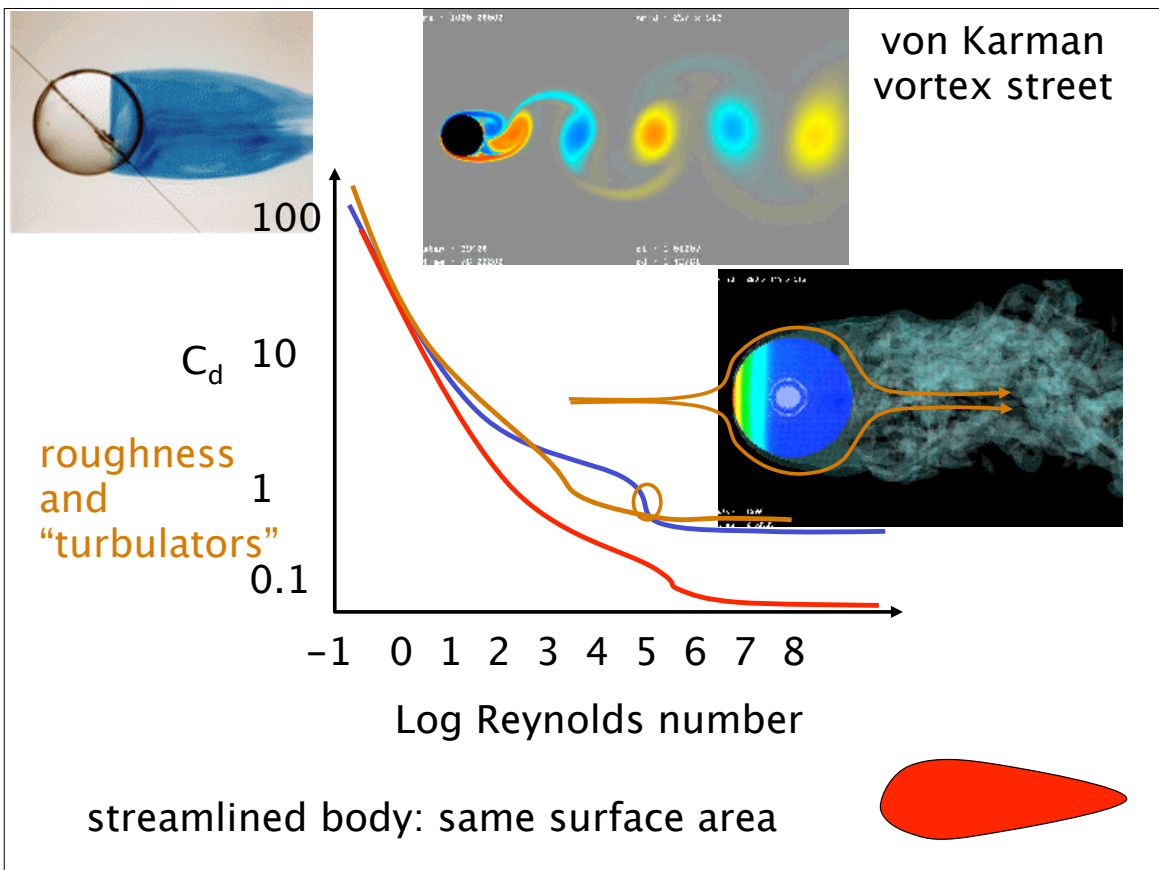
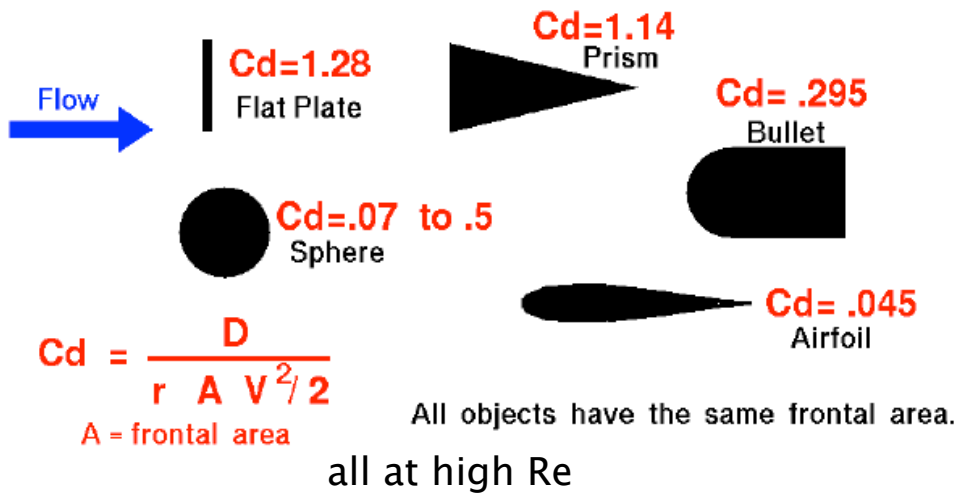
Pressure stress:  $\frac{P \sim \rho u^2}{\tau \sim \mu \quad u/L} = \text{Re} = \frac{\rho u L}{\mu}$



## Shape Effects on Drag

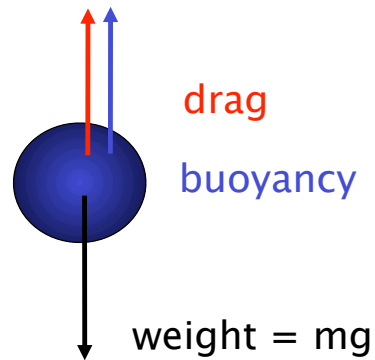
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Research  
Center

The shape of an object has a very great effect on the amount of drag.



$$D = 0.5 C_d \rho S u^2$$

a terminal situation



at equilibrium

$$mg = 0.5 C_d \rho S u^2 + V \rho g$$

$$V \rho_c g = 0.5 C_d \rho S u^2 + V \rho g$$

$$4\pi r^3 \rho_c g/3 = 0.5 C_d \rho \pi r^2 u^2 + 4\pi r^3 \rho g/3$$

$$4r (\rho_c - \rho)g/3 = 0.5 C_d \rho u^2$$

Consequences to porpoising?

$$\text{Fluid Dynamic Power} = D u = 0.5 C_d \rho S u^3$$



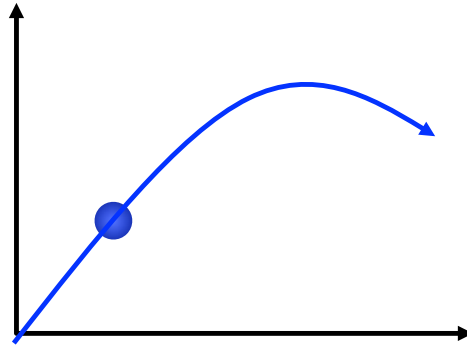
$$\text{Power} = 0.5 C_d \rho S (u_{\text{vertical}}^3 + u_{\text{horizontal}}^3) \text{ in water phase} \\ \sim 0 \text{ in air phase}$$

How do aerodynamic forces affect biological processes?

- Spores illustrated
- Small running creatures

$$m \frac{du_x}{dt} = -0.5 \rho S C_d (u_x^2 + u_y^2)$$
$$m \frac{du_y}{dt} = -m g - 0.5 \rho S C_d (u_x^2 + u_y^2)$$

$$C_d = 24/Re + 1/(1+Re^5) + 0.4$$



How do aerodynamic forces affect biological processes?

- Spores illustrated
- Small running creatures

Mathematica

DEMO

How do aerodynamic forces affect biological processes?

- Spores illustrated
- Small running creatures

How do aerodynamic forces affect running energetics?



$$\begin{aligned} \text{Power} &= (1 + \gamma) M g^2 b / 8V + m V^3 / 2(a+b) \\ &= (1 + \gamma) M g^2 a j / 8V + m V^3 / 2a(1+j) \\ &\quad + 0.5 \rho S C_d(V) V^3 \end{aligned}$$

How does Power vary with  
 foot mass (m)?  
 body mass (M)?  
 body velocity (V)?  
 leg length (~a)?  
 jumpiness (j)?  
 Drag?

DEMO

