Queuing

CEE 320
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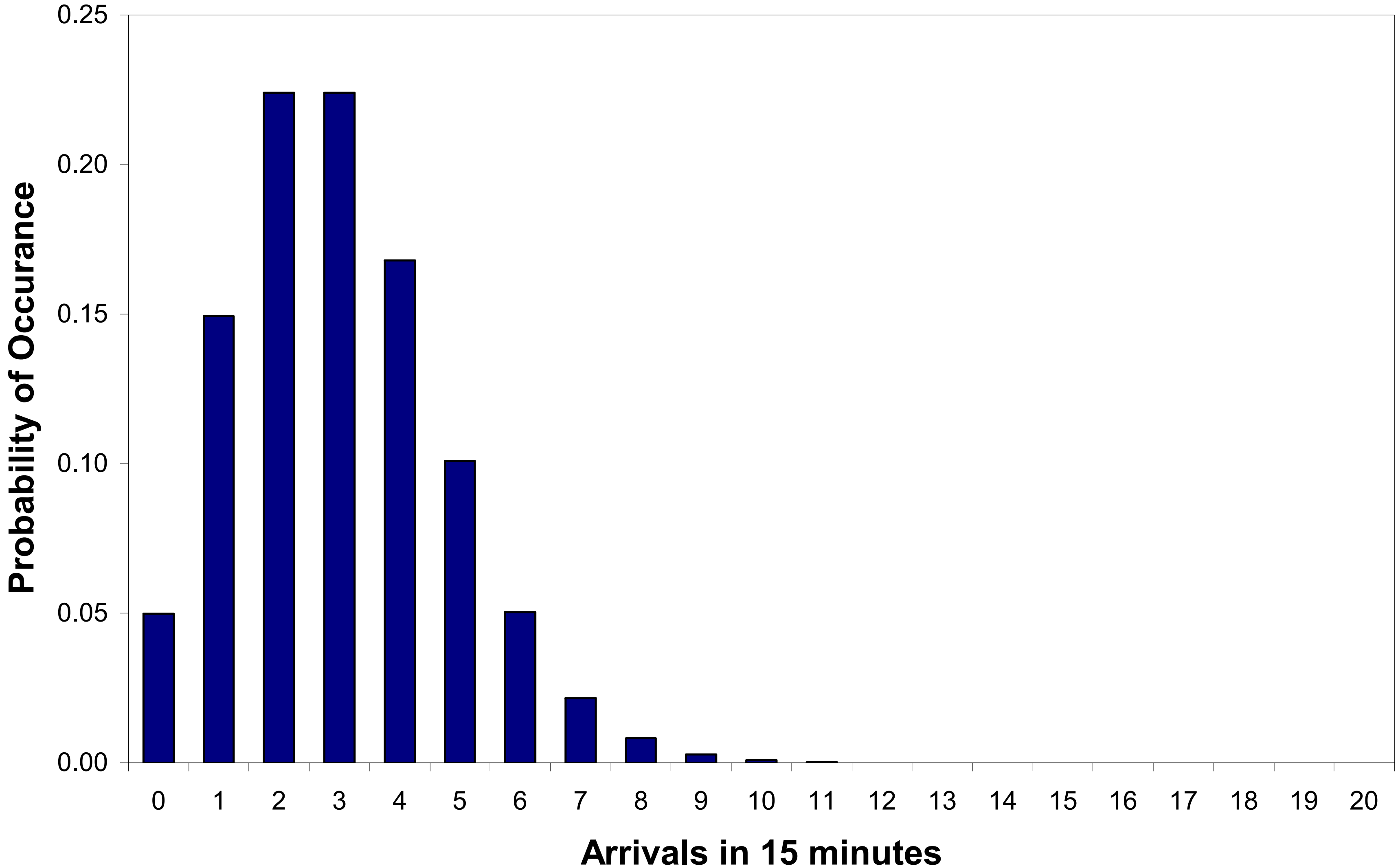
Poisson Distribution

- **Good for modeling random events**
- **Discrete values**
- **Only one parameter**

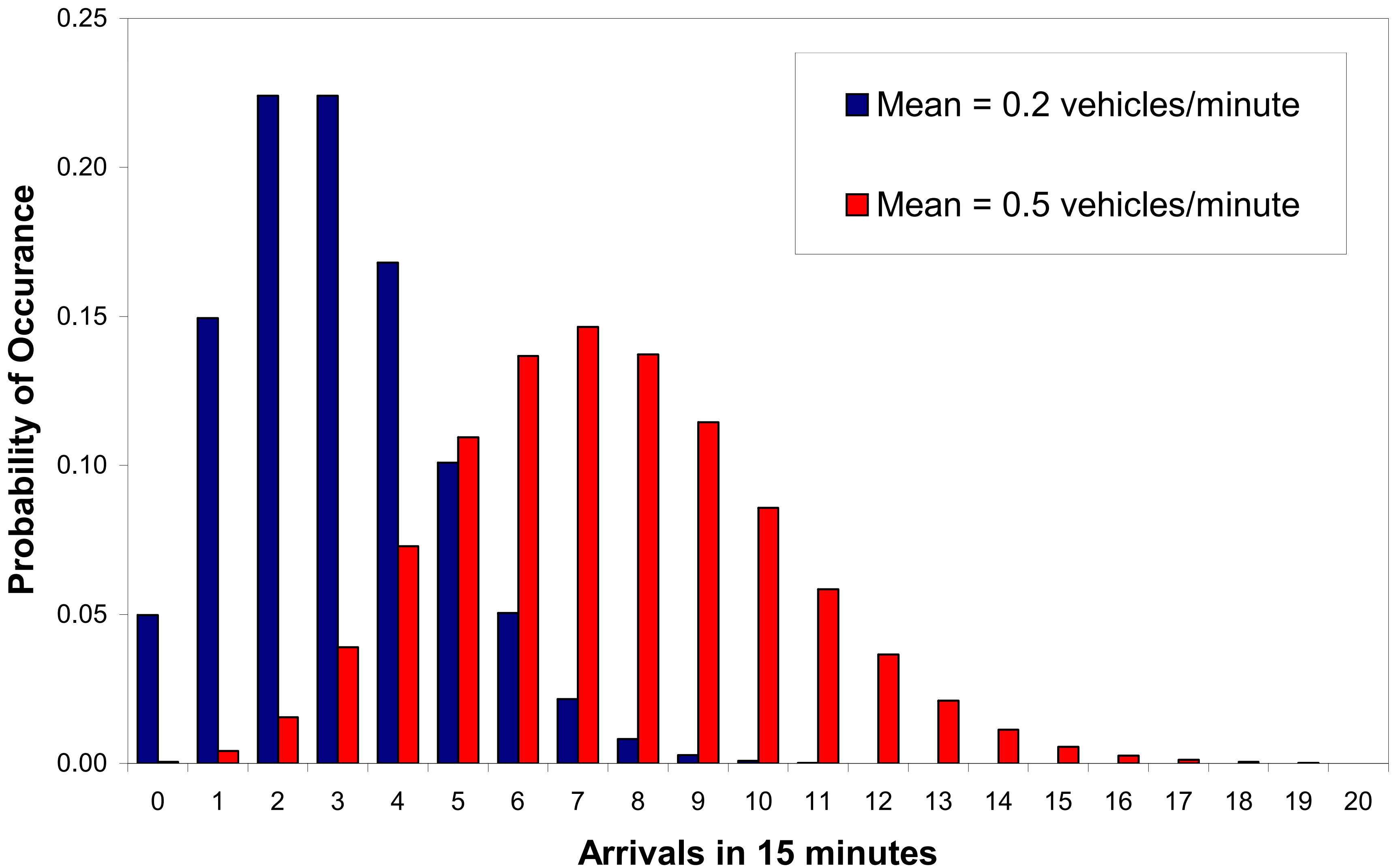
$$P(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

P(n)	=	probability of exactly n vehicles arriving over time t
n	=	number of vehicles arriving over time t
λ	=	average arrival rate and variance
t	=	duration of time over which vehicles are counted

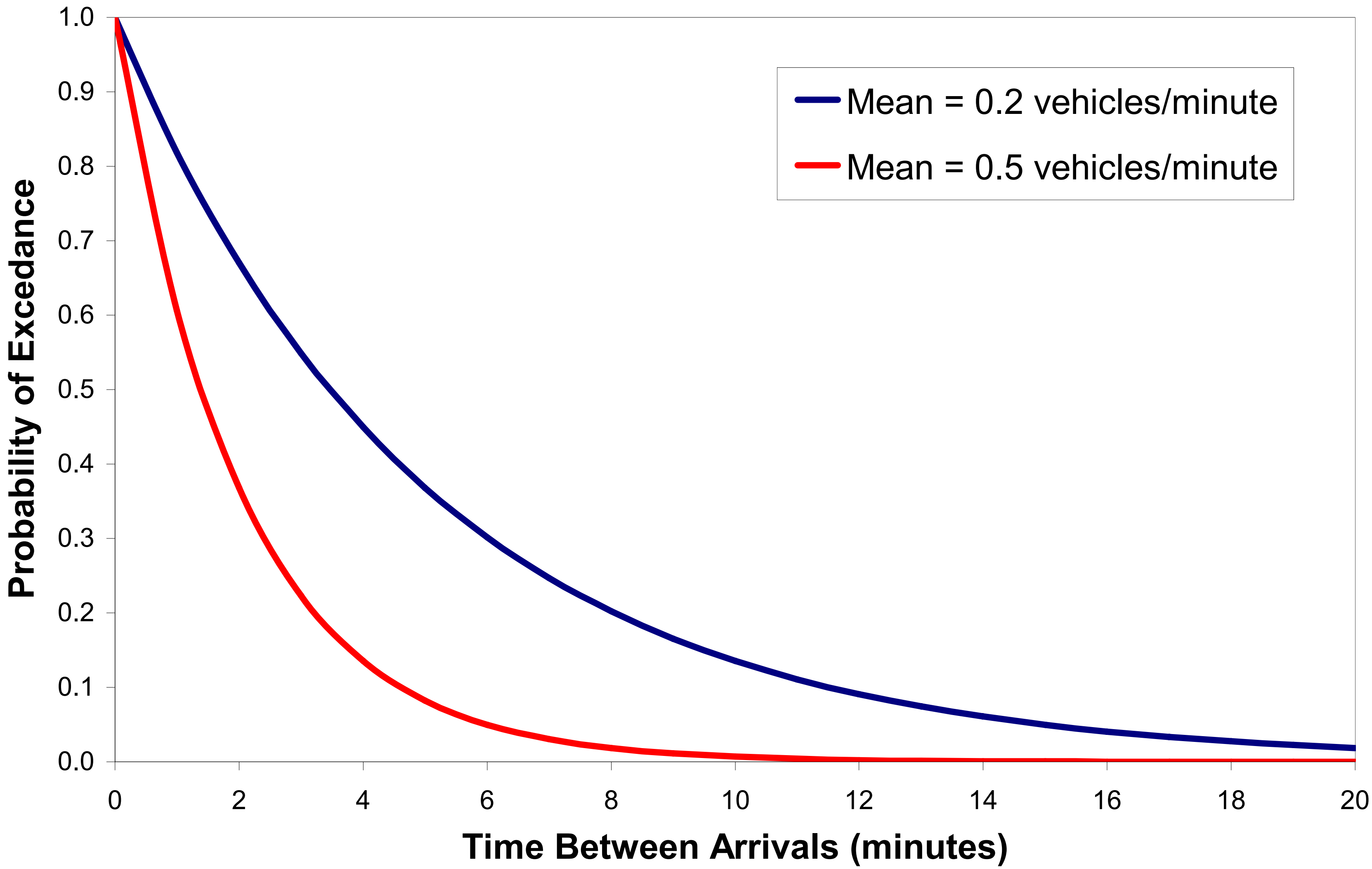
Poisson distribution



Different means



Example: Arrival Intervals



Poisson Ideas

- **Probability of exactly 4 vehicles arriving**
  - **$P(n=4)$**
- **Probability of less than 4 vehicles arriving**
  - **$P(n<4) =$**
- **Probability of 4 or more vehicles arriving**
  - **$P(n\geq 4) =$**
- **Probability of no vehicles arriving**
  - **$P(n=0) =$**

Poisson Distribution Example

Vehicle arrivals at the Olympic National Park main gate are assumed Poisson distributed with an average arrival rate of 1 vehicle every 5 minutes. What is the probability of the following:

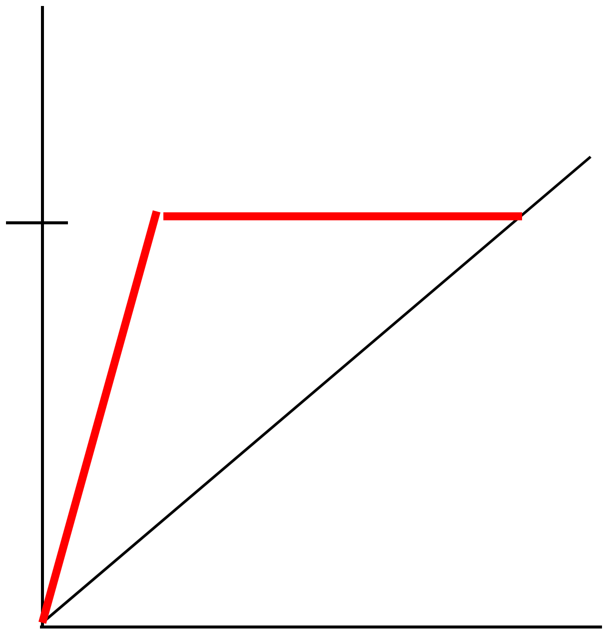
1. Exactly 2 vehicles arrive in a 15 minute interval?

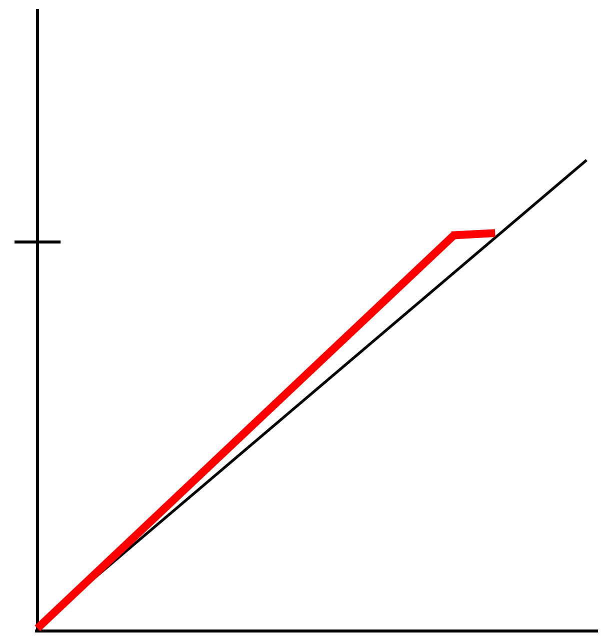
Example Calculations

Less than 2 vehicles arrive in a 15 minute interval?

More than 2 vehicles arrive in a 15 minute interval?

Queue times depend on variability





Steady state assumption

Queue Analysis – Numerical

$$\rho = \frac{\lambda}{\mu} \qquad \rho < 1.0$$

- **M/D/1**
  - **Average length of queue** $\overline{Q} = \frac{\rho^2}{2(1-\rho)}$
  - **Average time waiting in queue** $\overline{w} = \frac{1}{2\mu}\left(\frac{\rho}{1-\rho}\right)$
  - **Average time spent in <u>system</u>** $\bar{t} = \frac{1}{2\mu}\left(\frac{2-\rho}{1-\rho}\right)$

λ = arrival rate μ = departure rate ρ=traffic intensity

Queue Analysis – Numerical

$$\rho = \frac{\lambda}{\mu} \qquad \rho < 1.0$$

- **D/D/1**
  - **Average length of queue**
  - **Average time waiting in queue**
  - **Average time spent in <u>system</u>**

λ = arrival rate μ = departure rate ρ=traffic intensity

Queue Analysis – Numerical

$$\rho = \frac{\lambda}{\mu} \qquad \rho/N < 1.0$$

- **M/M/N**
  - **Average length of queue** $$\overline{Q} = \frac{P_0 \rho^{N+1}}{N!N}\left[\frac{1}{(1-\rho/N)^2}\right]$$
  - **Average time waiting in queue** $$\overline{w} = \frac{\rho + \overline{Q}}{\lambda} - \frac{1}{\mu}$$
  - **Average time spent in system** $$\bar{t} = \frac{\rho + \overline{Q}}{\lambda}$$

λ = arrival rate (all traffic), μ = departure rate (one server), ρ=traffic intensity, N=departure channels, n=vehicles

M/M/N – More Stuff

$\rho = \frac{\lambda}{\mu} \qquad \rho/N < 1.0$

– **Probability of having no vehicles in system**

$$P_0 = \frac{1}{\sum_{n_c=0}^{N-1} \frac{\rho^{n_c}}{n_c!} + \frac{\rho^N}{N!(1-\rho/N)}}$$

– **Probability of having *n* vehicles in the system**

$$P_n = \frac{\rho^n P_0}{n!} \quad \text{for n} \le \text{N} \qquad\qquad P_n = \frac{\rho^n P_0}{N^{n-N} N!} \quad \text{for n} \ge \text{N}$$

– **Probability of being in a queue**

$$P_{n>N} = \frac{P_0 \rho^{N+1}}{N! N(1-\rho/N)}$$

λ = arrival rate μ = departure rate ρ=traffic intensity

Example

You are entering Bank of America Arena at Hec Edmunson Pavilion to watch a basketball game. There is only one ticket line to purchase tickets. Each ticket purchase takes an average of 18 seconds. The average arrival rate is 3 persons/minute.

Find the average length of queue and average waiting time in queue assuming M/M/1 queuing.

Example

You are now in line to get into the Arena. There are 3 operating turnstiles with one ticket-taker each. On average it takes 3 seconds for a ticket-taker to process your ticket and allow entry. The average arrival rate is 40 persons/minute.

Find the average length of queue, average waiting time in queue assuming M/M/N queuing.

Example

You are now inside the Arena. They are passing out Harry the Husky doggy bags as a free giveaway. There is only one person passing these out and a line has formed behind her. It takes her exactly 6 seconds to hand out a doggy bag and the arrival rate averages 9 people/minute.

Find the average length of queue, average waiting time in queue, and average time spent in the system assuming M/D/1 queuing.