Problem 5.2

Water flows from reservoir A to B. The water temperature in the system is 10 °C, the pipe diameter D is 1 m, and the pipe length L is 300 m. If $H = 16$ m, $h = 2$ m, and if the pipe is steel, what will be the discharge in the pipe? In your solution, sketch hydraulic and energy grade lines. What will be the pressure at point P halfway between the two reservoirs?

Solution:

Write the energy equation from the water surface in A to the water surface in B.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + H = 0 + 0 + 0 + f\frac{L}{D}\frac{V_p^2}{2g} + \frac{V_p^2}{2g} \tag{1}$$

$$\frac{k_s}{D} \sim 0.00004 \quad \text{(from Fig. 5-5)}$$

therefore, first assume $f = 0.011$ (from Fig. 5-4). Rewriting Eq. (1) then gives us

$$V_p = \sqrt{\frac{H}{\left(f\frac{L}{D} + 1\right)}2g} = \sqrt{\frac{16\text{ m}}{\left(0.011\frac{300\text{ m}}{1\text{ m}} + 1\right)} \cdot 2 \cdot 9.81\text{ m/s}^2} = 8.54\text{ m/s}.$$

Check f:

$$\text{Re} = \frac{V_p D}{\nu} = \frac{8.54\text{ m/s} \cdot 1\text{ m}}{1.3 \cdot 10^{-6}\text{ m}^2/\text{s}} = 7 \cdot 10^6$$

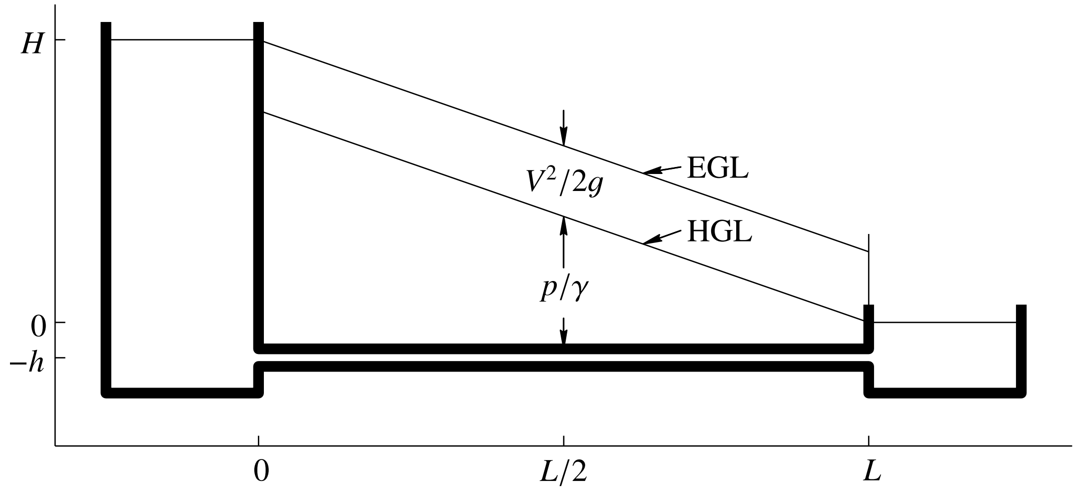
so from Fig. 5-4 $f = 0.010$. Solve for V_p again for better value of f

$$V_p = \sqrt{\frac{H}{\left(f\frac{L}{D} + 1\right)}2g} = \sqrt{\frac{16\text{ m}}{\left(0.010\frac{300\text{ m}}{1\text{ m}} + 1\right)} \cdot 2 \cdot 9.81\text{ m/s}^2} = 8.86\text{ m/s}.$$

$$Q = VA = V_p\frac{D^2\pi}{4} = 8.86\text{ m/s} \cdot \frac{(1\text{ m})^2\pi}{4} = 6.96\text{ m}^3/\text{s}.$$

To determine P_p write the energy equation between the water surface in A and point P:

$$0 + 0 + H = \frac{P_p}{\gamma} + \frac{V_p^2}{2g} - h + f\frac{\frac{1}{2}L}{D}\frac{V_p^2}{2g}$$



Then

$$P_p = \gamma\left(H + h - \frac{V_p^2}{2g}\left(1 + \frac{fL}{2D}\right)\right) = 78480 \text{ Pa} = 78.5 \text{ kPa}$$

Problem 5.3

In a problem like Prob. 5-2, if an error of 10% is made in choosing an f value, what would be the error in Q?

Solution:

In Prob. 5-2 the discharge varies as the first power of V but V varies inversely to the $\frac{1}{2}$ power of f.

$$Q = kf^{-\frac{1}{2}}$$

Then

$$dQ = -\frac{1}{2}kf^{-\frac{3}{2}}df$$

$$\frac{dQ}{Q} = \frac{-\frac{1}{2}kf^{-\frac{3}{2}}df}{kf^{-\frac{1}{2}}}$$

$$\frac{dQ}{Q} = -\frac{1}{2}\frac{df}{f}$$

So a 10% overestimation in f would underestimate Q by 5% and vice versa.

Problem 5.8

What horsepower must be supplied to the water to pump 2.5 cfs at 68 °F from the lower to the upper reservoir? Assume the pipe is steel. Sketch the hydraulic and energy grade lines.

Solution:

$$A = \frac{1}{4}D^2\pi = \frac{1}{4}(8\text{ in})^2 \cdot \pi = 50.3\text{ in}^2 = 0.349\text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{2.5\text{ ft}^3/\text{s}}{349\text{ ft}^2} = 7.16\text{ ft/s}.$$

$$\text{Re} = \frac{VD}{\nu} = \frac{7.16\text{ ft/s}\cdot\frac{2}{3}\text{ ft}}{1.1\cdot 10^{-5}\text{ ft}^2/\text{s}} = 4.3\cdot 10^5$$

$$\frac{k_s}{D} = 0.0002 \quad \text{(from Fig. 5-5)}$$

Then

$$f = 0.016 \quad \text{(from Fig. 5-4)}$$

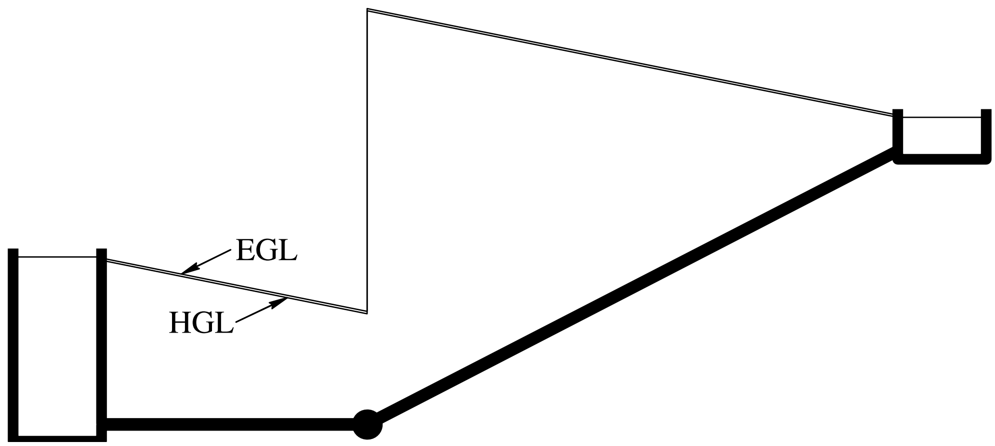
Assume entrance loss coefficient of 0.5 and exit loss coefficient of 1.0. Then

$$h_L = \left(f\frac{L}{D} + 1.5\right)\frac{V^2}{2g} = \left(0.016\frac{3000\text{ ft}}{\frac{2}{3}\text{ ft}} + 1.5\right)\frac{(7.16\text{ ft/s})^2}{2\cdot 32.2\text{ ft/s}^2} = 58.5\text{ ft}$$

Write energy equation from water surface to water surface

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + 100\text{ ft} + h_p = 0 + 0 + 150\text{ ft} + 58.5\text{ ft}.$$



so h_p = 108.5 ft. Power supplied:

$$P = Q\gamma h_p = 2.5 \text{ cfs} \cdot 62.4 \text{ lbs/ft}^3 \cdot 108.5 \text{ ft} = 16900 \text{ ft} \cdot \text{lb/s} = 30.8 \text{ horsepower.}$$

Problem 5.27

a) Determine the discharge of water through the system shown.

b) Draw the hydraulic and energy grade lines for the system.

c) Locate the point of maximum pressure.

d) Locate the point of minimum pressure.

e) Calculate the maximum and minimum pressures in the system.

Solution:

$$\frac{k_s}{D} = 0.004$$

Assume $f = 0.028$ and $r/D \sim 2$ giving $K_b \sim 0.2$.

a) $$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \sum h_L$$

$$0 + 100 \text{ ft} + 0 = 0 + 64 \text{ ft} + \left(1 + 0.5 + K_b + f\frac{L}{D}\right)\frac{V_2^2}{2g}$$

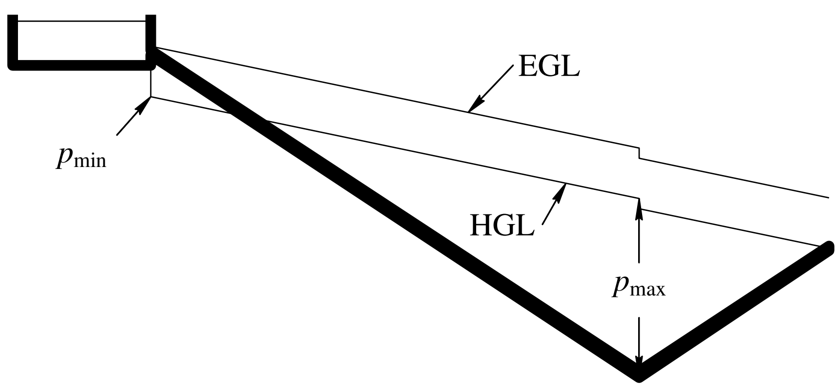
$$\frac{V_2^2}{2g} = \frac{100 \text{ ft} - 64 \text{ ft}}{1 + 0.5 + 0.2 + 0.028\frac{72 \text{ ft}+28 \text{ ft}}{1 \text{ ft}}} = 8.0 \text{ ft}$$

$$V_2 = \sqrt{8 \text{ ft} \cdot 2 \cdot 32.2 \text{ ft/s}^2} = 22.7 \text{ ft/s}.$$

$$\text{Re} = \frac{VD}{\nu} = \frac{22.7 \text{ ft/s} \cdot 1 \text{ ft}}{1.22 \cdot 10^{-5} \text{ ft}^2\text{/s}} = 1.9 \cdot 10^6, \quad f = 0.028$$

$$Q = VA = 22.7 \text{ ft/s} \cdot \frac{\pi}{4} \text{ ft}^2 = 17.8 \text{ cfs}$$

b) HGL and EGL sketch



c) Maximum pressure occurs where the HGL rises the highest above the pipe. See figure above.

d) Minimum pressure occurs where the HGL goes the furthest below the pipe. See figure above.

e) $$\frac{p_{\min}}{\gamma} = 100 \text{ ft} - 95 \text{ ft} - \frac{V^2}{2g}(1 + 0.5) = -7 \text{ ft}$$

$$p_{\min} = 7 \text{ ft} \cdot 62.4 \text{ lbs/ft}^3 = -437 \text{ lbs/ft}^2 = -3.03 \text{ psi gauge}$$

$$\frac{p_{\max}}{\gamma} + \frac{V_b^2}{2g} + z_b = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g} + \sum h_L$$

where subscript b refers to the bend. Then

$$\frac{p_{\max}}{\gamma} = 64 \text{ ft} - 44 \text{ ft} + 8 \text{ ft}\left(0.2 + 0.028 \cdot \frac{28 \text{ ft}}{1 \text{ ft}}\right) = 27.9 \text{ ft}$$

$$p_{\max} = 27.9 \text{ ft} \cdot 62.4 \text{ lbs/ft}^3 = 1736 \text{ lbs/ft}^2 = 12.1 \text{ psi gauge}$$

Problem 5.33

Assuming $f = 0.020$, determine the discharge in the pipes. Neglect minor losses.

Solution:

Continuity:

$$Q_{AB} + Q_{AC} = Q_{BD}$$

Head loss:

$$h_f = f\frac{L}{D}\frac{Q^2}{2gA^2} \quad \text{or} \quad Q = k\sqrt{h_f}$$

where

$$k = \sqrt{\frac{g\pi^2 D^5}{8fL}}.$$

Calculating this for value for each pipe we get $k_{AB} = 0.283 \text{ m}^{5/2}/\text{s}$, $k_{CB} = 0.256 \text{ m}^{5/2}/\text{s}$ and $k_{BD} = 0.630 \text{ m}^{5/2}/\text{s}$. We also know that

$$h_{f,AB} = h_A - h_B, \quad h_{f,CB} = h_C - h_B \quad \text{and} \quad h_{f,BD} = h_B - h_D$$

So by assuming a value for h_b, we can calculate Q for each pipe and check against the continuity equation, and then iterate until we get everything to match.

h_B ft	$h_{f,AB}$ ft	Q_{AB} cfs	$h_{f,CB}$ ft	Q_{CB} cfs	$h_{f,BD}$ ft	Q_{BD} cfs	$Q_{AB} + Q_{CB} - Q_{BD}$ cfs
50.0	100.0	2.83	50.0	1.81	50.0	4.46	0.18
55.0	95.0	2.75	45.0	1.72	55.0	4.67	-0.21
52.5	97.5	2.79	47.5	1.76	52.5	4.57	-0.01
52.3	97.7	2.79	47.7	1.77	52.3	4.56	0.00

Problem 5.37

This manifold is used to discharge heated effluent from a power plant into the Columbia River. There are ten discharge pipes spaced 10 ft apart and the end pipe is to discharge water at a rate of 2.00 cfs. Determine the water surface elevation required in the reservoir and the total discharge.

Solution:

First write the energy equation from the center of the main pipe to the water surface of the river.

$$\frac{p_p}{\gamma} + \frac{V_p^2}{2g} + z_p = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + z_s + \sum h_L$$

$$\frac{p_p}{\gamma} + 0 + 490 \text{ ft} = 0 + 0 + 500 \text{ ft} + \frac{V_r^2}{2g}\left(0.5 + 1.0 + f\frac{L}{D}\right) \tag{1}$$

where

$$V_r = \frac{Q}{A} = \frac{2 \text{ cfs}}{\frac{\pi}{4}\left(\frac{4 \text{ in}}{12 \text{ in/ft}}\right)^2} = 22.92 \text{ ft/s}$$

is the velocity in the riser pipe.

$$\text{Re}_r = \frac{V_r D}{\nu} = \frac{22.92 \text{ ft/s} \cdot \frac{4 \text{ in}}{12 \text{ in/ft}}}{0.74 \cdot 10^{-5} \text{ ft}^2/\text{s}} = 1.03 \cdot 10^6$$

Note: the kinematic viscosity was based upon an assumed effluent water temperature of 100 °F.

$$\frac{k_s}{D} = 0.00045 \quad \text{(From Fig. 5-5, assumed steel pipe)}$$

$$f_r = 0.017 \quad \text{(From Fig. 5-4)}$$

Then the head loss in the end 4-inch riser pipe is given as

$$h_{L,r} = \frac{V_r^2}{2g}\left(0.5 + 1.0 + f\frac{L}{D}\right) = \frac{(22.92 \text{ ft/s})^2}{2 \cdot 32.2 \text{ ft/s}^2} \cdot \left(1.5 + 0.017 \cdot \frac{2 \text{ ft}}{\frac{4 \text{ in}}{12 \text{ in/ft}}}\right) = 13.07 \text{ ft.} \qquad (2)$$

Then from Eq. (1)

$$\frac{p_p}{\gamma} = 500 \text{ ft} - 490 \text{ ft} + 13.07 \text{ ft} = 23.07 \text{ ft}$$

Also from Eq. (1) and Eq. (2)

$$h_p - 10 \text{ ft} = \frac{Q^2}{2gA^2}\left(1.5 + f\frac{L}{D}\right)$$

so

$$Q_r = 0.533 \text{ m}^{5/2}/\text{s} \cdot \sqrt{h_p - 10 \text{ ft}}$$

Now consider the head loss in the main pipe between the last two riser pipes.

$$Q = 2 \text{ cfs} \quad \text{so} \quad V = \frac{Q}{A} = 2.55 \text{ ft/s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{2.55 \text{ ft/s} \cdot 1 \text{ ft}}{0.74 \text{ ft}^2/\text{s}} = 3.5 \cdot 10^5$$

$$\frac{k_s}{D} = 0.00016, \quad f = 0.016 \quad \text{(Fig. 5-4)}$$

Then

$$h_f = 0.016 \cdot \frac{10 \text{ ft}}{1 \text{ ft}} \cdot \frac{(2.55 \text{ ft/s})^2}{2 \cdot 32.2 \text{ ft/s}^2} = 0.02 \text{ ft}$$

The remainder of the calculations are shown in tabular form below to yield the head in the main pipe at riser number 10.

Junction	h_p ft	h_f ft	Q_r cfs	Q_p cfs	V ft/s	Re 10^6	f —
1	23.07		2.00				
		0.02		2.00	2.55	0.34	0.016
2	23.08		2.00				
		0.06		4.00	5.09	0.69	0.015
3	23.14		2.00				
		0.13		6.00	7.65	1.03	0.014
4	23.27		2.01				
		0.23		8.02	10.21	1.38	0.014
5	23.50		2.03				
		0.36		10.05	12.80	1.73	0.014
6	23.85		2.06				
		0.52		12.11	15.42	2.08	0.014
7	24.37		2.10				
		0.66		14.21	18.09	2.44	0.013
8	25.03		2.14				
		0.87		16.35	20.82	2.81	0.013
9	25.90		2.21				
		1.13		18.55	23.62	3.19	0.013
10	27.03		2.28				
				20.84	26.53	3.59	0.013

Now determine the water surface elevation in the reservoir.

$$\frac{p_r}{\gamma} + \frac{V_r^2}{2g} + z_r = \frac{p_p}{\gamma} + \frac{V_p^2}{2g} + z_p + \sum h_L$$

$$0 + 0 + z_r = 27.03 \text{ ft} + \frac{(26.53 \text{ ft/s})^2}{2 \cdot 32.2 \text{ ft/s}^2} + 490 \text{ ft} + \frac{(26.53 \text{ ft/s})^2}{2 \cdot 32.2 \text{ ft/s}^2} \cdot \left(1 + 0.5 + f\frac{L}{D}\right)$$

$$z_r = 27.03 \text{ ft} + 490 \text{ ft} + \frac{(26.53 \text{ ft/s})^2}{2 \cdot 32.2 \text{ ft/s}^2} \cdot \left(0.5 + 0.013\frac{500 \text{ ft}}{1 \text{ ft}}\right) = 604 \text{ ft}.$$

Problem 5.48

This 30° vertical bend in a pipe having a 2-ft diameter carries water at a rate of 31.4 cfs. If the pressure p_1 is 10 psi at the lower end of the bend where the elevation is 100 ft, and p_2 is 8 psi at the upper end where the elevation is 103 ft, what will be the vertical component of force that must be exerted by the "anchor" on the bend to hold it in position? The bend itself weighs 300 lb, and the length L is 4 ft.

Solution:

$$V = \frac{Q}{A} = \frac{31.4 \text{ cfs}}{\pi \cdot (1 \text{ ft})^2} = 9.99 \text{ ft/s}.$$

$$\sum F_y = \rho Q \left(V_{2y} - V_{1y} \right)$$

$$F_{\text{anchor}} - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ = \rho Q (V \sin 30^\circ - V \sin 0^\circ)$$

$$F_{\text{anchor}} = \pi \cdot (1 \text{ ft})^2 \cdot 4 \text{ ft} \cdot 62.4 \text{ lbs/ft}^3 + 300 \text{ lbs} + 8 \text{ psi} \cdot \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 \pi \cdot (1 \text{ ft})^2 \cdot \sin 30^\circ$$

$$+1.94 \text{ slug/ft}^3 \cdot 31.4 \text{ cfs} \cdot 9.99 \text{ ft/s} \cdot \sin 30^\circ = 3198 \text{ lbs}$$

Problem 5.52

The pipe diameter D is 30 cm, d is 15 cm, and the atmospheric pressure is 100 kPa. What is the maximum allowable discharge before cavitation occurs at the throat of the venturi meter if $H = 5$ m.

Solution:

Take section 1 at reservoir surface and section 2 at a section of diameter d.

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + + \frac{V_2^2}{2g} + z_2$$

$$0 + 0 + H = \frac{p_{2,\text{vapor}}}{\gamma} + \frac{V_2^2}{2g} + 0$$

where $p_{2,\text{vapor}}$ = 2340 Pa abs = −97660 Pa gauge. Then

$$\frac{V_2^2}{2g} = H - \frac{p_{2,\text{vapor}}}{\gamma} = 5 \text{ m} - \frac{-97660 \text{ Pa}}{9790 \text{ N/m}^3} = 14.98 \text{ m}$$

giving $V = 17.1$ m/s and

$$Q = V_2 A_2 = 17.1 \text{ m/s} \cdot \frac{\pi}{4} \cdot (0.15 \text{ m})^2 = 0.303 \text{ m}^3/\text{s}.$$

Problem 5.55

Determine the head loss per 1000 ft in this tunnel that is lined with concrete and is to have a water discharge of 1000 cfs.

Solution:

$$h_f = f \frac{L}{4R} \frac{V^2}{2g}$$

Assume granular or brushed surface in fairly good condition. Then we have

$$k_s = 0.001 \text{ ft} \quad \text{(From Table 5-6)}$$

$$A = \frac{1}{2}\pi(5 \text{ ft})^2 + 8 \text{ ft} \cdot 10 \text{ ft} = 119.3 \text{ ft}^2$$

$$V = \frac{Q}{A} = \frac{1000 \text{ cfs}}{119.3 \text{ ft}^2} = 8.38 \text{ ft/s}$$

$$P = \pi \cdot 5 \text{ ft} + 2 \cdot 8 \text{ ft} + 10 \text{ ft} = 41.71 \text{ ft}$$

$$R = \frac{A}{P} = \frac{119.3 \text{ ft}^2}{41.71 \text{ ft}} = 2.86 \text{ ft}$$

$$\text{Re} = \frac{4RV}{\nu} = \frac{4 \cdot 2.86 \text{ ft} \cdot 8.38 \text{ ft/s}}{1.2 \cdot 10^{-5} \text{ ft}^2/\text{s}} = 8.0 \cdot 10^6$$

$$\frac{k_s}{4R} = \frac{0.001 \text{ ft}}{4 \cdot 2.86 \text{ ft}} = 8.7 \cdot 10^{-5}$$

$$f = 0.013 \quad \text{(from Fig. 5-4)}$$

Then

$$\frac{h_f}{L} = \frac{f}{4R} \frac{V^2}{2g} = \frac{0.013}{4 \cdot 2.86 \text{ ft}} \frac{(8.38 \text{ ft/s})^2}{2 \cdot 32.2 ft/s^2} = 0.0012$$

So friction loss over 1000 ft is 1000 ft · 0.0012 = 1.2 ft.