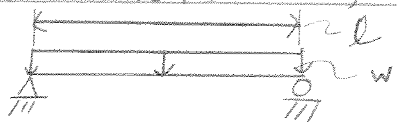
Homework #4 CEE 454 | 6.23, 6.31, 6.34, 6.38 (c)

(6.23) Given:



$w_D = 18$ psf, $w_L = 50$ psf, $1.2D + 1.6L$, $\ell = 14'$ (168"), 16" o.c., No. 1 H-F

$l_u = 0$, MC $\leq 19\%$, $\Delta_{L\,allow} \leq \ell/360$, $\Delta_{D+L\,(allow)} \leq \ell/240$

Find: Adequate section size and size of notches on tension & compression side at (a) supports and (b) interior of member and (c) determine V_r' if 1" deep notch used at support.

Solution: $\boxed{M_n' \geq M_u,\ V_n' \geq V_u,\ \Delta_{allow} \geq \Delta_{actual}}$ Then find V_r' and check NDS specs 4.4.3

1) $M_u = \dfrac{w_u \ell^2}{8} = \dfrac{\left(\overset{21.6}{1.2(18\,psf)} + \overset{80.0}{1.6(50\,psf)}\right)\left(\overset{1.3'}{16/12'}\right)(14')^2}{8} = 3311$ ft-# $= \underline{\underline{39.7 \text{ in-k}}}$

$V_u = \dfrac{w_u \ell}{2} = \dfrac{0.135(14')}{2} = \underline{\underline{0.95^k}}$ $\qquad W_u = 135$ #/ft

$\Delta_{actual\,L} = \dfrac{5w\ell^4}{384EI} = \dfrac{5\left(\frac{0.067^{k/'}}{12^{"/'}}\right)(168")^4}{384EI}$

$\Delta_{actual\,D+L} = \dfrac{5w\ell^4}{384EI} = \dfrac{5\left(\frac{0.090^{k/'}}{12^{"/'}}\right)(168")^4}{384EI}$

2) $M_n' = S_x F_{bn}' = S_x (0.975^{ksi})(C_r = 1.15)(2.16)(0.8)(C_F \overset{?}{=} 1.1)$

$\Rightarrow S_x(2.131\,ksi) = 39.7$ in-k

$(S_x)_{min} = 18.6\ in^3 < S_x = 21.39\ in^3$ for 2x10 ($1\frac{1}{2} \times 9\frac{1}{4}$")

$A = 13.88\ in^2$, $I = 98.93\ in^4$

Also assume $W_D = 3.854$ #/ft $= 0.0039$ k/ft

M_u additional $= \dfrac{1.2(0.0039^{k/'})(14')^2}{8} = 0.115$ ft-k $= \underline{\underline{1.38 \text{ in-k}}}$

$\therefore$ $M_{u\,total} = 39.7 + 1.38 = \underline{\underline{41.1 \text{ in-k}}}$

$V_u = 0.95^k + \dfrac{1.2(.0039)(14)}{2} = \underline{\underline{0.98^k}}$

$M_n' \Rightarrow C_F = 1.1$ for 2x10 so $M_n' = 21.39\ in^3(2.131\,ksi) = \underline{\underline{45.6 \text{ in-k}}}$

$M_n' = 45.6$ in-k > 41.1 in-k OK

$V_n' = \frac{2}{3} A (F_{vn}' = 0.150\,ksi)(2.16)(0.8) = \frac{2}{3}(13.88\ in^2)(0.259) = \underline{\underline{2.41}}^k > \underline{\underline{0.98}}^k$

OK

$I = 98.93\ in^4$, $E = 1500$ ksi so from before:

$$\Delta_{actual_L} = \frac{5(0.006)(168)^4}{384(1500)(98.93)} = \underline{\underline{0.42''}} < \Delta_{allow_L} = \ell/360 = 0.47'' \text{ OK}$$

$$\Delta_{actual_{D+L}} = \frac{5(0.0075)(168)^4}{384(1500)(98.93)} = 0.52'' < \Delta_{allow_{D+L}} = \ell/240 = 0.7'' \text{ OK}$$

Now check section 4.4.3 : $d/4 = 9.25''/4 = \underline{\underline{2.32''}}$ deepest notch at support

Notch depth $\leq d/6$ in outer thirds of the span; $\frac{d}{6} = \frac{9.25}{6} = \underline{\underline{1.54''}}$

Notches not permitted in middle third of span.

$$V_r' = \left(\frac{2}{3} F_v' \, b \, d_n\right)\left(\frac{d_n}{d}\right)^2 = \frac{2}{3} \underbrace{(0.150)(2.16)(0.8)}_{0.2592}(1.5'')(8.25)\underbrace{\left\{\frac{8.25}{9.25}\right\}^2}_{0.795}$$

$= \underline{\underline{1.71^k}}$

Before, $V_n' = \underline{\underline{2.41^k}}$.

6.31 *Given:* The rafter connection in Fig. 6.F. The load is a combination of $P_D = 140$ lb and $P_S = 560$ lb. Consider ~~the D + S (ASD)~~ or 1.2D + 1.6S (LRFD) load combination. Lumber is No. 1 Spruce-Pine-Fir (South). $C_M = 1.0$ and $C_t = 1.0$.

Find:
- *d.* Factored bearing load P_u in the rafter and in the top plate of the wall (LRFD).
- *e.* The adjusted LRFD bearing resistance $P'_{n\perp}$ in the top plate.
- *f.* The adjusted LRFD bearing resistance $P'_{n\theta}$ in the rafter.

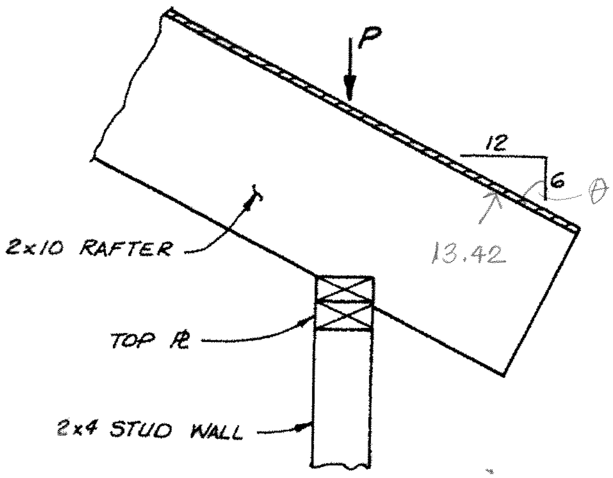


Figure 6.F

Table 1B:

$2 \times 10 = 1\tfrac{1}{2}'' \times 9\tfrac{1}{4}''$

$2 \times 4 = 1\tfrac{1}{2} \times 3\tfrac{1}{2}''$

Table 4A:

S-P-F (S) No. 1

$F_{c\perp} = .335$ ksi

$F_c = 1.050$ ksi

Solution:

$$P_u = 1.2D + 1.6S = 1.2(0.140^k) + 1.6(0.560^k) = 0.168 + 0.896 = 1.064^k$$

$$P'_{n\perp} = F'_{c_{n\perp}}(A_b)$$

$$A_b = \underbrace{\ell_b}_{\text{bearing length}} \underbrace{d_b}_{\text{bearing width}} = (1.5'')(3.5'') = 5.25 \text{ in}^2$$

$$\ell_b = 1.5'', \quad C_b = \frac{\ell_b + 0.375}{\ell_b} = 1.25$$

$$F'_{c_{n\perp}} = (F_{c\perp})(C_M)(C_t)(C_i)(C_b)\underbrace{K_F\, \phi_c \lambda}_{1.875\,(0.8)}$$

Table 4.3.1 Specs

Tables N1–N3 (p. 170) Specs

$$= (0.335 \text{ ksi})(1.25)(1.875)(0.8)$$

$$= 0.628 \text{ ksi}$$

$$P'_{n\perp} = (0.628\text{ksi})(5.25 \text{ in}^2) = 3.3^k > 1.064^k \quad \text{OK}$$

$P'_{n\theta} = A_b F'_{n\theta}$: must use Hankinson formula p. 22 Specs 3.10.3 section

$$F'_\theta = \frac{F_c^* F'_{c\perp}}{F_c^* \sin^2\theta + F'_{c\perp}\cos^2\theta}$$

$F_c^* = F_c (C_m)(C_t)(C_F)(C_i)(\underline{N_o C_p})(K_F \phi_c \lambda)$ Table 4.3.1

$C_F = 1.0$ p. 30 Table 4A Suppl.

$K_F \phi_c \lambda = 2.16\,(0.8)$ p. 170 Specs

$\therefore$ $\underline{\underline{F_c^*}} = 1.050$ ksi $(2.16)(0.8) = \underline{\underline{1.81}}$ ksi

$\theta \Rightarrow$ figure 3I p. 22 Specs $\theta = \tan^{-1}\left(\frac{12}{6}\right) = 63.43°$

$\sin\theta = 0.8944$
$\cos\theta = 0.4473$

$$\underline{\underline{F'_{\theta n}}} = \frac{1.81\,(0.628)}{1.81(0.8944)^2 + 0.628\,(0.4473)^2}$$

$$= \frac{1.14}{1.45 + 0.1256}$$

$$= \underline{\underline{0.72 \text{ ksi}}}$$

$\underline{\underline{P'_{\theta n}}} = 0.72$ ksi $(5.25 \text{ in}^2) = \underline{\underline{3.78^k}}$

6.33 *Given:* The beam-to-column connection in Fig. 6.G. The gravity reaction from the simply supported beam is transferred to the column by bearing (not by the bolts). Assume the column and the metal bracket have adequate strength to carry the load. $C_t = 1.0$.

Find: Using ASD, the maximum allowable beam reaction governed by bearing stresses for the following conditions:

a. The beam is a 4 × 12 No. 1 DF-L. MC ≤ 19 percent, and the dimensions are $A = 12$ in. and $B = 5$ in. Loads are (D + S).

b. The beam is a $5\frac{1}{8} \times 33$ DF glulam Combination 24F-V4. MC = 18 percent, and the dimensions are $A = 0$ and $B = 12$ in. Loads are (D + S).

c. The beam is a 6 × 16 No. 1 DF-L. MC = 20 percent, and the dimensions are $A = 8$ in. and $B = 10$ in. Loads are (D + L_r).

d. What deformation limit is associated with the bearing stresses used in parts *a* to *c*?

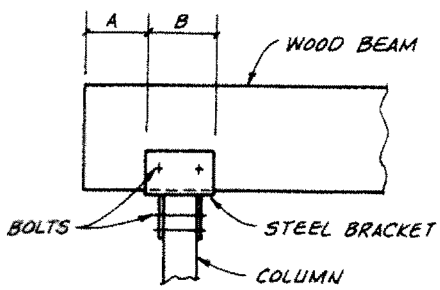
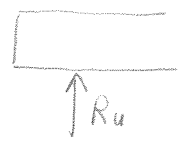


Figure 6.G

In all cases,

$$R_u = A_b F'_{c\perp n}$$



6.34 Repeat Prob. 6.33 using LRFD and determine the maximum factored beam reaction governed by bearing for each of the conditions.

Solution:

a. 4 x 12 No. 1 DF-L, $A = 12''$, $B = 5''$, $1.2D + 1.6S$

$3\frac{1}{2}'' \times 11\frac{1}{4}''$, $F_{c\perp} = 625$ psi, $A = 12'' > 3''$, $B = 5'' = \ell_b < 6''$

$$C_b = \frac{\ell_b + 0.375}{\ell_b} = \frac{5'' + 0.375}{5''} = 1.075 \qquad (b = 3.5'')$$

$$F'_{c\perp n} = 0.625(1.875)(0.8)(1.075) = 1.01 \text{ ksi}$$

$$R_u = F'_{c\perp n} A_b = 1.01 \text{ ksi } \underbrace{(3.5'')(5'')}_{17.5 \text{ in}^2} = \underline{17.7^k}$$

b. $5\frac{1}{8}'' \times 33''$ DF 24F-V4, $A = 0$, $B = 12''$, $1.2D + 1.6S$

$A_b = 5\frac{1}{8}''(12'') = 61.5 \text{ in}^2$; $\ell_b > 6''$

$F_{c\perp n} = 0.650$ ksi, ($C_b = 1.0$ see Table 3.10.4 p. 22 of Specs)

$$F'_{c\perp n} = 0.650 \overset{C_M}{(0.53)}(1.0)(1.0)(1.875)(0.8) = 0.516 \text{ ksi}$$ (C_M p. 59 of suppl. Table 5A)

$$\underline{R_u} = F'_{c\perp n} A_b = 0.516 \text{ ksi } (61.5 \text{ in}^2) = \underline{31.7^k}$$

Find the min req'd beam size for 20 psf = Lr.
Given 20F − 1.5E DF gluelam, D = 16 psf (assume it includes beam weight).

$F_{bx}^{+} = F_{bx}^{-} = 2000$ psi, use balanced layup, $F_{vx} = 210$ psi, $E_x = 1.5 \times 10^6$ psi

Spacing of girder = 24′ ∴ $D = 16\text{ psf}(24') = 384$ #/ft
$L_r = 20\text{ psf}(24) = 480$ #/ft

optimum Cantilever $L_c = 0.172 l = 0.172(40') = 6.88' = 82.56''$

$M^{+} = M^{-} = 0.086\,wL^2$ p. 6.79 of the text

∴ $M^{+} = M^{-} = 0.086(1.2(0.384) + 1.6(0.48))(40')^2 = 169$ ft-k $= 2028$ in-k

$M_n' = S_x F_{bn}' = S_x(2.0)(2.16)(0.8)(0.9 \text{ assumed}) = 2028$ in-k

$S_{x\,min} = 652 \text{ in}^3$

∴ Select from Table 1C $5\tfrac{1}{8} \times 28\tfrac{1}{2}$, $A = 146.1$, $S_x = 693.8 \text{ in}^3$, $I_x = 9886.6 \text{ in}^4$

For lateral stability, use $l_u = 6.88'$: Table 3.3.3 Specs

$$\frac{l_u}{d} = \frac{6.88(12''/')}{28.5''} = 2.89 < 7$$

Use $l_e = 2.06\,l_u = 2.06(82.56'') = 170''$

$$R_B = \sqrt{\frac{170''(28.5'')}{(5.125'')^2}} = \sqrt{184} = 13.6 < 50 \quad \underline{OK}$$

$$F_{bE} = \frac{1.20\,E'_{min}}{R_B^2} = \frac{1.20(620\text{ ksi})(1.875)}{184} = 7.58 \text{ ksi}$$

$$F_{bx}^{*} = 2.0(2.16)(0.8) = 3.46 \text{ ksi}$$

$$\frac{F_{bE}}{F_{bx}^{*}} = \frac{7.58}{3.46} = 2.19$$

$$C_L = \frac{1+2.19}{1.9} - \sqrt{\left(\frac{1+2.19}{1.9}\right)^2 - \frac{2.19}{0.95}}$$

$$= 1.679 - \sqrt{2.819 - 2.305}$$

$$= 1.679 - 0.717$$

$$= \underline{0.96}$$

OK, try $C_V \Rightarrow l = 6.88'$ + extra on mmt diagram for "0 to 0" length

Using figure 18, $\ell(1 - a^2/\ell^2) =$ I.P. length $\rightarrow 40'(1 - (6.88/40)^2) = 38.8'$

$"\ell" = (40 - 38.8) + 6.88 = 8.08'$

$$C_V = \left(\frac{21}{8.08}\right)^{0.1} \left(\frac{12}{28.5}\right)^{0.1} \left(1\right)^{0.1} = 1.1(0.92) = \underline{1.00} \text{ use } C_L$$

$F'_{b_n} = 0.96(3.46 \text{ ksi}) = 3.32 \text{ ksi}$

$\underline{M'_n} = S_x F'_{b_n} = 693.8 \text{ in}^3 (3.32 \text{ ksi}) = 2304 \text{ in-k} > 2028 \text{ in-k}$ $\underline{\underline{\text{OK}}}$

$$\underline{V'_n} = \frac{2}{3} A F_V' = \frac{2}{3}(146.1 \text{ in}^2)(\overbrace{0.210)(2.16}^{0.363})(0.8)$$

$= \underline{\underline{35.5^k}}$

$$\underline{\underline{V_{u\,max}}} = \frac{W_u}{2\ell}(\ell^2 + a^2) \text{ from figure 18}$$

$$= \frac{1.23(40^2 + 6.88^2)}{2(40)}$$

$= 25.3^k < V'_n$ $\underline{\underline{\text{OK}}}$