

## Ex Problem 10 - Countercurrent Packed Tower for Absorption of Ammonia Gas (Pilat)

### Given

Gas (air flow) upward of 4,000 acfm at 68F or 528R and 1 atm pressure  
 Inlet gaseous conc. of 6,000 ppm NH<sub>3</sub> (dry basis)  $Y_{in} = 0.006$   
 Inlet water conc of 0.0006 mole fraction NH<sub>3</sub> in water  $x_{in} = 0.0006$   
 Liquid flow rate = 500 lb liquid/ ft<sup>2</sup> hr Mol Wt H<sub>2</sub>O =  $M_L = 18.016$  lb/lbmole  
 Gaseous flow rate of 500 lb gas (air) / ft<sup>2</sup> hr Mol Wt Gas (air) = 29 lb/lbmole  
 atomic wt N = 14.0067 atomic wt H = 1.0079  
 Assume Govt Emission Std is 1,000ppm NH<sub>3</sub> (dry basis) so  $y_{out} = 0.001$

Need to define unit lbmole

$$\text{lbmole} := (\text{mole}) \cdot (453.6)$$

$$M_L := \frac{18.016 \cdot \text{lb}}{\text{lbmole}}$$

$$M_{\text{NH}_3} := \frac{17.031 \cdot \text{lb}}{\text{lbmole}}$$

$$M_G := 29 \cdot \frac{\text{lb}}{\text{lbmole}}$$

$$Y_{out} := 0.001 \quad X_{in} := 0.0006$$

$$Y_{in} := 0.006 \quad X_{out} = \text{Unknown}$$

$$Q_{\text{Gas}} := 4000 \cdot \text{ft}^3 \cdot \text{min}^{-1}$$

$$P_{total} := 1 \cdot \text{atm} \quad P := 1 \cdot \text{atm}$$

$$T := 528 \cdot \text{R} \quad \text{mmHg} := \frac{\text{atm}}{760}$$

Six NH<sub>3</sub> vapor pressures

$$i := 0..5$$

### Find

- a)  $H$  = Henry's law constant from solubility data of NH<sub>3</sub> in water at 20°C
- b)  $H_{OG}$  from ammonia-water data of Fellingner for 1.5" Raschig ring packing
- c) Liquid molar flow rate  $L_m$  (lb moles/ft<sup>2</sup> hr)
- d) Gaseous molar flow rate  $G_m$  (lb moles/ft<sup>2</sup> hr)
- e)  $X_{out}$  via Mass Balance around the absorption tower
- f) Number of gas phase mass transfer units  $N_{OG}$
- g) Height of packed section in tower  $Z = (H_{OG}) (N_{OG})$  h) Tower Diameter (ft)
- i) Gas phase overall mass transfer coefficient  $K_{Ga}$  (lb moles/ft<sup>3</sup> hr atm)
- j) Gas velocity in tower (ft/sec)
- k) Gas residence time in packed section of tower (seconds)
- L)  $L/G$  = Liquid/Gas flow rate ratio in gallons water / ft<sup>3</sup> 1000 gas
- m) Draw Y-X graph with equilibrium curve and tower operating line.
- n) Ammonia collection efficiency

a) Calc Henry's law constant  $H$  using equilibrium solubility data for NH<sub>3</sub> in water at temperatures in 20°C range from Table B-4 page 682 Cooper & Alley (3rd Ed)

| (lb NH <sub>3</sub> per 100 lb H <sub>2</sub> O) | NH <sub>3</sub> Partial Pressure (mm Hg) |
|--|--|
| 2  | 12.0                                     |
| 3  | 18.2                                     |
| 4  | 24.9                                     |
| 5  | 31.7                                     |
| 7.5  | 50.0                                     |
| 10   | 69.6                                     |

$$Wt_{\text{NH}_3} := \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 7.5 \\ 10 \end{pmatrix} \cdot \text{lb}$$

$$P_{\text{NH}_3} := \begin{pmatrix} 12.0 \\ 18.2 \\ 24.9 \\ 31.7 \\ 50.0 \\ 69.6 \end{pmatrix} \cdot \text{mmHg}$$

$$P_{\text{NH}_3} = \begin{pmatrix} 12 \\ 18.2 \\ 24.9 \\ 31.7 \\ 50 \\ 69.6 \end{pmatrix} \text{ torr}$$

Mole fraction NH<sub>3</sub> in liquid phase =  $X_{\text{NH}_3}$   
 Mole fraction NH<sub>3</sub> in gas phase =  $Y_{\text{NH}_3}$   
 Henry's Law Constant =  $H$

$$x_{\text{NH}_3} := \frac{\text{molesNH}_3 \text{ in Water}}{\text{Total Moles in Water}}$$

$$y_{\text{NH}_3, i} := \frac{P_{\text{NH}_3, i}}{P_{total}}$$

$$X_{\text{NH}_3, i} := \frac{\frac{Wt_{\text{NH}_3, i}}{17.031 \cdot \text{lb}}}{\frac{Wt_{\text{NH}_3, i}}{17.031 \cdot \text{lb}} + \frac{100 \cdot \text{lb}}{18.016 \cdot \text{lb}}}$$

$$Y_{\text{NH}_3} = \begin{pmatrix} 0.016 \\ 0.024 \\ 0.033 \\ 0.042 \\ 0.066 \\ 0.092 \end{pmatrix}$$

$$X_{\text{NH}_3} = \begin{pmatrix} 0.021 \\ 0.031 \\ 0.041 \\ 0.05 \\ 0.074 \\ 0.096 \end{pmatrix}$$

$$H = \begin{pmatrix} 0.7621 \\ 0.7786 \\ 0.8071 \\ 0.8303 \\ 0.895 \\ 0.9573 \end{pmatrix}$$

$$H_i := \frac{Y_{\text{NH}_3, i}}{X_{\text{NH}_3, i}}$$

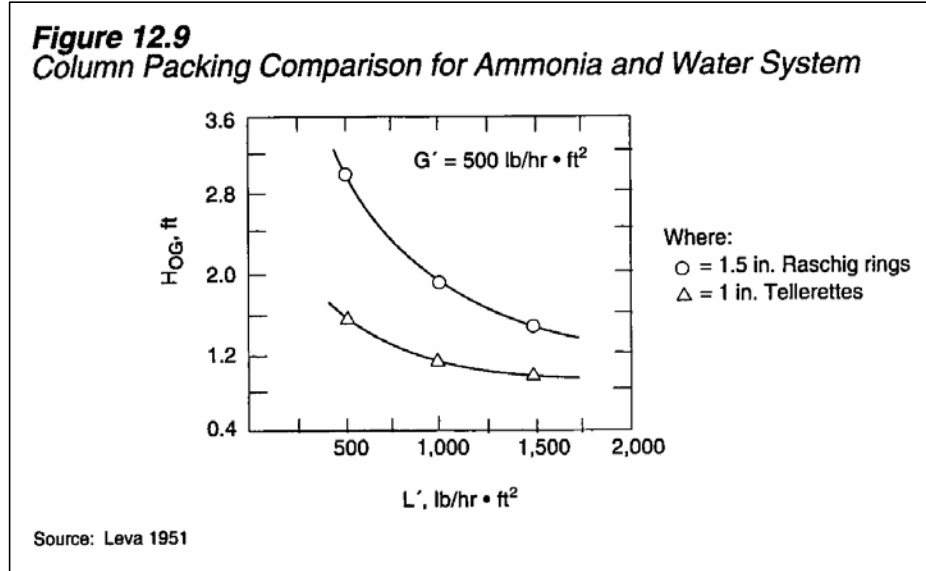
Assume  $H = 0.7614 = y_{\text{NH}_3} / X_{\text{NH}_3}$  because the inlet liquid phase NH<sub>3</sub> concentration  $X_{in}$  is low at 0.0006

Henry's law constant  $H$  is in units of (mole fraction NH<sub>3</sub> in gas phase) / (mole fraction NH<sub>3</sub> in liquid phase)

**(a)  $H := 0.7614$**

b) Obtain height of gas phase mass transfer unit  $H_{OG} = 3.0$  ft from graph (shown below) for 1.5" Raschig rings,  $L = 500$  lb liquid /  $ft^2$  hr,  $G = 500$  lb air /  $ft^2$  hr. pp 456 Noll

(b)  $H_{OG} := 3.0 \cdot ft$



c) Liquid flow rate in lbmole units =  $L_m$   
Liquid flow rate in lb units =  $L$

$L := 500 \cdot lb \cdot ft^{-2} \cdot hr^{-1}$

$M_L = 18.016 \text{ gm} \cdot \text{mole}^{-1}$

d) Gaseous molar flow rate =  $G_m$   
Gaseous mass flow rate =  $G$

$L_m := \frac{L}{M_L}$

$M_G = 29 \text{ gm} \cdot \text{mole}^{-1}$

$G := 500 \cdot lb \cdot ft^{-2} \cdot hr^{-1}$

(c)  $L_m = 27.753 \frac{\text{lbmole}}{\text{ft}^2 \cdot \text{hr}}$

$Y_{in} = 0.006$

$Y_{out} = 0.001$

$X_{in} = 0.0006$

$G_m := \frac{G}{M_G}$

(d)  $G_m = 17.241 \frac{\text{lbmole}}{\text{ft}^2 \cdot \text{hr}}$

e) The mass balance equation  
 $NH_3$  into tower =  $NH_3$  out of tower  
 $Y_{in} G_m + X_{in} L_m = Y_{out} G_m + X_{out} L_m$   
 $X_{in}$  was given as 0.0006 to can solve for the unknown  $X_{out}$

$X_{in} = 0.0006$

$X_{out} := \left( \frac{G_m}{L_m} \right) \cdot (Y_{in} - Y_{out}) + X_{in}$

$H = 0.7614$

$X_{out2} := \left( \frac{17.241 \cdot \frac{\text{lbmole}}{\text{ft}^2 \cdot \text{hr}}}{27.753 \cdot \frac{\text{lbmole}}{\text{ft}^2 \cdot \text{hr}}} \right) \cdot (.006 - .001) + .0006$

(e)  $X_{out} = 0.003706$

$X_{out2} = 0.003706$

Note that liquid conc  $X_{out} = 0.003706$  is in mole fraction units, not ppm (in liquids, ppm means parts per million by weight whereas in gases, ppm means ppm by mole or by gaseous volume)

To graph the operating line using the liquid phase concentration  $X$  as the  $X$  axis variable we will need a linear equation relating the tower gas phase  $NH_3$  conc  $Y$  in terms of the liquid phase  $NH_3$  conc  $X$ . Starting at  $X_{in} = 0.0006$  at the top of the tower and increasing  $X$  to  $X_{out}$  one can obtain the linear operating equation for  $Y$ . Note that this equation represents the actual gas phase and liquid phase conc in the absorption tower (one could take a sample of the tower gases & liquids & measure the  $NH_3$  conc).

Operating line linear equation  $\rightarrow$

$Y := \left( \frac{L_m}{G_m} \right) \cdot (X - X_{in}) + Y_{out}$

**f.) Number of Gas Phase Mass Transfer Units or  $N_{OG}$**

$P_{total} = 1 \text{ atm}$

Because it is difficult to put an asterisk to denote the equilibrium conc for Y or Y\* in Mathcad, let  $Y_{starin}$  be the gas phase mole fraction conc of  $NH_3$  in equilibrium with the liquid water at the gaseous inlet (bottom of this tower) and let  $Y_{starout}$  be the gas phase conc. of  $NH_3$  in equilibrium with the liquid water at the gaseous outlet (top of tower). Using nomenclature of  $Y_{in}$ ,  $Y_{out}$ ,  $X_{in}$ ,  $X_{out}$ ,  $Y_{starin}$ , and  $Y_{starout}$  should help to reduce confusion as to where these concentrations occur and make it easier to label diagrams and tower illustrations.

$Y_{starin} := (H) \cdot X_{out}$

$Y_{starin} = 0.00282$

$Y_{starout} := H \cdot X_{in}$

$Y_{starout} = 0.00046$

$$N_{OG} := \frac{Y_{in} - Y_{out}}{(Y_{in} - Y_{starin}) - (Y_{out} - Y_{starout})} \cdot \ln \left( \frac{Y_{in} - Y_{starin}}{Y_{out} - Y_{starout}} \right)$$

**(f)**  $N_{OG} = 3.352$

there are 3.352 gas phase mass transfer units

Note that the above equation for  $N_{OG}$  assumes a straight equilibrium line and a straight operating line (which is an OK assumption for the absorption of dilute gas concentrations and air pollutants are usually in the dilute concentration range).

**g.) Height of Packed Tower Section =  $Z = H_{OG} N_{OG}$**

$Z := H_{OG} \cdot N_{OG}$

$Z = 10.057 \text{ ft}$

**(g)**

**h) Tower Diameter**

$Z_2 := (3.0 \cdot \text{ft}) \cdot (3.352)$

$Z_2 = 10.056 \text{ ft}$

$RG := 0.082054 \cdot \frac{\text{liter} \cdot \text{atm}}{\text{mole} \cdot \text{K}}$

$Mg := \frac{29 \cdot \text{gm}}{\text{mole}}$

The height of the 1.5 inch Raschig ring packed section in the absorption tower = 10.057 ft

$G = 500 \frac{\text{lb}}{\text{ft}^2 \cdot \text{hr}}$

$\rho_g := \frac{P \cdot Mg}{RG \cdot T}$

$T = 528 \text{ R}$

$P := 1 \cdot \text{atm}$

$\text{Area} := \frac{Q_{Gas} \cdot \rho_g}{G}$

Area = tower cross-sectional area

gas density =  $\rho_g$

$\rho_g = 0.075217 \text{ lb} \cdot \text{ft}^{-3}$

$$\text{Area} := \left( \frac{4000 \cdot \text{ft}^3 \cdot \text{min}^{-1}}{500 \cdot \frac{\text{lb}}{\text{ft}^2 \cdot \text{hr}}} \right) \cdot \left( 0.07522 \cdot \frac{\text{lb}}{\text{ft}^3} \right)$$

$\text{Area} = 36.106 \text{ ft}^2$

$\text{Diameter} := \left( \frac{\text{Area} \cdot 4}{\pi} \right)^{0.5}$

**Diameter = 6.78 ft**

**(h)**

**Tower Diameter = 6.78 ft**

**i) Gas Phase Overall Mass Transfer Coefficient  $K_{GA}$**

$G_m = 17.241 \text{ lbmole} \cdot \text{ft}^{-2} \cdot \text{hr}^{-1}$

Graphs of the *measured*  $K_{GA}$  versus liquid or gas flow rates are provided by equipment manufacturers for various absorption packing types and sizes.

$KGA := \frac{17.241 \cdot \frac{\text{lbmole}}{\text{ft} \cdot \text{hr}}}{(3 \cdot \text{ft}) \cdot (1 \cdot \text{atm})}$

$KGA := \frac{G_m}{H_{OG} \cdot P}$

$H_{OG} = 3 \text{ ft}$

$P = 1 \text{ atm}$

**(i)**

$KGA = 1.752 \text{ m} \frac{\text{lbmole}}{\text{ft}^3 \cdot \text{hr} \cdot \text{atm}}$

**j) Gas Velocity (superficial) in packed tower**

$\text{Velocity} := (Q_{Gas}) \cdot (\text{Area}^{-1})$

$\text{Vel}_2 := (4000 \cdot \text{ft}^3 \cdot \text{min}^{-1}) \cdot (36.106 \cdot \text{ft}^2)^{-1}$

$\text{Vel}_2 = 1.846 \text{ ft} \cdot \text{sec}^{-1}$

**(j)**

**Velocity = 1.846  $\text{ft} \cdot \text{sec}^{-1}$**

**k) Gas Residence time in packed absorption tower section**

$\text{ResidenceTime} := Z \cdot (\text{Velocity}^{-1})$

$\text{ResidenceTime}_2 := (10.056 \cdot \text{ft}) \cdot (1.846 \cdot \text{ft} \cdot \text{sec}^{-1})^{-1}$

**(k)**

**ResidenceTime = 5.447 sec**

**ResidenceTime<sub>2</sub> = 5.447 sec**

L ) Liquid to gas flow rate ratio L/G in gallons/1000 cf gas

L/G = 9.019 gallons water  
per 1000 ft<sup>3</sup> gas

$$LG = 9.019 \frac{\text{gal}}{1000 \cdot \text{ft}^3}$$

$$LG := \frac{L \cdot \left(8.34 \cdot \frac{\text{lb}}{\text{gal}}\right)^{-1}}{G \cdot \rho_g^{-1}} \quad (l)$$

m) Graph of Y-X equilibrium curve and Mass Balance Operating Line

$$j := 0..7$$

Ystar = Gas phase equilibrium NH<sub>3</sub> cond  
in mole fraction units

$$Y_{\text{out}} = 0.001$$

$$X_{\text{in}} = 0.0006$$

$$Y_{\text{in}} = 0.006$$

$$X_{\text{out}} = 0.003706$$

The 8 X values shown in the matrix  
to the right are for the horizontal axis  
in the graph and span the range from  
X<sub>in</sub> to X<sub>out</sub>

X :=

.0005  
.0006  
.001  
.002  
.0025  
.0030  
.0035  
.003706

Now to put the **operating line** equation  
on an Y-X graph

$$YY_j := \text{OpSlope} \cdot X_j$$

$$\text{OpSlope} = 1.6096803$$

$$\text{OpSlope} := \frac{L_m}{G_m}$$

$$H = 0.761$$

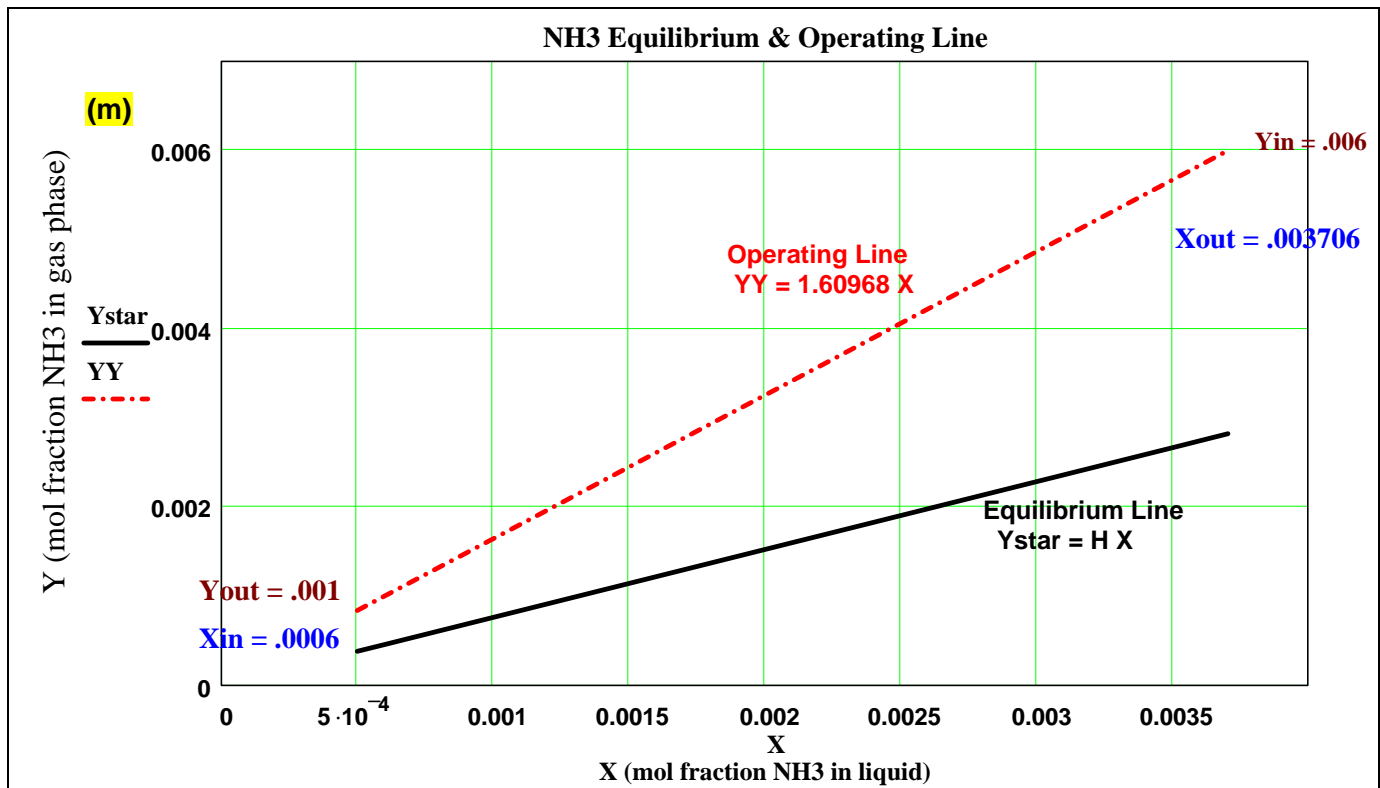
$$Y_{\text{star}_j} := H \cdot (X_j)$$

$$YY_j := (\text{OpSlope}) \cdot (X_j - X_{\text{in}}) + Y_{\text{out}}$$

$$YY_7 = 0.006$$

$$X_7 = 0.003706$$

The operating line equation  $YY = 1.60968 X$  is upper dash-dot line shown in below graph. The difference between the operating line & the lower solid equilibrium equation  $Y_{\text{star}} = H X$  line shows the NH<sub>3</sub> concentration gradient driving force; the concentration gradient ( $y-y^*$ ) causes the NH<sub>3</sub> to move from the gas into the liquid.



$$Y_{\text{in}} = 0.006$$

$$X_{\text{out}} = 0.003706$$

$$Y_{\text{out}} = 0.001$$

$$X_{\text{in}} = 0.0006$$

Govt Emission Std is 1,000ppm NH<sub>3</sub> (dry basis) so Y<sub>out</sub> = 0.001

Remember, the **operating line** represents the *actual* concentrations of NH<sub>3</sub> in the gas and liquid inside the tower and thus these are *measurable* parameters. Y<sub>out</sub> and X<sub>in</sub> are at the top of the tower and Y<sub>in</sub> and X<sub>out</sub> are at the bottom of the tower for countercurrent flow (see illustration on next page)

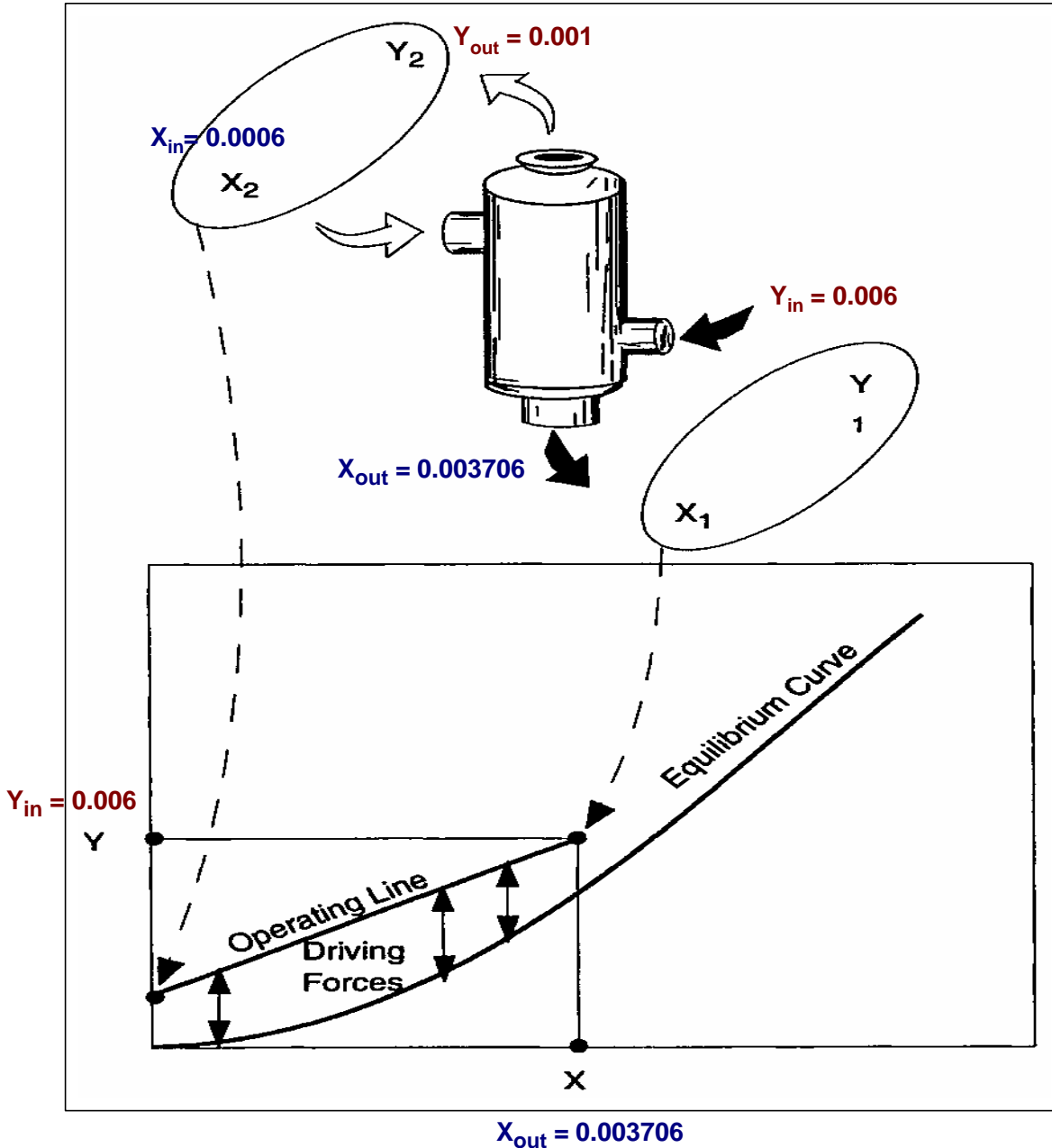
n) Ammonia collection efficiency

$$\text{PercentCollectionEff} := \frac{(Y_{\text{in}} - Y_{\text{out}}) \cdot 100}{Y_{\text{in}}}$$

PercentCollectionEff = 83.333

(n)

Illustration below shows the ammonia absorption inlet and outlet concentrations for a countercurrent absorption tower.



Y = gas phase pollutant concentrations (mole fraction)  
 X = liquid phase pollutant concentrations (mole fraction)

**Pilat Comment:**

I prefer to use  $Y_{\text{in}}$ ,  $Y_{\text{out}}$ ,  $X_{\text{in}}$ , and  $X_{\text{out}}$  rather than  $Y_1$ ,  $Y_2$ ,  $X_1$ , and  $X_2$  because I think it is too easy to forget whether the subscripts 2 and 1 indicate the top or bottom of an absorption tower.