

Kleinschmidt Equation for Calculation of Particle Collection by Objects

Kleinschmidt (who had published journal articles with chemical engineering Professor H. Johnstone, U. of Illinois) developed an equation while working at Arthur D. Little on a project for the Pease-Anthony Equipment Co. of Cambridge, Mass. and published an article entitled "Factors in Spray Scrubber Design" in 1939 in the journal *Chemical and Metallurgical Engineering*. This equation relates the overall scrubber particle collection efficiency η_o to f the fraction of gas swept by the collecting bodies (water droplets, fibers, particles of sand, etc.) and $\eta_{\text{single body}}$ (which is the particle collection efficiency of a single object) and was written for droplet scrubbers with R the distance the drop travels with respect to the gas, L the liquid flow rate, D_d the drop diameter, G the gas vol. flow rate, and f the fraction of gas swept by the collecting drops or objects.

$$\eta_o = 1 - e^{-\left(\frac{3RL}{2D_d G}\right)\eta_{\text{single drop}}} = 1 - e^{-f\eta_{\text{single body}}} \quad (1)$$

For the Pease Anthony cyclonic scrubber Kleinschmidt assumed that the distance the droplet traveled with respect to the gas was approximately the radius of the scrubber and that the single droplet particle collection efficiency was 100% or 1.0 fractionally. He discussed the variation of the single droplet particle collection efficiency with droplet diameter and particle diameter. Kleinschmidt referenced a publication by R. R. Harmon "Removal of Suspended Matter from Industrial Gases", *Institute of Fuels*, (London), April 28, 1938 which presented a similar equation (however, this reference could not be located).

The Kleinschmidt equation can be used for spray droplet scrubbers, for fiber filters, for wire mist eliminators, and essentially for any particle collection device which uses a multiple of the same type of the collection body. For fiber filters with h = filter depth in direction of gas flow, D_f the fiber diameter, and α the filter packing density = (fiber volume)/(filter volume), the fraction of gas swept f is given by:

$$f = \frac{\text{gas volume swept by collecting bodies / time}}{\text{gas volumetric flow rate (gas volume / time)}} \quad \text{For Filters} \quad f = \frac{4\alpha h}{\pi D_f (1-\alpha)} \quad (2)$$

Spray Droplet Scrubbers:

For spray droplet scrubbers the gas volume swept by a spherical drop is $V = (\pi D_d^2 R)/4$ where R is the distance the droplet travels with respect to the gas, D_d the drop diameter, and L the liquid flow rate. The distance the drop travels with respect to the gas $R = (\text{drop velocity})(\text{residence time in scrubber})$. A droplet sprayed from a nozzle has an initial velocity of $(\text{spray nozzle liquid flow rate}/\text{nozzle opening area})$ and changes with time as the gas fluid drag usually slows the droplet until it reaches the sides or bottom of the scrubber.

$$\text{The number of spray droplets introduced } n = \frac{\text{no drops}}{\text{unit time}} = \frac{\text{liquid flow rate}}{\text{volume 1 drop}} = \frac{6L}{\pi D_d^3} \quad (3)$$

$$\text{Gas vol swept by } n \text{ droplets / time} = nV = \left(\frac{6L}{\pi D_d^3}\right)\left(\frac{\pi D_d^2 R}{4}\right) = \frac{3RL}{2D_d} \quad (4)$$

Substituting into the f equation for R in ft, L in gal/min, D_d in microns, and G in ft³/min.

$$f = \left(\frac{3RL}{2D_d}\right) / G = \frac{3RL}{2D_d G} = \frac{3(R, \text{ft})(3.05 \times 10^5 \text{ microns / ft})(L, \text{gal / min})}{2(D_d, \text{microns})(G, \text{ft}^3 / \text{min})(7.48 \text{ gallon / ft}^3)} = 61,200 \frac{RL}{D_d G} \quad (5)$$

Fiber Filters or Wire Demisters:

Given: h = filter thickness in direction of gas flow b = length of fiber perpendicular to gas flow

L = filter height b = filter width V_g = Gas velocity upstream of filter

f = fraction of gas swept by collecting objects (fibers or wires)

$$\text{Packing Density} = \alpha = \frac{\text{fiber volume}}{\text{filter volume}} = (1 - \text{fractional free volume}) \quad D_f = \text{fiber diameter}$$

$$f = \frac{(\text{gas volumetric flow swept by fibers, gas vol./time})}{(\text{gas volumetric flow through entire filter, gas vol./time})} = \frac{4 \alpha h}{\pi D_f (1 - \alpha)} \quad (6)$$

Presenting equations for the parameters as the gas flow through the filter:

$$\text{Gas velocity inside filter} = \frac{V_g}{1 - \alpha} \quad (7)$$

$$\text{Gas swept by fibers} = (\text{fiber x-sectional area } \perp \text{ to gas flow, cm}^2)(\text{gas vel in fiber filter, cm/s}) = \text{cm}^3/\text{s} \quad (8)$$

$$\text{fiber x-sectional area } \perp \text{ to flow} = (\text{x-sect area 1 fiber, cm}^2)(\text{no. fibers / entire filter}) = \text{cm}^2 / \text{filter} \quad (9)$$

$$\text{x-sect. area 1 fiber} = (D_{\text{fiber}}, \text{ cm})(\text{fiber length, cm}) = (D_f)(b) = \text{cm}^2 \quad (10)$$

$$\alpha = \frac{\text{fiber volume}}{\text{filter volume}} = \frac{(\text{volume of 1 fiber, cm}^3) (\text{no. fibers / entire filter})}{(\text{filter height, cm})(\text{filter depth, cm})(\text{filter width, cm})} \quad (11)$$

Solving for the no. of fibers/entire filter using eq. 11

$$\frac{\text{no. fibers}}{\text{entire filter}} = \frac{(\alpha) (\text{Filter Height, cm}) (\text{Filter Width, cm}) (\text{Filter Depth, cm})}{(\text{volume of 1 fiber, cm}^3)} = \frac{\alpha L b h}{\pi (D_f / 2)^2 (b)} \quad (12)$$

Substituting for no. fibers/entire filter in eq. 9 to get fiber x-section and then substituting into eq. 8 to get gas swept by all the fibers

$$\text{Gas swept by fibers} = (D_f b, \text{ cm}^2) \left(\frac{\alpha 4 L b h}{\pi D_f^2 b} \right) \left(\frac{V_g, \text{ cm/s}}{1 - \alpha} \right) = \left(\frac{\alpha 4 L b h}{\pi D_f} \right) \left(\frac{V_g}{1 - \alpha} \right) = \frac{\text{cm}^3}{\text{s}} \quad (13)$$

Substituting into eq. 6 for the gas swept by fibers/ gas volumetric flow and f

$$f = \frac{(\text{gas swept by fibers})}{(\text{gas flow filter})} = \frac{\left[\frac{\alpha 4 L b h V_g}{\pi D_f (1 - \alpha)} \right]}{(V_g L b)} = \frac{[4 \alpha h]}{\pi D_f (1 - \alpha)} \quad (14)$$

Equation 14 is the same as eq. 2 for the fraction of gas swept by fibers