Control of Particulate Air Pollutant Emissions

A. Equation for Overall Collection Efficiency of Air Pollution Control Device

$$\eta_{\mathrm{Overall}} := \frac{(\mathrm{WeightParticlesCollected})}{(\mathrm{WeightParticlesEnteringControlDevice})}$$

$$\eta_{\mathrm{Overall}} := \frac{\left(C_{\mathrm{in}}\cdot \mathrm{GasFlow}_{\mathrm{in}}\right) - \left(C_{\mathrm{out}}\cdot \mathrm{GasFlow}_{\mathrm{out}}\right)}{\left(C_{\mathrm{in}}\cdot \mathrm{GasFlow}_{\mathrm{in}}\right)}$$

Assuming GasFlowin = GasFlowout, the Gas Flow terms cancel out to give:

$$\eta_{\mathrm{Overall}} := \frac{\left(C_{\mathrm{in}}\right) - \left(C_{\mathrm{out}}\right)}{\left(C_{\mathrm{in}}\right)}$$

B. Kleinschmidt Equation

Kleinschmidt (chemical engineer) developed an equation (published in 1939) to relate the particle collection efficiency of a "Pease Anthony Wet Scrubber" to the scrubber operating parameters. This equation can be shown to relate the particle overall particle collection efficiency to the fraction of gas swept by the collecting objects (droplets for wet scrubbers, fibers for filters) and the particle collection of a single collecting object (ie drops or filter fibers).

$$\eta_{\mathrm{Overall}} := 1 - \exp\left[-(\mathrm{FractionGasSwept})\cdot\left(\eta_{\mathrm{SingleObject}}\right.\right]$$

The fraction of gas swept by the collecting bodies (filter fibers or scrubbing liquid droplets) is given by:

$$\mathrm{FractionGasSwept} := \frac{(\mathrm{GasVolFlowSweptByCollectingBodies})}{(\mathrm{TotalGasVolFlow})}$$

The Fraction of Gas Swept is commonly identified as the parameter f

For Fiber Filters

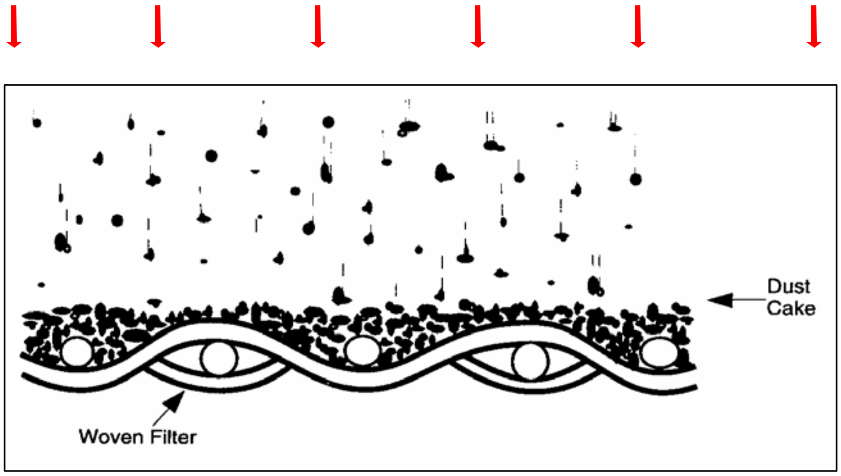
$$f_{\mathrm{FiberFilters}} := \frac{4\cdot\alpha\cdot h}{\pi\cdot D_{\mathrm{Fiber}}\cdot(1-\alpha)}$$

h = Filter Depth in gas flow direction

D_{Fiber} = Fiber Diameter

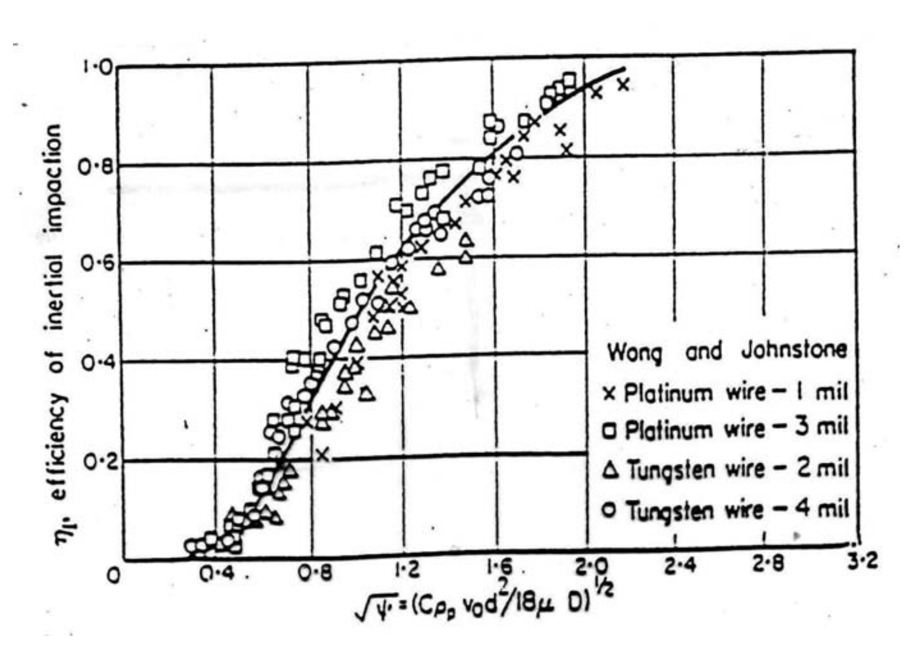
α = packing density = (fiber volume/filter volume)

Note that this fraction of gas swept does not include the duct cake filtration. It only includes the gas filtration by the filter fibers.



C Particle Collection Efficiency of single fibers by the inertial impaction mechanism

A graduate student Wong (Chemical Engr), performing research under the direction of Professor Johnstone, measured the particle collection efficiency of single wires (platinum and tungsten wires) in the 1 to 4 mil diameter range. Their research results were published in an U of Illinois Engineering Experimentation Report (1953). The single wire particle collection efficiencies versus the square root of the inertial impaction parameter are shown in the graph below.



1 inch = 1000 mil

1 mil = 0.001 inch

D_f = fiber diameter

$$\psi := \frac{(\text{"particle stop distance"})}{(\text{"fiber diameter"})}$$

The inertial impaction parameter Ψ, a dimensionless parameter, is defined as

$$\Psi := \frac{C \cdot \rho_{particle} \cdot u_{gas} \cdot \left(d_{particle}\right)^2}{18 \cdot \mu_{gas} \cdot D_{Fiber}}$$

Where
u_{gas} = gas velocity past the stationary filter fiber
$d_{particle}$ = particle diameter
C = Cunningham slip Correction factor of the particle
μ_{gas} = gas dynamic viscosity
D_{Fiber} = fiber diameter

Inertial Impaction Parameter = ψ

$i := 0 .. 11$

Wong & Johnstome Measured Particle Collection Efficiencies on Single Wires = η_m

"Equation for Inertial Particle Collection on a Single Fiber at Collector Re = 10" by Landahl and Hermann, *J. Colloid Science*, Vol 4 pp 103 (1949)

This calculation is to compare the single fiber collection efficiency calculated using the Landahl & Hermann equation with data measured by Professor Johnstone & student Wong at U of Illinois and reported in U. of Illinois Engr Exp Station Report #11, 1953.

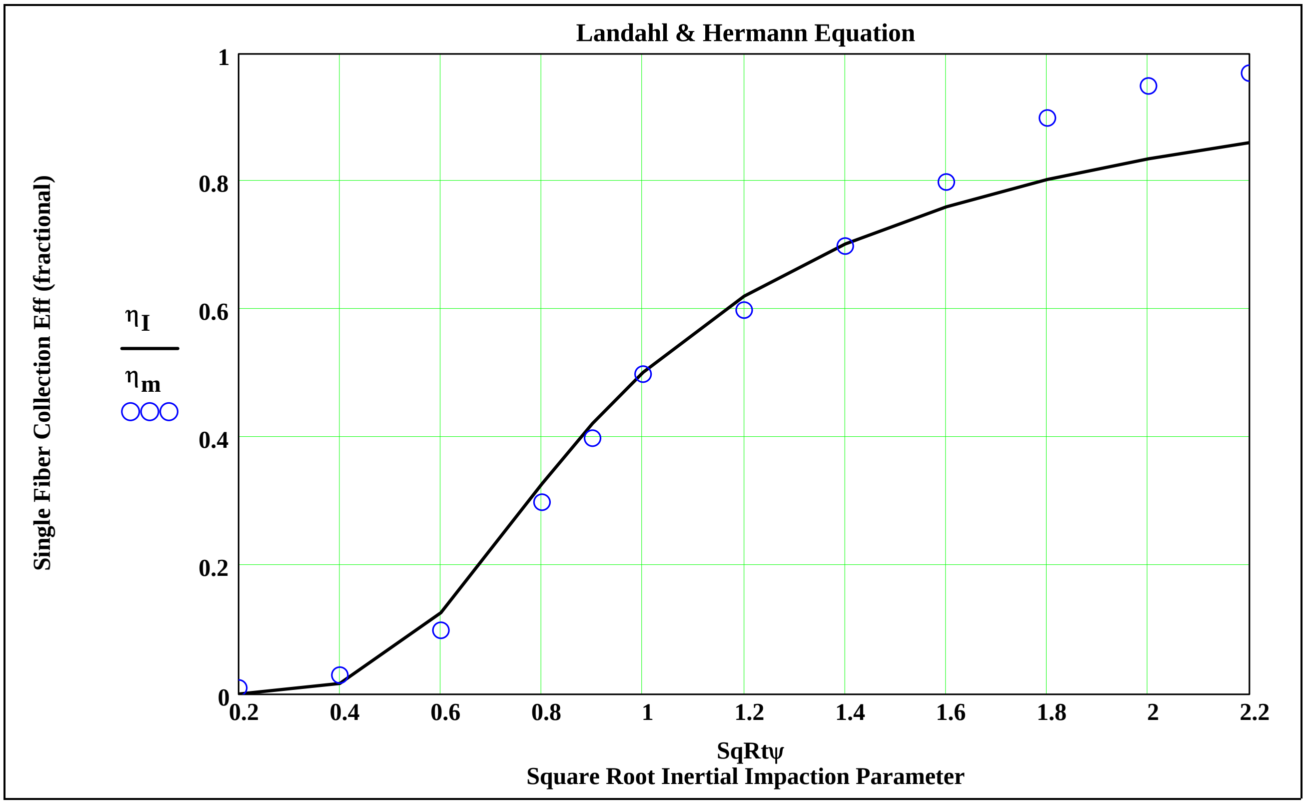
$$\eta_m := \begin{pmatrix} .01 \\ .03 \\ .1 \\ .3 \\ .4 \\ .5 \\ .6 \\ .7 \\ .8 \\ .9 \\ .95 \\ .97 \end{pmatrix} \qquad \psi := \begin{pmatrix} .04 \\ .16 \\ .36 \\ .64 \\ .81 \\ 1.0 \\ 1.44 \\ 1.96 \\ 2.56 \\ 3.24 \\ 4.0 \\ 4.84 \end{pmatrix}$$

Landahl & Hermann Equation for Single Fiber or Single Wire Particle Collection Efficiency due to Inertial Impaction = η_I

$$\eta_{I_i} := \frac{(\psi_i)^3}{(\psi_i)^3 + 0.77 \cdot (\psi_i)^2 + 0.22}$$

$$SqRt\psi_i := \sqrt{\psi_i}$$

Note that the inertial impaction parameter Ψ (used in US) and the Stokes number Stk (used in Europe) have sometimes been mistakenly identified (ie asssumed they were identical)



The comparison shown in the above graph indicates that the Wong & Johnstone measured data agrees with the Landahl and Hermann empirical equation for the collection of aerosol particles on single wires. Note that the books and journal articles which show the Stokes number Stk in the Landahl & Hermann equation have possibly misidentifed the definition for Stk.

$\mathrm{Stk} := 2 \cdot \psi$

Stokes Number = Stk

$$\mathrm{Stk} := \frac{(\text{"particle stop distance"})}{(\text{"fiber radius"})}$$

The collector Reynolds number = $\mathrm{Re}_{\mathrm{Fiber}}$

$$\mathrm{Re}_{\mathrm{Fiber}} := \frac{D_{\mathrm{Fiber}} \cdot u_{\mathrm{gas}} \cdot \rho_{\mathrm{gas}}}{\mu_{\mathrm{gas}}}$$

$$\mathrm{Re}_{\mathrm{Fiber}} := \frac{D_{\mathrm{Fiber}} \cdot u_{\mathrm{gas}}}{\nu_{\mathrm{gas}}}$$

Where

u_{gas} = gas velocity past fiber

ρ_{gas} = gas density

μ_{gas} = gas dynamic viscosity

ν_{gas} = gas kinematic viscosity

$\mu\mathrm{m} := \mathrm{m} \cdot 10^{-6}$

D. Cunningham Slip Correction Factor

Cunningham Slip Correction Factor as f (Temp = 20 & 100ºC) at 1 atm.

Gas Kinematic Viscosity = $\mu/\rho = \nu$

T=Temp. Deg. K

$RG := 0.082054 \cdot \frac{liter \cdot atm}{mole \cdot K}$

$T := \begin{pmatrix} 293.33 \\ 373.33 \end{pmatrix} \cdot K$

$j := 0 .. 1$

$i := 0, 1 .. 23$

$M_{gas} := 29 \cdot \frac{gm}{mole}$

RG = universal gas constant

$A := \left(\frac{8 \cdot RG}{\pi \cdot M_{gas}} \right)$

$T = \begin{pmatrix} 527.994 \\ 671.994 \end{pmatrix} R$

dia = particle diameter

$\nu := \begin{pmatrix} 0.151 \\ 0.231 \end{pmatrix} \cdot \frac{cm^2}{(sec)}$

$U_j := (A \cdot T_j)^{\frac{1}{2}}$

$U = \begin{pmatrix} 4.628 \times 10^4 \\ 5.221 \times 10^4 \end{pmatrix} \frac{cm}{sec}$

Gas molecule mean free path = λ in cm

$\lambda_j := \frac{(\nu_j)}{(0.499) \cdot (U_j)}$

$\lambda = \begin{pmatrix} 0.0654 \\ 0.0887 \end{pmatrix} \mu m$

Mean velocity of air molecule of MW = 29 = U in cm/sec

U = sq.rt [(8RT) / (πM)]

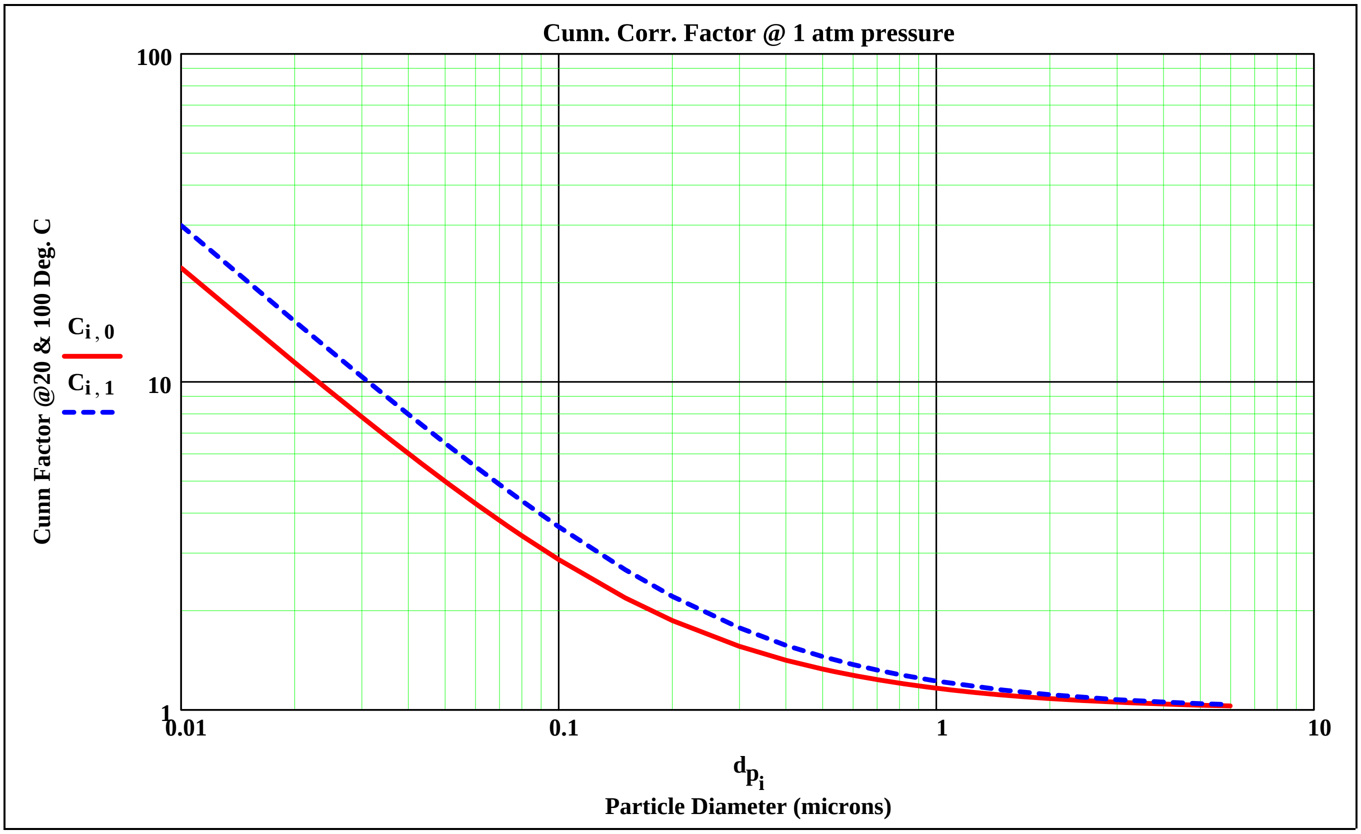
Cunningham Slip Correction Factor = C

$$C_{i,j} := 1 + \left(\frac{\lambda_j}{dia_i} \right) \cdot \left[2.514 + 0.80 \cdot e^{-(.55) \cdot \frac{(dia_i)}{(\lambda_j)}} \right]$$

$dia := \begin{pmatrix} 0.01 \\ 0.02 \\ 0.03 \\ 0.04 \\ 0.05 \\ 0.06 \\ 0.07 \\ 0.08 \\ 0.10 \\ 0.15 \\ 0.20 \\ 0.30 \\ 0.40 \\ 0.50 \\ 0.60 \\ 0.70 \\ 0.80 \\ 1.0 \\ 1.5 \\ 2.0 \\ 3.0 \\ 4.0 \\ 5.0 \\ 6.0 \end{pmatrix} \cdot \mu m$

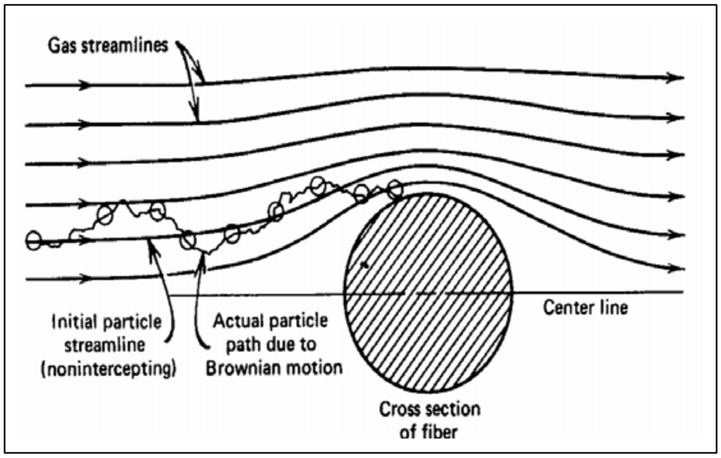
In the graph below, the upper dashed curve is for T = 100ºC (212ºF) and the lower solid curve is for T = 20ºC (68ºF)

$d_p := 10^6 dia$



E. Particle Collection Efficiency of a Single Fiber due to Brownian Diffusion

There are a number of equations for the single fiber particle collection due to the Brownian Diffusion Mechanism



Ku = Kuwabara Hydrodynamic factor

Pe = Peclet Number

Sc = Schmidt Number

Δ = Particle Diffusivity

k = Boltzmann Constant

$$k := \left(1.38032 \cdot 10^{-18}\right) \cdot \left(\frac{cm^2 \cdot gm}{K \cdot sec^2}\right)$$

$$\Delta := \frac{C \cdot k \cdot T}{3 \cdot \pi \cdot \mu_g \cdot d_p}$$

$$Pe := \frac{D_{Fiber} \cdot u_{gas}}{\Delta}$$

$$i := 1 .. 18$$

Example Calculation

α = .05 19 particle diameters d_p

T = 80F Gas velocity = u_g = 9 ft/min

D_{Fiber} = 0.001cm = 10 μm

$$u_g := 9 \cdot ft \cdot min^{-1}$$

$$T := 540 \cdot R$$

$$\alpha := 0.05$$

$$T = 300 K$$

$$D_{Fiber} := 0.001 \cdot cm$$

$$d_p := \begin{pmatrix} 30 \\ 15 \\ 9 \\ 7 \\ 5 \\ 3 \\ 1.5 \\ .95 \\ .85 \\ .75 \\ .65 \\ .55 \\ .45 \\ .35 \\ .25 \\ .15 \\ .075 \\ .035 \\ .015 \end{pmatrix} \cdot \mu m$$

The calculation of the single wire particle collection efficiency over the particle diameter range from 0.015 to 30 microns diameter all for the same conditions (gas velocity, fiber diameter, temp,etc.)

gas density = ρ_g

Gas Kinematic Viscosity = $\mu_g/\rho_g = \nu_g$

$$\nu_g := 0.157 \cdot \frac{cm^2}{(sec)}$$

$$M_g := 29 \cdot gm \cdot mole^{-1}$$

$$\mu_g := 0.000185 \cdot \frac{gm}{cm \cdot sec}$$

$$\rho_g := \frac{\mu_g}{\nu_g}$$

Gas molecular wt = M

$$\rho_g = 1.178 \times 10^{-3} gm \cdot cm^{-3}$$

Mean velocity of air molecule of MW = 29 = U in cm/sec

$$RG := 0.082054 \cdot \frac{liter \cdot atm}{mole \cdot K}$$

$$U_{Air} := \left(\frac{8 \cdot RG \cdot T}{\pi \cdot M_g}\right)^{\frac{1}{2}}$$

$$\lambda := \frac{(\nu_g)}{(0.499) \cdot (U)}$$

$$U = \begin{pmatrix} 4.628 \times 10^4 \\ 5.221 \times 10^4 \end{pmatrix} \frac{cm}{sec}$$

Gas molecule mean free path = λ in cm

$$\lambda = \begin{pmatrix} 0.068 \\ 0.0603 \end{pmatrix} \mu m$$

Cunningham Slip Correction Factor = Cc

$$Cc_i := 1 + \left(\frac{\lambda}{d_{p_i}}\right) \cdot \left[2.514 + 0.80 \cdot e^{-(.55) \cdot \frac{(d_{p_i})}{(\lambda)}}\right]$$

$$\Delta_i := \frac{Cc_i \cdot k \cdot T}{3 \cdot \pi \cdot \mu_g \cdot d_{p_i}}$$

$$Pe_i := \frac{D_{Fiber} \cdot u_g}{\Delta_i}$$

Ku = Kuwabara Hydrodynamic factor

$$Ku := -0.5 \cdot \ln(\alpha) - 0.75 + \alpha - 0.25 \cdot \alpha^2$$

$$Ku = 0.797$$

$$\eta_{BDLeeLiu_i} := (2.58)\cdot\left(\frac{1-\alpha}{Ku}\right)\cdot\left(Pe_i\right)^{-.6667}$$

Lee, K. W. and Professor Ben Y. H. Liu (U of Minnesota Dept of Mech Engr) "Theoretical Study of Aerosol Filtration" *Aerosol Science & Technology*, Vol 1, pp147-161 (1982)

$$Sc_i := \frac{\nu_g}{\Delta_i}$$

$$Re_{Fiber} := \frac{D_{Fiber}\cdot u_g}{\nu_g}$$

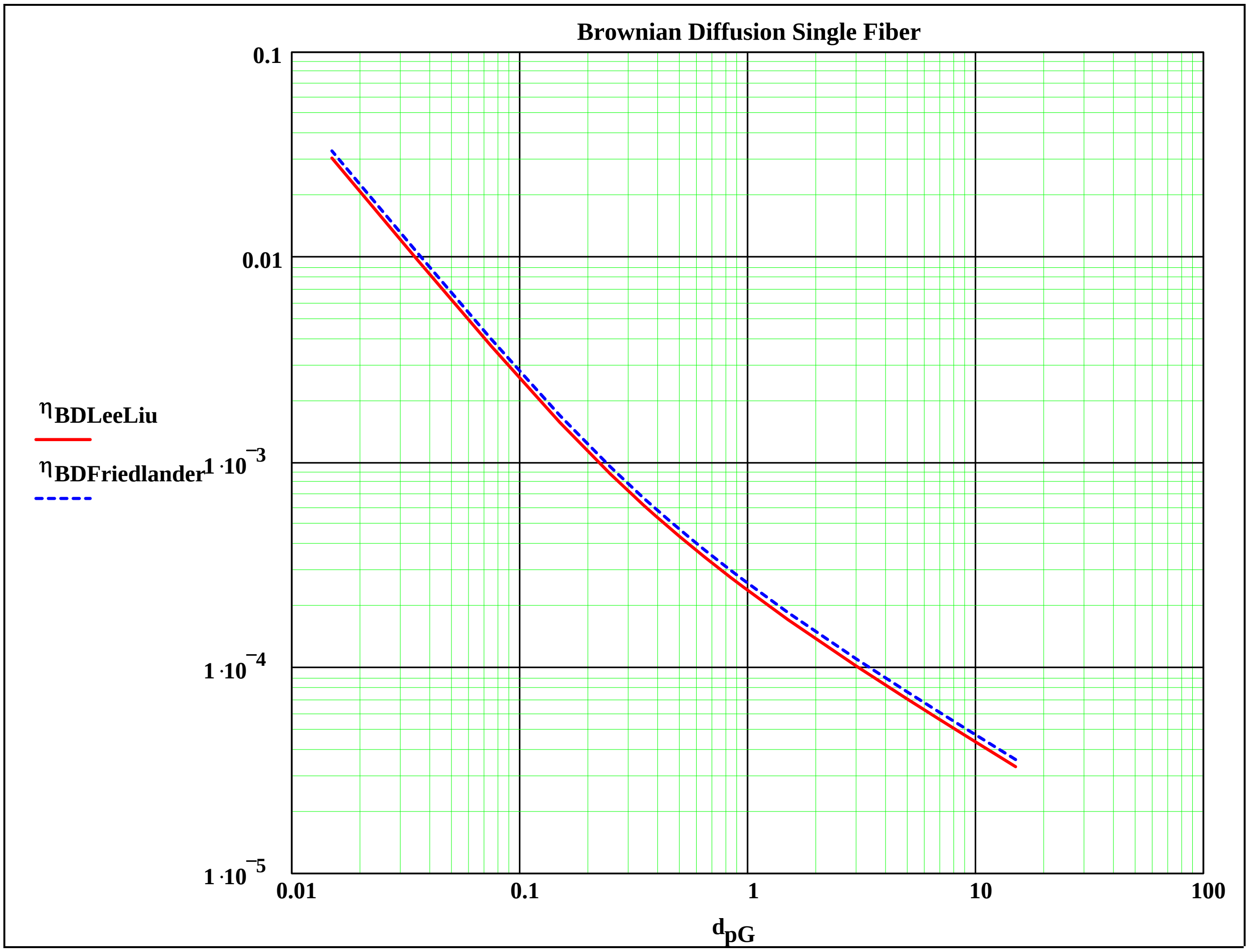
$$Re_{Fiber} = 0.029$$

$\eta_{BDLeeLiu}$ is the single fiber particlecollection efficiency due to Brownian motion

$$d_{pG} := 10^6 d_p$$

$$\eta_{BDFriedlander_i} := 6\cdot\left(Sc_i\right)^{-\left(\frac{2}{3}\right)}\cdot\left(Re_{Fiber}{}^{-0.5}\right)$$

$\eta_{BDFriedlander}$ equation reported by Professor Sheldon Friedlander & R Pasceri, *Canadian Journal of Chemical Engineering* Vol 38, page 212 (1960).



In the graph above, the solid red line is the fractional single fiber collection efficiency calculated using the Lee and Liu equation and the upper dashed blue curve was calculated using the Friedlander and Pasceri equation (both for the Brownian Diffusion mechanism). The Friedlander equation results are slightly larger than the Lee & Liu results.

$$\Psi_i := \frac{Cc_i\cdot\rho_{particle}\cdot u_g\cdot\left(d_{p_i}\right)^2}{18\cdot\mu_g\cdot D_{Fiber}}$$

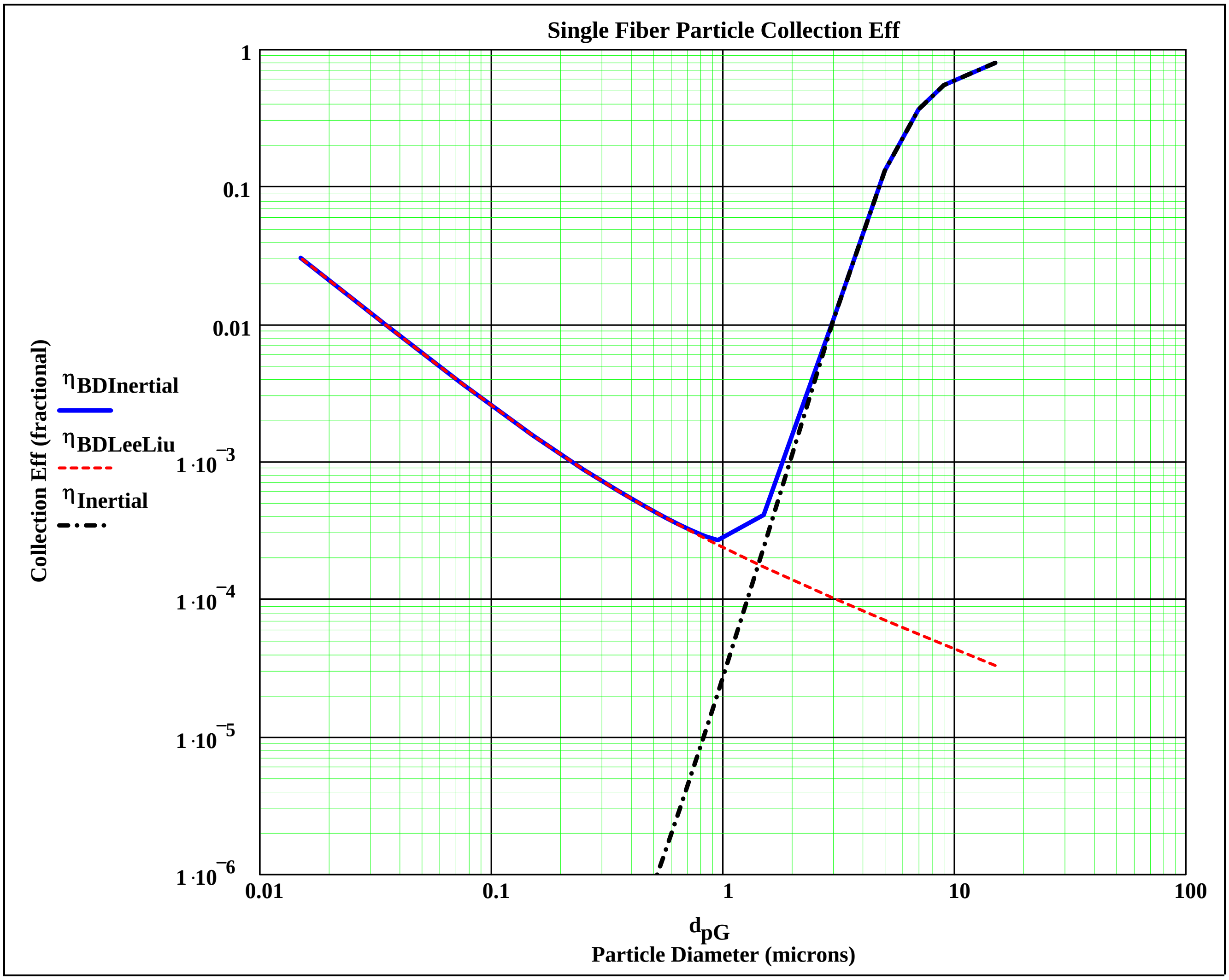
$$\rho_{particle} := 1.0\cdot gm\cdot cm^{-3}$$

To compare the inertial & Brownian diffusion particle collection mechanisms, & the combined mechanisms collection efficiency $\eta_{BDInertial}$, the single fiber particle collection due to inertial impaction $\eta_{Inertial}$ will be calculated

$$\eta_{Inertial_i} := \frac{\left(\Psi_i\right)^3}{\left(\Psi_i\right)^3 + .77\cdot\left(\Psi_i\right)^2 + .22}$$

$$\eta_{BDInertial_i} := 1 - \left(1 - \eta_{Inertial_i}\right)\cdot\left(1 - \eta_{BDLeeLiu_i}\right)$$

In the graph below the dashed red line is for particle collection on a single fiber due to the Brownian diffusion mechanism and the solid dash-dot curve on the right side is particle collection due to inertial impaction. The solid blue curve is for the single fiber particle collection efficiency due to the combined Brownian diffusion & Inertial impaction mechanisms.



There also can be particle collection by the interception mechanism. However, the classic interception of particles by single fibers and single spherical drops was first described by equations because the true finite diameters of the aerosol particles were not considered during particle numerical trajectory calculations because the early computers did not have sufficient memory and computing capability to handle the particle finite diameter. So they only calculated the trajectories of the center of the particles and later corrected for the actual particle diameter and named this correction "interception". My graduate student Anil Prem (Anil works for a metallurgical company near Albany Oregon) did the particle trajectory calculations including the particle finite diameters and we published these results.

Example Problem: Particle Collection by York Model 421 Wire Mesh Demister

Given York Style 421 wire mesh mist eliminator (Demister)
Wire diameter = Dwire = Dcollector = 0.01 inches
Demister thickness in direction of gas flow h = 3 inches
Demister Wire surface are = 110 square ft / cubic ft
Free volume = 97.7% Volume occupied by wire = 100 - 97.7 = 2.3%

Gas Velocity = Vgas = 10 ft/sec Gas temperature = T = 171°F

Particle density = ρ_p = 1.00 grams/cm^3

Gas dynamic viscosity = μ_g = 0.000207777 gm/cm sec

Gas density = ρ_g = 0.001007 gm/cm^3

wire mesh filter packing density = α = 1 - fractional free volume
α = (volume wire fibers)/(volume entire mesh filter)
Landahl & Hermann equation for calculating $\eta_{inertial}$ for single fiber or wire
"Wire Mesh Mist Eliminators" by York & Poppele (1963)

$$\mu m := m \cdot 10^{-6}$$

$$T := (171 + 460) \cdot R$$

$$T = 350.556\,K$$

$$\alpha := 0.023 \qquad i := 0 .. 3$$

$$d_p := \begin{pmatrix} 2.0 \\ 3.0 \\ 4.0 \\ 5.0 \end{pmatrix} \cdot \mu m$$

$$\rho_p := 1.0 \cdot \frac{gm}{cm^3}$$

$$\mu_g := \left(2.07777 \cdot 10^{-4}\right) \cdot \frac{gm}{cm \cdot sec} \qquad \rho_g := 0.001007 \cdot \frac{gm}{cm^3} \qquad Vgas := 10.0 \cdot \frac{ft}{sec}$$

$$\nu_g := \frac{\mu_g}{\rho_g}$$

Find: a) Fraction of gas swept by wires in demister = f
b) Single wire particle collection efficiency = $\eta_{inertial}$
c) Mesh mist eliminator overall particle collection eff. as f (dp) = η_o.
d) *Graph overall & single wire collection eff on provided graph.*

$$\nu_g = 0.2063 \frac{cm^2}{sec}$$

The Cunningham Slip Correction factors C of the particles are needed.

Gas (air) molecular wt = M

$$M := 29 \cdot gm \cdot mole^{-1} \qquad R_G := 0.082054 \cdot \frac{liter \cdot atm}{mole \cdot K} \qquad D_{wire} := 0.01 \cdot in$$

The universal gas constant = R_G

Mean velocity of air molecule of MW = 29 = U in cm/sec

$$U := \left(\frac{8 \cdot R_G \cdot T}{\pi \cdot M}\right)^{\frac{1}{2}} \qquad U = 50589.1835 \frac{cm}{sec}$$

Gas molecule mean free path = λ in cm

$$\lambda := \frac{(\nu_g)}{(0.499) \cdot (U)} \qquad \lambda = 0.0817\,\mu m$$

Cunningham Slip Correction Factor = CC

$$CC_i := 1 + \left(\frac{\lambda}{d_{p_i}}\right) \cdot \left[2.514 + 0.80 \cdot e^{-(.55) \cdot \frac{(d_{p_i})}{(\lambda)}}\right]$$

$$CC = \begin{pmatrix} 1.103 \\ 1.068 \\ 1.051 \\ 1.041 \end{pmatrix} \qquad d_p = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \mu m$$

$$\Psi_i := \frac{CC_i \cdot \rho_p \cdot Vgas \cdot \left(d_{p_i}\right)^2}{18 \cdot \mu_g \cdot D_{wire}}$$

$\Psi_i =$
0
3.156
1.152
0.704

Inertial Impaction Parameter = Ψ

$$\eta_{interial_i} := \frac{(\Psi_i)^3}{(\Psi_i)^3 + .77 \cdot (\Psi_i)^2 + .22}$$

Landahl & Hermann equation J. Colloid Science Vol 4, p. 103 (1949)
Single Wire Particle Collection Efficiency due to Inertial Impaction = $\eta_{inertial}$

Note that the single wire particle collection efficiency due to Brownian Diffusion is not being calculated because the gas velocity is too large to enable the particle Brownian diffusion mechanism to be significant.

(b)

$$\eta_{interial} = \begin{pmatrix} 0.011898 \\ 0.091034 \\ 0.261387 \\ 0.434824 \end{pmatrix}$$

Fraction of Gas Swept by Demister Filter Wire Fibers = f

$$f := \frac{4\cdot\alpha\cdot h}{\pi\cdot D_{wire}\cdot(1-\alpha)}$$

$\alpha = 0.023$

$D_{wire} = 0.01\,in$

$h := 3\cdot in$

Kleinschmidt Equation
η = fractional overall particle collection efficiency for gas flow through multiple collection objects (in this case the gas flows around many wires located perpendicular to the gas flow direction)

(a) $f = 8.992$

h = Demister thickness in direction of gas flow

$$\eta_{o_i} := 1 - e^{-f\cdot\eta_{interial_i}}$$

$$Pt_i := 100\cdot\left(1-\eta_{o_i}\right)$$

Pt = % particle penetration through demister

$$Pt = \begin{pmatrix} 89.85328 \\ 44.10515 \\ 9.53275 \\ 2.00402 \end{pmatrix}$$

dpp is to show units in microns (be careful here, Mathcad default graph units differ from version to version)

$$dpp_i := \left(10^6\right)\cdot d_{p_i}$$

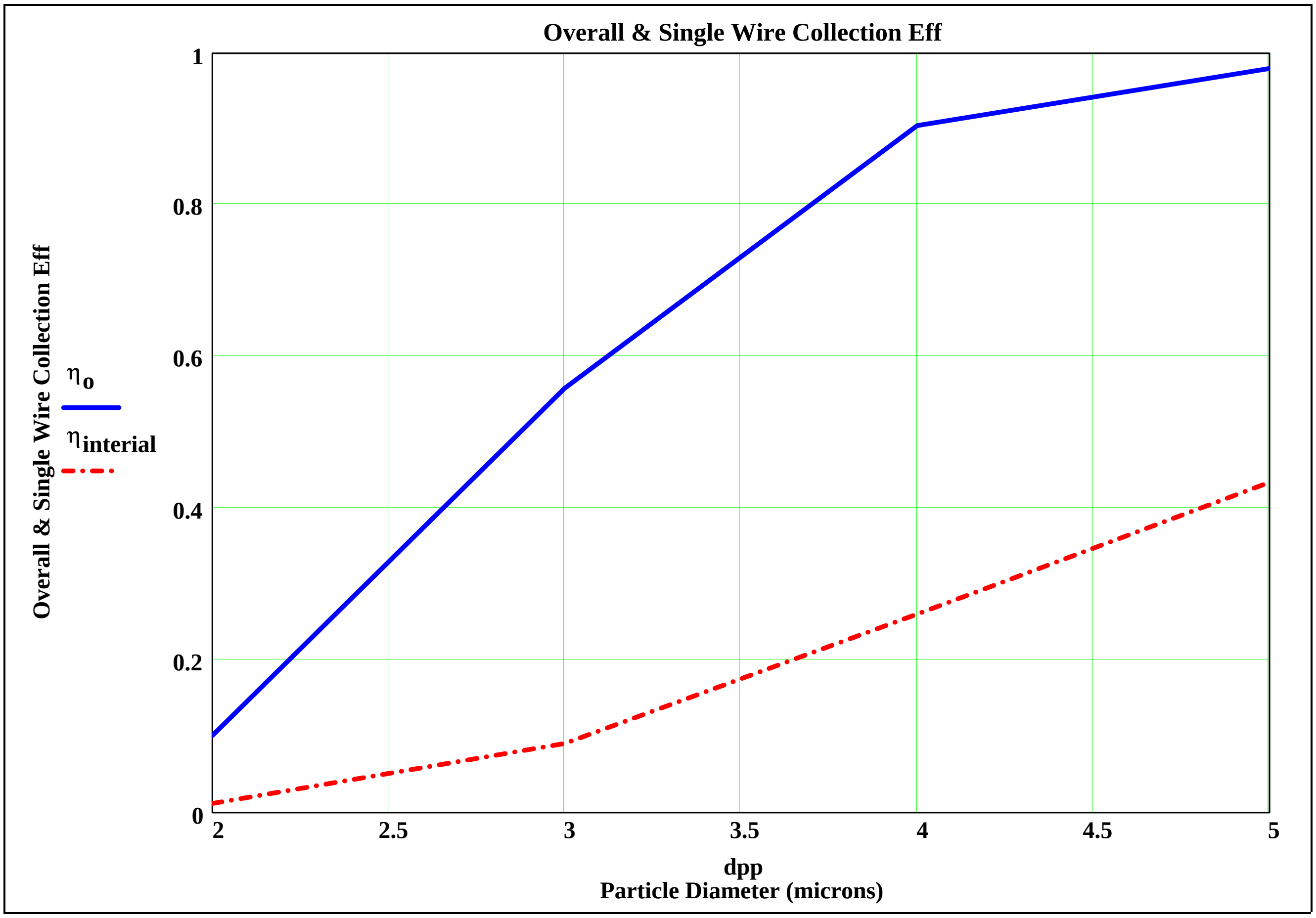
(c)

$$\eta_o = \begin{pmatrix} 0.10147 \\ 0.55895 \\ 0.90467 \\ 0.97996 \end{pmatrix}$$

$$d_p = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}\mu m$$

(d)

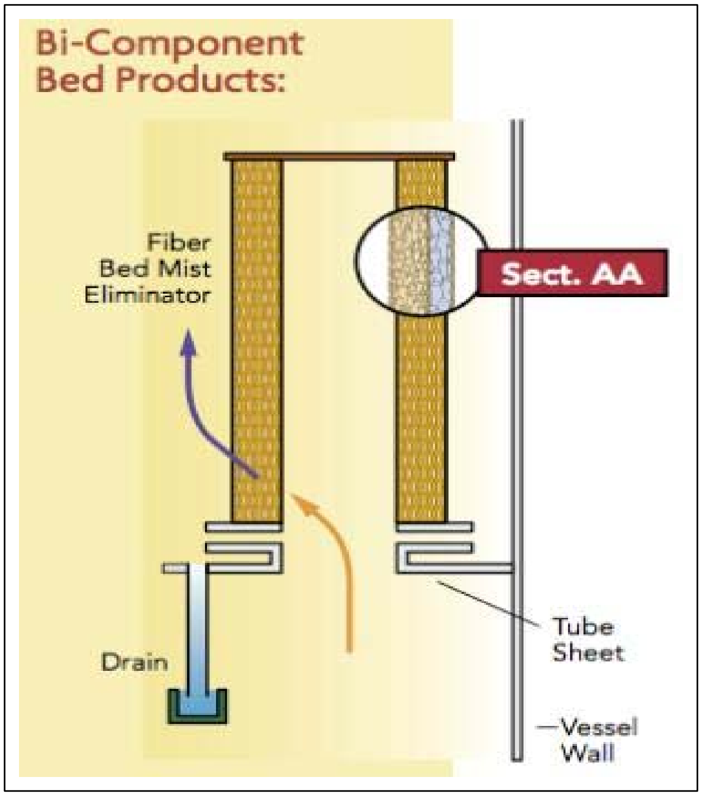
In the graph below the upper solid blue curve is the fractional overall particle collection efficiency and the lower red dash dot curve is the fractional single wire particle collection efficiency.

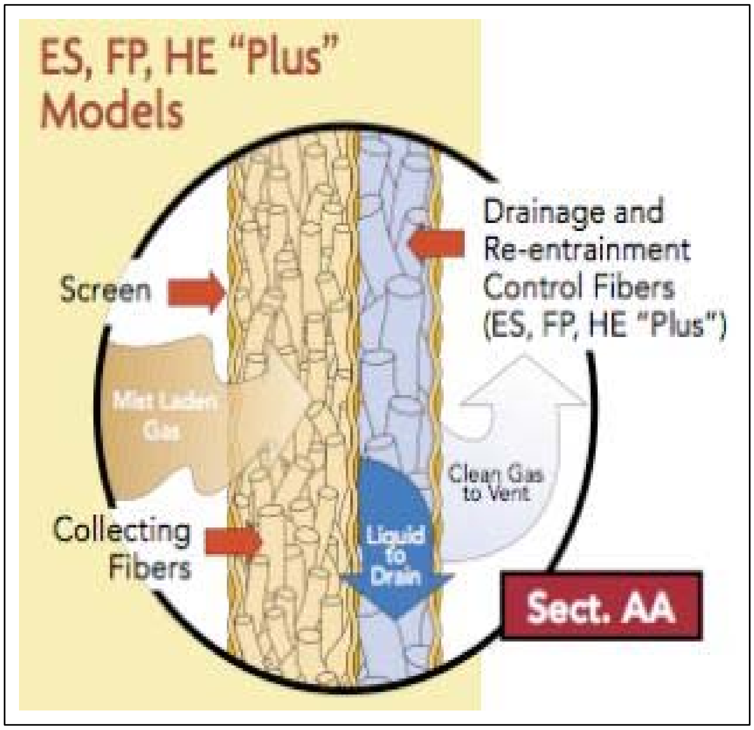


e) Note that there are graphs of the pressure drop vs gas velocity in the article "Wire Mesh Mist Eliminators" by O. H. York & E. W. Popple, Chem Engr Progress Vol 59 No 6 pp 2-7 (June 1960).

Demister or Mist Eliminator

Flexifiber Mist Eliminator





MECS Inc is apparently the newer name for Monsanto Enviro-Chem. The history of MECS, Inc. dates back to 1969 when Monsanto Enviro-Chem Systems Inc. was formed as a wholly owned subsidiary of Monsanto Company through a consolidation of several existing process technology, product and services businesses. Some of these businesses were established as early as 1917.

Diffusion Fiber Bed Mist Eliminator

The ES (Energy Saver) Fiber BedMist, Elminator shown in the diagram to the left, incorporates a special wound fiber, computer controlled quality, and a bi-component design. ES elements are made with our proprietary, one-of-kind wrapping machine. The machine wraps the fiber in a unique angled pattern, which facilitates drainage. In addition, the pressure drop across the element is monitored during the entire wrapping process ensuring that all mist eliminators in a set are packed to a uniform pressure drop.

The heart of the ES is its bi-component fiber bed design.

In a conventional fiber bed, liquid draining on the downstream face of the fiber bed is sometimes re-entrained in the exiting clean gas. This re-entrainment becomes worse with higher loadings of mist and/or higher flow velocities.

The HE Fiber Bed Mist Eliminator is the original design that was developed in 1960 by Joe Brink (hence it was called the Brink mist eliminator; later Brink was a chemical engineering professor at Wash State U) .

HE elements are still in use in many applications, and are typically used for soluble salt applications.

The HE Element consists of fibers which are packed between two concentric cylindrical screens. HE elements vary in diameter from 8.5 inches to 24+ inches (216mm to 610+ mm), and are available in lengths up to 288 inches (7315 mm).

Fibers may be glass, polypropylene , polyester or ceramic. The screens can be fabricated from any weldable metal or can be made of fiberglass reinforced plastic or polypropylene. The HE "Plus" adds a re-entrainment control layer similar to the ES design.