1) The probability of contracting a certain disease (event D) during one's lifetime is very small, with probability P(D)=0.001. However, if left untreated the disease is always fatal. Fortunately, modern medical science has provided a diagnostic test T to detect the presence of the disease; however, the test is not always correct. If a person has the disease, there is only 85% probability that the test will be positive, i.e. $P(T \mid D) = 0.85$. Also, there is a small probability of 2% that the test will be positive even when a patient does not have the disease, $P(T \mid \overline{D}) = 0.02$.
   - Draw and label the probability trees.
   - What is the probability of a positive test result?
   - If a person's test result is positive, what is the probability that he has the disease?

2) The size in millimeters of a crack in a structural weld is described by a random variable X with the following pdf:

$$f_X(x) = \begin{cases} \frac{x}{8} & 0 < x \le 2 \\ \frac{1}{4} & 2 < x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

   - Sketch the pdf and cdf on a piece of paper.
   - Determine the mean crack size.
   - What is the probability that a crack will be smaller than 4 mm?
   - Determine the median crack size.

3) Using the timber strength data provided to the class, we define the following events:

$$A \equiv \left\{25 < MOR < 50 \, {}^{N}\!/_{mm^2}\right\}$$

$$B \equiv \left\{35 < MOR < 55 \, {}^{N}\!/_{mm^2}\right\}$$

Show on a plot and calculate the following:

$$P(A), P(B), P(AB \text{ [intersection]}), P(A + B \text{ [union]}).$$

4) Consider the occurrence of earthquakes in a region for which the cdf is the exponential distribution

$$F_X(x) = 1 - e^{-\lambda x}$$

where
λ is a parameter and
$0 \le x \le \infty$.

Assume $\lambda = 0.2$.

Let x range from 0 to 25 and plot $F_X(x)$ and $f_X(x)$.

What is the probability of the intensity X exceeding 10 units?

5) In class, we examined data on the compressive strength of concrete.
- Find the mean and standard deviation of the sample.
- Assume that the data have a normal distribution and use the standard normal tables.
    i. What value of the compressive strength is exceeded in 19 tests out of 20?
    ii. What is the probability that a test core will withstand a compressive strength greater than $45 \, {}^{N}\!/_{mm^2}$?
    iii. What is the probability that the compressive strengths are in the range of 50.11 to 70.19 N/mm^2, that is, two standard deviations from the mean?

6) 4.7 from the text. (p. 100).

7) An existing reinforced concrete building must be tested for possible obsolescence. Based on professional judgment, the engineer classifies concrete quality as 35 to 40, 40 to 45, 45 to 50 or 50 to 60 N/mm$_2$ based on a 28-day test of compressive strength of concrete cylinders. The relative likelihoods assigned to these states are 0.2, 0.3, 0.4 and 0.1, respectively. Concrete cores are to be cut and tested to ascertain the true state, although the engineer knows that results from test cores are not conclusive. Therefore, conditional probabilities are estimated to account for the uncertainties involved in examining the cores. These probabilities describe the likelihood that the value of core strength indicated predicts a given unknown state. For example, if the true state also lies between 35 and 40 N/mm^2, but there is a 20% chance that it will lie between 40 and 45 N/mm^2, and a 10 % chance that it lies in the range of 45 to 50 N/mm^2. The conditional probabilities are tabulated below.

If the engineer takes three subsequent cores and the lab tests yield sample $z_1 = 41 \, {}^{N}\!/_{mm^2}$; sample $z_2 = 49 \, {}^{N}\!/_{mm^2}$; and sample $z_3 = 44 \, {}^{N}\!/_{mm^2}$ what are the posterior probabilities of the four states at the end of the experiment? The required posterior probability is given by $P(\text{state } x_i | \text{sample } z_3 = y_2)$.

	State			
Core Strength	x_1 $35-40\ N/mm^2$	x_2 $40-45\ N/mm^2$	x_3 $45-50\ N/mm^2$	x_4 $50-60\ N/mm^2$
y_1 $35-40\ N/mm^2$	0.7	0.2	0.1	0.0
y_2 $40-45\ N/mm^2$	0.2	0.6	0.2	0.1
y_3 $45-50\ N/mm^2$	0.1	0.1	0.6	0.2
y_4 $50-60\ N/mm^2$	0.0	0.1	0.1	0.7