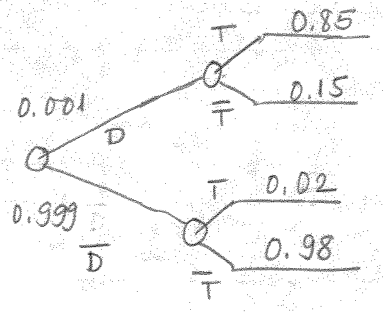
1) $P(D) = 0.001$

$P(T|D) = 0.85$

$P(T|\overline{D}) = 0.02$



0.0009 T D

0.0002 $\overline{T}$ D

0.0199 T $\overline{D}$

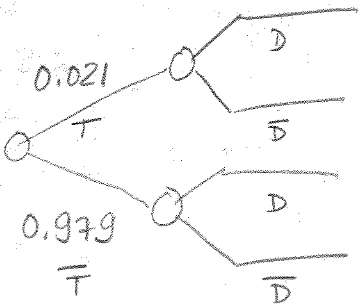
0.979 $\overline{T}$ $\overline{D}$

$\boxed{0.0009 + 0.0199 = 0.021 = P(T)}$

$0.0002 + 0.979 = = \boxed{P(\overline{T}) = 0.9792}$

a) P(T) = 0.021 from tree above

b) P(D|T) = flip tree – see below



0.0009 → $\frac{0.0009}{0.021} = \boxed{0.043 = P(D|T)}$

0.0199 → $0.948 = P(\overline{D}/T)$

Weld Problem HW 2

x	fX(x)	FX(x)
0	0	0
1	0.125	0.0625
2	0.25	0.25
3	0.25	0.5
4	***0.25***	***0.75***
5	0.25	1

Mean [mm] using the table

3 →

$$\sum_{i=1}^{6} x f_X(x) = 0(0) + 1(0.125) + 2(0.25) + 3(0.25) + 4(0.25) + 5\left(\frac{.25}{2}\right) = \boxed{3}$$

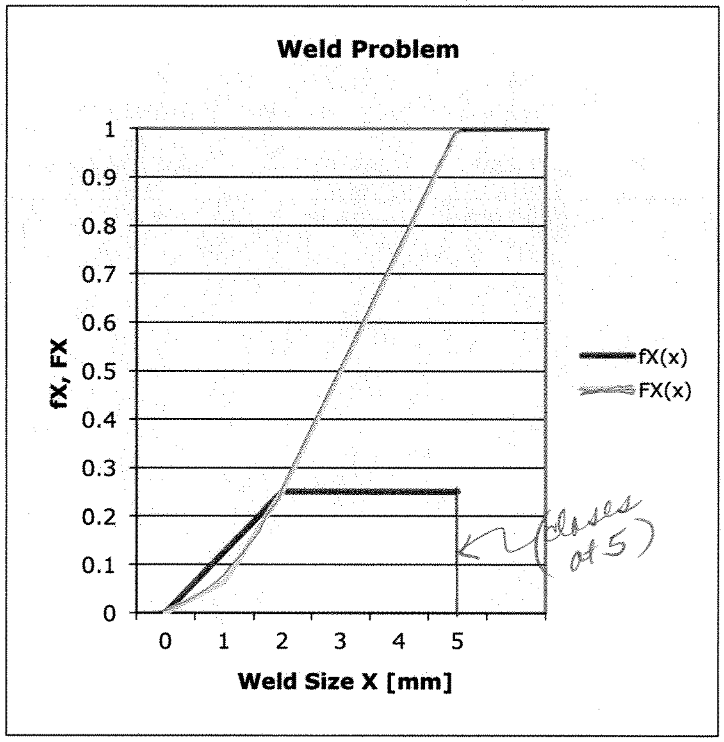
– or –

Integration:

$$\int_0^2 x f_X(x)dx + \int_2^5 x f_X(x)\,dx =$$

$$\left(\frac{x^3}{24}\Big|_0^2 + \left(\frac{x^2}{8}\Big|_2^5 =\right.\right.$$

$$\frac{2^3}{24} + \left(\frac{25}{8} - \frac{4}{8}\right) = \boxed{2.96} \sim 3$$



		Answer from Table above
P[X<=4mm]	FX(X<=4mm)	0.75 ←
median	FX(0.5)	3 mm

By integration

$$F_X(X \le 4\,mm) = \int_{-\infty}^{4} f_X(x)\,dx$$

$$= \frac{1}{8}\int_0^2 x\,dx + \frac{1}{4}\int_2^4 dx$$

$$= \frac{1}{8}\left(\frac{x^2}{2}\Big|_0^2 + \frac{1}{4}\left(x\Big|_2^4\right.\right.$$

$$= \frac{1}{4} + \frac{2}{4}$$

$$\boxed{= \frac{3}{4}}$$

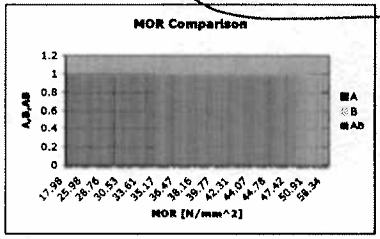
Problem 3

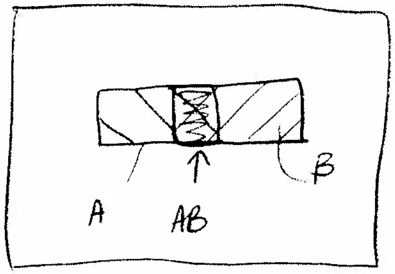
Table E.1.1. Kottagoda & Rosso 50 by 150 mm Swedish redwood and whitewood timber test [abbreviated at 100 observations]

Conversion 145.0377

Number	Modulus of Rupture "MOR" [N/mm^2]	MOR [psi]	Sorted Data [N/mm^2]	A	B	AB
1	48.78	7074.94	17.98			
2	41.64	6039.37	22.67			
3	39.77	5768.15	22.75			
4	35.89	5205.40	23.16			
5	54.04	7837.84	23.19			
6	35.43	5138.69	24.84			
7	22.75	3299.61	25.39	1		
8	44.78	6494.79	25.98	1		
9	43.99	6380.21	26.63	1		
10	47.74	6924.10	27.31	1		
11	25.39	3682.51	27.9	1		
12	38	5511.43	27.93	1		
13	25.98	3768.08	28.46	1		
14	36.84	5343.19	28.71	1		
15	34.49	5002.35	28.76	1		
16	38.71	5614.41	28.83	1		
17	30.53	4428.00	28.98	1		
18	32.02	4644.11	29.9	1		
19	39.34	5705.78	29.93	1		
20	46.33	6719.60	30.05	1		
21	46.99	6815.32	30.33	1		
22	22.67	3288.00	30.53	1		
23	39.62	5746.39	31.33	1		
24	44.78	6494.79	32.02	1		
25	26.63	3862.35	32.4	1		
26	43.48	6306.24	33.06	1		
27	30.05	4358.38	33.18	1		
28	30.33	4398.99	33.47	1		
29	39.21	5686.93	33.61	1		
30	51.39	7453.49	33.71	1		
31	50.91	7383.87	34.12	1		
32	49.65	7201.12	34.44	1		
33	47.83	6937.15	34.49	1		
34	44.07	6391.81	34.63	1		
35	45.54	6605.02	35.03	1	1	1
36	34.12	4948.69	35.17	1	1	1
37	45.92	6660.13	35.3	1	1	1
38	36.47	5289.52	35.43	1	1	1
39	28.98	4203.19	35.67	1	1	1
40	40.85	5924.79	35.89	1	1	1
41	56.6	8209.13	36	1	1	1
42	28.76	4171.28	36.38	1	1	1
43	47.42	6877.69	36.47	1	1	1
44	33.61	4874.72	36.84	1	1	1
45	44.36	6433.87	36.85	1	1	1
46	35.3	5119.83	36.88	1	1	1
47	33.18	4812.35	36.92	1	1	1
48	36.85	5344.64	38	1	1	1
49	17.98	2607.78	38.05	1	1	1
50	27.9	4046.55	38.16	1	1	1
51	44.36	6433.87	38.71	1	1	1
52	32.4	4699.22	39.21	1	1	1
53	33.06	4794.95	39.33	1	1	1
54	33.47	4854.41	39.34	1	1	1
55	35.67	5173.49	39.6	1	1	1
56	28.46	4127.77	39.62	1	1	1
57	23.16	3359.07	39.77	1	1	1
58	44.51	6455.63	39.93	1	1	1
59	42.47	6159.75	40.39	1	1	1
60	48.39	7018.37	40.71	1	1	1
61	38.05	5518.68	40.85	1	1	1
62	35.03	5080.67	41.64	1	1	1
63	31.33	4544.03	41.72	1	1	1
64	27.31	3960.98	42.31	1	1	1
65	53.99	7830.59	42.47	1	1	1
66	43.41	6296.09	43.33	1	1	1
67	28.71	4164.03	43.41	1	1	1
68	58.34	8461.50	43.48	1	1	1
69	48.37	7015.47	43.99	1	1	1
70	29.93	4340.98	44	1	1	1
71	36.38	5276.47	44.07	1	1	1
72	46.86	6796.47	44.3	1	1	1
73	36	5221.36	44.36	1	1	1
74	23.19	3363.42	44.36	1	1	1
75	36.88	5348.99	44.51	1	1	1
76	44.3	6425.17	44.59	1	1	1
77	44.59	6467.23	44.78	1	1	1
78	44	6381.66	44.78	1	1	1
79	40.39	5858.07	45.54	1	1	1
80	41.72	6050.97	45.92	1	1	1
81	29.9	4336.63	46.33	1	1	1
82	35.17	5100.98	46.5	1	1	1
83	34.44	4995.10	46.86	1	1	1
84	27.93	4050.90	46.99	1	1	1
85	50.98	7394.02	47.42	1	1	1
86	40.71	5904.48	47.74	1	1	1
87	34.63	5022.66	47.83	1	1	1
88	24.84	3602.74	48.37	1	1	1
89	28.83	4181.44	48.39	1	1	1
90	42.31	6136.55	48.78	1	1	1
91	39.33	5704.33	49.65	1	1	1
92	39.93	5791.36	50.91		1	
93	39.6	5743.49	50.98		1	
94	38.16	5534.64	51.39		1	
95	43.33	6284.48	51.9		1	
96	69.07	10017.75	53.99		1	
97	51.9	7527.46	54.04		1	
98	33.71	4889.22	56.6			
99	46.5	6744.25	58.34			
100	36.92	5354.79	69.07			

	P(A)	P(B)	P(AB)
sum	65	63	57
frequency	0.65	0.63	0.57
P(A+B)=	0.91		





actual A values	Actual B Values	Actual AB Values
25.39	35.03	35.03
25.98	35.17	35.17
26.63	35.3	35.3
27.31	35.43	35.43
27.9	35.67	35.67
27.93	35.89	35.89
28.46	36	36
28.71	36.38	36.38
28.76	36.47	36.47
28.83	36.84	36.84
28.98	36.85	36.85
29.9	36.88	36.88
29.93	36.92	36.92
30.05	38	38
30.33	38.05	38.05
30.53	38.16	38.16
31.33	38.71	38.71
32.02	39.21	39.21
32.4	39.33	39.33
33.06	39.34	39.34
33.18	39.6	39.6
33.47	39.62	39.62
33.61	39.77	39.77
33.71	39.93	39.93
34.12	40.39	40.39
34.44	40.71	40.71
34.49	40.85	40.85
34.63	41.64	41.64
35.03	41.72	41.72
35.17	42.31	42.31
35.3	42.47	42.47
35.43	43.33	43.33
35.67	43.41	43.41
35.89	43.48	43.48
36	43.99	43.99
36.38	44	44
36.47	44.07	44.07
36.84	44.3	44.3
36.85	44.36	44.36
36.88	44.36	44.36
36.92	44.51	44.51
38	44.59	44.59
38.05	44.78	44.78
38.16	44.78	44.78
38.71	45.54	45.54
39.21	45.92	45.92
39.33	46.33	46.33
39.34	46.5	46.5
39.6	46.86	46.86
39.62	46.99	46.99
39.77	47.42	47.42
39.93	47.74	47.74
40.39	47.83	47.83
40.71	48.37	
40.85	48.39	
41.64	48.78	
41.72	49.65	
42.31	50.91	
42.47	50.98	
43.33	51.39	
43.41	51.9	
43.48	53.99	
43.99	54.04	
44	35.03	
44.07		
44.3		
44.36		
44.36		
44.51		
44.59		
44.78		
44.78		
45.54		
45.92		
46.33		
46.5		
46.86		
46.99		
47.42		
47.74		
47.83		
48.37		
48.39		
48.78		
49.65		
25.39		
25.98		
26.63		
27.31		
27.9		
27.93		

$$P(A+B) = P(A) + P(B) - P(AB)$$

Problem 4. Exponential distribution.

Lambda 0.2

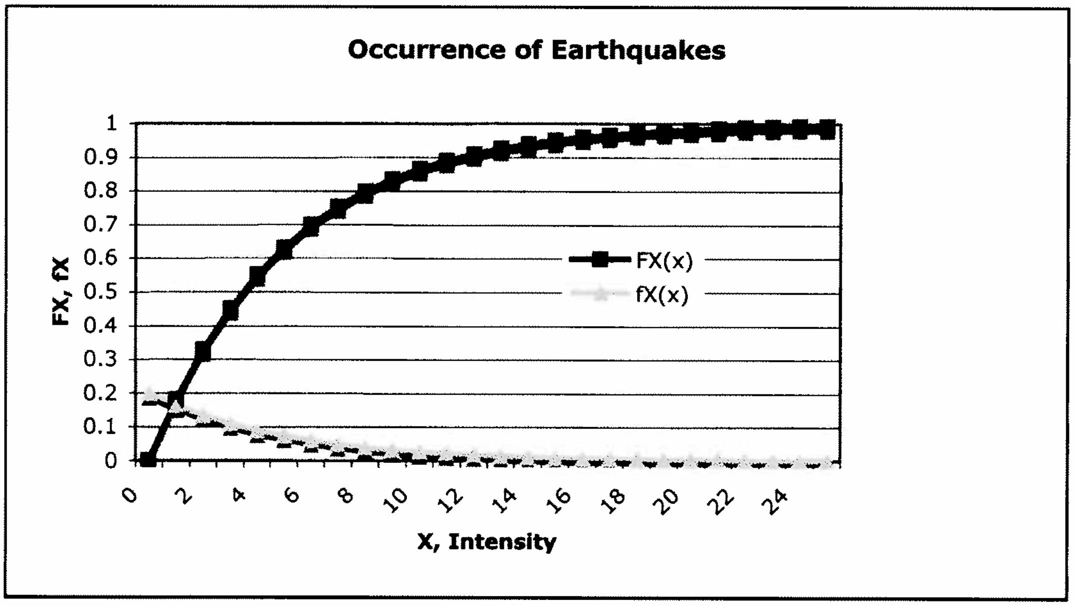
x	FX(x)	fX(x)
0	0	0.2
1	0.181	0.164
2	0.330	0.134
3	0.451	0.110
4	0.551	0.090
5	0.632	0.074
6	0.699	0.060
7	0.753	0.049
8	0.798	0.040
9	0.835	0.033
10	0.865	0.027
11	0.889	0.022
12	0.909	0.018
13	0.926	0.015
14	0.939	0.012
15	0.950	0.010
16	0.959	0.008
17	0.967	0.007
18	0.973	0.005
19	0.978	0.004
20	0.982	0.004
21	0.985	0.003
22	0.988	0.002
23	0.990	0.002
24	0.992	0.002
25	0.993	0.001

FX(10)	**0.865**
1-FX(10)	**0.135**
	1.000

$F_X = 1 - e^{-\lambda x}$

$f_X = -(-\lambda e^{-\lambda x}) = \lambda e^{-\lambda x}$

$$\left.\begin{aligned} &F(X \leq 10) = 0.865 \\ &1 - F(X \leq 10) = F(X > 10) = 0{,}135 \end{aligned}\right\}$$



Problem 5

Table 2.2.1 p. 60 Kottegoda & Rosso
Density and Compressive Strength at 28 days from examination of 40 concrete cube records

Conversions		0.00003613		145.0377	
	Density [kg/m^3]	Density [lbs/in^3]	Compressive Strength [N/mm^2]	Compressive Strength [psi]	Sorted
1	2437	0.08805	60.5	8774.78	69.5
2	2437	0.08805	60.9	8832.80	68.9
3	2425	0.08762	59.8	8673.25	68.3
4	2427	0.08769	53.4	7745.01	68.1
5	2428	0.08772	56.9	8252.65	67.3
6	2448	0.08845	67.3	9761.04	67.2
7	2456	0.08874	68.9	9993.10	65.7
8	2436	0.08801	49.9	7237.38	64.9
9	2435	0.08798	57.8	8383.18	64.9
10	2446	0.08837	60.9	8832.80	63.4
11	2441	0.08819	61.9	8977.83	63.3
12	2456	0.08874	67.2	9746.53	61.9
13	2444	0.08830	64.9	9412.95	61.5
14	2447	0.08841	63.4	9195.39	61.1
15	2433	0.08790	60.5	8774.78	60.9
16	2429	0.08776	68.1	9877.07	60.9
17	2435	0.08798	68.3	9906.07	60.5
18	2471	0.08928	65.7	9528.98	60.5
19	2472	0.08931	61.5	8919.82	60.5
20	2445	0.08834	60.5	8774.78	60.5
21	2436	0.08801	59.6	8644.25	60.2
22	2450	0.08852	60.5	8774.78	59.8
23	2454	0.08866	59.8	8673.25	59.8
24	2449	0.08848	56.7	8223.64	59.6
25	2441	0.08819	57.9	8397.68	59
26	2457	0.08877	60.2	8731.27	58.9
27	2447	0.08841	55.8	8093.10	58.8
28	2436	0.08801	53.2	7716.01	57.9
29	2458	0.08881	61.1	8861.80	57.8
30	2415	0.08725	50.7	7353.41	56.9
31	2448	0.08845	59	8557.22	56.7
32	2445	0.08834	63.3	9180.89	56.3
33	2436	0.08801	52.5	7614.48	55.8
34	2469	0.08920	54.6	7919.06	54.6
35	2455	0.08870	56.3	8165.62	54.4
36	2473	0.08935	64.9	9412.95	53.4
37	2488	0.08989	69.5	10080.12	53.2
38	2454	0.08866	58.9	8542.72	52.5
39	2427	0.08769	54.4	7890.05	50.7
40	2411	0.08711	58.8	8528.22	49.9
mean	**2445**	**0.08834**	**60.2**	**8724.02**	
Standard deviation	**16**	**0**	**5**	**727**	

cdf	0.95	
FX(0.95)	68.4	using norminv
0.05		
FX(0.05)	51.90	
FX(45)	0.0013	
1-FX(45)	0.9987	
FX(50.11)	0.02265	
FX(70.19)	0.97735	
delta	*0.95471*	

<u>notes:</u>

$$\frac{19}{20} = 0.95 \rightarrow \Phi\left(\frac{X_{0.05} - 60.2}{5}\right) = 0.05$$

$$\frac{X_{0.05} - 60.2}{5} = \Phi^{-1}(0.05) =$$

$$X_{0.05} - 60.2 = -1.64(5)$$

$$X_{0.05} = \underline{52 \text{ N/mm}^2} \approx \underline{51.9 \text{ (excel)}}$$

Using Appendix I

$$\Phi\left(\frac{70.19 - 60.2}{5}\right) - \Phi\left(\frac{50.11 - 60.2}{5}\right) = ?$$

$$\Phi(1.998) - \Phi(-2.018) =$$

$$0.97670 - (\overset{0.0222}{1 - 0.97778}) = \underline{\underline{0.9555}}$$

Using Appendix I

(close to excel

$\mu_x = 300^k$, $COV(X) = 0.2$, $\sigma_x = 0.2(300^k) = 60^k$

$\zeta_x^2 = \ln(1+\delta_x^2) = \ln(1+0.2^2) = 0.0392$

$\zeta_x = 0.198 \approx 0.2$

$\lambda_x = \ln\mu_x - \frac{1}{2}\zeta_x^2 = \underset{5.7038}{\ln(300)} - \underbrace{\frac{1}{2}(.0392)}_{0.0196} = 5.6842$

$X \sim LN(5.6842, 0.198)$

a. $\ln(100^k) = 4.6052$

$$\Phi\left(\frac{4.6052-5.6842}{0.198}\right) = \Phi(-5.4495) = \overset{1-\Phi(5.45)}{\underline{\underline{2.6216 \times 10^{-8}}}}$$

b. $\ln(250^k) = 5.52146$

$$\Phi\left(\frac{5.52146-5.6842}{0.198}\right) = \Phi(-0.8219) \cong 1-0.79389 = \underline{\underline{0.2061}}$$

c. $\ln(100+LL) = \ln(D)$

$\Phi(\quad) = 0.001$

$$\Phi\left(\frac{\ln(D)-5.6842}{0.198}\right) = 0.001$$

$$\frac{\ln D - 5.6842}{0.198} = \Phi^{-1}(0.001) = -\overbrace{\Phi(0.999)}^{-3.09}$$

$\ln D = 0.198(-3.09) + 5.6842$

$\ln D = 5.07238$

$D = 159^k = DL + LL$

$\boxed{LL = 59^k}$

Problem 7

Bayesian Updating Problem

	P(x)	range
B1	0.2	35-40
B2	0.3	40-45
B3	0.4	45-50
B4	0.1	50-60

z1	41
z2	49
z3	44

True State Bj

Measured State Bi	x1 35 - 40	x2 40 - 45	x3 45-50	x4 50 - 60
y1 35-40	0.7	0.2	0.1	0
y2 40-45	0.2	0.6	0.2	0.1
y3 45-50	0.1	0.1	0.6	0.2
y4 50 - 60	0	0.1	0.1	0.7

					sum
P(Bk|z1)	0.04	0.18	0.08	0.01	0.31
B1	0.129				
B2	0.581				
B3	0.258				
B4	0.032				

P(Bk|z2)	0.013	0.058	0.155	0.006	0.232
B1	0.056				
B2	0.250				
B3	0.667				
B4	0.028				

P(Bk|z3)	0.011	0.150	0.133	0.003	0.297
B1	0.037				
B2	***0.505***				
B3	0.449				
B4	0.009				

final updated probabilities