1) Problem 7.1 of text

SS beam, $\ell = 40'$, D = 80 psf, L = 50 psf, 10' o.c.
A36 steel; W21 x 73; $g(x) = M_R - M_S$

$M_{R_N} = 151 \times 36 = \underline{5436 \text{ in-k}}$
$M_{R_N} = \mu_R - 2\sigma_R$; $\delta_R = 0.13$

$D_N = 80 \text{ psf} = \mu_D + \sigma_D$; $\delta_D = 0.13 \rightarrow \mu_D(1 + 0.13) = 80$

$L_N = 50 \text{ psf} = \mu_L + 2\sigma_L$; $\delta_L = 0.37 \rightarrow \mu_L(1 + 2(.37)) = 50$

$\mu_D = \frac{80}{1.13} = \underline{70.8 \text{ psf}}$; $\sigma_D = \underline{9.2 \text{ psf}}$

$\mu_L = \frac{50}{1.74} = \underline{28.7 \text{ psf}}$; $\sigma_L = \underline{10.6 \text{ psf}}$

$S = D + L$

$\mu_S = \mu_D + \mu_L = 70.8 + 28.7 = \underline{99.5 \text{ psf}}$

$\sigma_S = \sqrt{\sigma_D^2 + \sigma_L^2} = \sqrt{9.2^2 + 10.6^2} = \underline{14.0 \text{ psf}}$

$\delta_S = \frac{14.0}{99.5} = \underline{0.14}$

$M_S = \frac{S\ell^2}{8}\left\{\frac{(10')(12''/')}{(1000 \#/k)}\right\}$

$\mu_{M_S} = \mu_S \frac{\ell^2}{8}\left(\frac{(120)}{1000}\right) = \frac{99.5(40')^2}{8(1000)}(120) = \underline{2388 \text{ in-k}}$

$\sigma_{M_S} = \delta_{M_S}(\mu_{M_S}) = 0.14(2388 \text{ in-k}) = \underline{334.3 \text{ in-k}}$

$\mu_R^M - 2\sigma_R = 5436 \text{ in-k}$

$\mu_R^M(1 - 2(0.13)) = 5436 \text{ in-k}$

$\mu_{M_R} = \frac{5436 \text{ in-k}}{0.74} = \underline{7346 \text{ in-k}}$; $\sigma_{M_R} = 0.13(7346) = \underline{955 \text{ in-k}}$

a) $p_f = ?$

$p_f = 1 - \Phi\left\{\frac{\mu_{M_R} - \mu_{M_S}}{\sqrt{\sigma_{M_R}^2 + \sigma_{M_S}^2}}\right\} = 1 - \Phi\left(\frac{7346 - 2388}{\sqrt{3372^2 + 955^2}}\right) = 1 - \Phi\left(\frac{4958}{1013}\right)$

$= 1 - \Phi(4.89) \cong 1 - \Phi(4.9) = 4.7987 \times 10^{-7}$ from Appendix I

(b) $\bar{\xi} = \frac{\mu_{ML}}{\mu_{MS}} = \frac{7346 \text{ in-k}}{2388 \text{ in-k}} = \underline{3.08}$ central safety factor

$\bar{\phi} = 1 - \varepsilon \beta \delta_R \quad (7.11a)$

$\bar{\gamma}_i = 1 + \varepsilon \varepsilon_{nn} \beta \delta_{Si} \quad (7.18a)$

$\varepsilon = \frac{\sqrt{\sigma_{ML}^2 + \sigma_{MS}^2}}{\sigma_{ML} + \sigma_{MS}} \quad (7.8)$

$\varepsilon = \frac{\sqrt{955^2 + 334^2}}{955 + 334} = \frac{1012}{1289} = 0.785 \cong \underline{0.79}$

$\varepsilon_{nn} = \frac{\sqrt{\sigma_{S1}^2 + \sigma_{S2}^2}}{\sigma_{S1} + \sigma_{S2}} = \frac{\sqrt{\sigma_D^2 + \sigma_L^2}}{\sigma_D + \sigma_L} = \frac{\sqrt{9.2^2 + 10.6^2}}{9.2 + 10.6} = \frac{14}{19.8} = \underline{0.71}$

$\therefore$

$\bar{\phi} = 1 - 0.79(4.90)(0.13) = \underline{0.50}$

$\bar{\gamma}_D = 1 + (0.79)(0.71)(4.90)(0.13) = \underline{1.36}$

$\bar{\gamma}_L = 1 + (0.79)(0.71)(4.90)(0.37) = \underline{2.02}$

c) $\phi = \frac{1 - \varepsilon \beta \delta_R}{1 - k_R \delta_R} = \frac{1 - \varepsilon \beta \delta_{ML}}{1 - k_{ML} \delta_{ML}} = \frac{1 - 0.79(4.9)(0.13)}{1 - 2(0.13)} = \frac{0.50}{0.74} = \underline{0.68}$

$\gamma_D = \frac{\bar{\gamma}_D}{1 + k_D \delta_D} = \frac{1.36}{1 + 0.13} = \underline{1.20}$

$\gamma_L = \frac{\bar{\gamma}_L}{1 + k_L \delta_L} = \frac{2.02}{1 + 2(.37)} = \underline{1.16}$

Normal page1

Don't use until lognormal case but shown here anyway

Table 7.5 p. 218 Expanded

Random Variable	Mean	COV	Standard Deviation	Zeta	Lambda
As [in2]	1.56	0.036	0.0562	0.0360	0.4440
Fy [ksi]	47.7	0.15	7.155	0.1492	3.8538
Fc' [ksi]	3.5	0.21	0.735	0.2077	1.2312
b [in]	8	0.045	0.36	0.0450	2.0784
d [in]	13.2	0.086	1.1352	0.0858	2.5765
eta [dimensionless]	0.59	0.05	0.0295	0.0500	-0.5289
M [kip-in]	326.25	0.17	55.46	0.1688	5.773

g(X) mean	865.56	Standard Deviation g(X)	155.63	beta-estimate	5.56

Started with "3.0" below

Step 1 g(X)	Mc-Mu = 0				
Step 2	beta - see below in step 7-9				
Step 3	As* [in2]	1.56	1.53	1.53	1.53
	Fy* [ksi]	47.7	32.21	28.40	25.62
	Fc'* [ksi]	3.5	3.15	3.23	3.28
	b *[in]	8	7.96	7.97	7.98
	d *[in]	13.2	11.35	11.47	11.53
	eta*	0.59	0.59	0.59	0.59
	M* [kip-in]	326.25	385.55	410.24	417.61
Step 4 MEANS	AsN [in2]	1.56	1.56	1.56	1.56
	FyN [ksi]	47.7	47.7	47.7	47.7
	Fc'N [ksi]	3.5	3.5	3.5	3.5
	bN [in]	8	8	8	8
	dN [in]	13.2	13.2	13.2	13.2
	etaN	0.59	0.59	0.59	0.59
	MN [kip-in]	326.25	326.25	326.25	326.25
Sigmas	AsN [in2]	0.05616	0.05616	0.05616	0.05616
	FyN [ksi]	7.155	7.155	7.155	7.155
	Fc'N [ksi]	0.735	0.735	0.735	0.735
	bN [in]	0.36	0.36	0.36	0.36
	dN [in]	1.1352	1.1352	1.1352	1.1352
	eta-N	0.0295	0.0295	0.0295	0.0295
	MN [kip-in]	55.4625	55.4625	55.4625	55.4625
Step 5	dg/dAs	480.056	290.544	268.674	249.911
	dg/dfy	15.70	13.81	14.52	14.97
	dg/df'c	33.34	18.22	13.52	10.64
	dg/db	14.58	7.21	5.48	4.38
	dg/dd	74.41	49.30	43.59	39.32
	dg/deta	-197.76	-96.81	-73.73	-59.02
	dg/dM	-1.00	-1.00	-1.00	-1.00
Step 6	"Downstairs"	155.63	128.19	129.06	129.65
	alpha As	0.1732	0.1273	0.1169	0.1083
	alpha fy	0.7218	0.7708	0.8051	0.8263
	alpha f'c	0.1574	0.1045	0.0770	0.0603
	alpha b	0.0337	0.0203	0.0153	0.0122
	alpha d	0.5428	0.4366	0.3834	0.3443
	alpha eta	-0.0375	-0.0223	-0.0169	-0.0134
	alpha M	-0.3564	-0.4327	-0.4297	-0.4278
Step 7-9	beta estimate	3.000	3.500	3.833	3.833 ← β
g(X)		539.31	116.66	45.84	0.90

final values

Notice these don't change for the normal case.

Derivatives provided in attached sheet.

project:

CEE518 Derivatives for Table 7.6 p.219

$$g(X) = M_c - M_u$$

$$M_c = A_s f_y d\left\{1 - \frac{\eta A_s f_y}{b d f_c'}\right\} = A_s f_y d - \frac{\eta A_s^2 f_y^2}{b f_c'}$$

$$\mu_{M_c} = \mu_{A_s}\mu_{f_y}\mu_d\left\{1 - \frac{\mu_\eta \mu_{A_s}\mu_{f_y}}{\mu_b \mu_d \mu_{f_c'}}\right\}$$

Evaluate derivatives of M_n for σ_{M_n}:

$$\frac{\partial M_n}{\partial A_s} = f_y d - \frac{2\eta A_s f_y^2}{b f_c'}$$

$$\frac{\partial M_n}{\partial f_y} = A_s d - \frac{2\eta A_s^2 f_y}{b f_c'}$$

$$\frac{\partial M_n}{\partial f_c'} = 0 - \frac{\eta A_s^2 f_y^2}{b}\left(-\frac{1}{f_c'^2}\right)$$

$$\frac{\partial M_n}{\partial b} = \frac{\eta A_s^2 f_y^2}{b^2 f_c'}$$

$$\frac{\partial M_n}{\partial d} = A_s f_y$$

$$\frac{\partial M_n}{\partial \eta} = -\frac{A_s^2 f_y^2}{b f_c'}$$

$$\therefore \sigma_{M_n}^2 = \left(\frac{\partial M_n}{\partial A_s}\right)^2\sigma_{A_s}^2 + \left(\frac{\partial M_n}{\partial f_y}\right)^2\sigma_{f_y}^2 + \left(\frac{\partial M_n}{\partial f_c'}\right)^2\sigma_{f_c'}^2 + \left(\frac{\partial M_n}{\partial b}\right)^2\sigma_b^2 + \left(\frac{\partial M_n}{\partial d}\right)^2\sigma_d^2 + \left(\frac{\partial M_n}{\partial \eta}\right)^2\sigma_\eta^2$$

$$\sigma_{M_u}^2 = \left(\frac{\partial g}{\partial M}\right)^2\sigma_{M_u}^2 = (-1)^2\sigma_{M_u}^2 = \sigma_{M_u}^2$$

Table 7.5 p. 218 Expanded

Random Variable	Mean	COV	Standard Deviation	Zeta	Lambda
As [in2]	1.56	0.036	0.05616	0.03599	0.4440
Fy [ksi]	47.7	0.15	7.155	0.14917	3.8538
Fc' [ksi]	3.5	0.21	0.735	0.20774	1.2312
b [in]	8	0.045	0.36	0.04498	2.0784
d [in]	13.2	0.086	1.1352	0.08584	2.5765
eta [dimensionless]	0.59	0.05	0.0295	0.04997	-0.5289
M [kip-in]	326.25	0.17	55.46	0.16879	5.7734

g(X) mean	865.56	Standard Devation g(X)	1486.56	beta-estimate	0.5823

start w/ "3.6" below

Step 1 g(X)	Mc-Mu = 0				
Step 2	beta - see below in step 7-9				
Step 3	As* [in2]	1.56	1.53	1.53	1.53
	Fy* [ksi]	47.7	31.78	34.24	33.08
	Fc'* [ksi]	3.5	3.08	3.12	3.07
	b *[in]	8	7.18	7.19	7.05
	d *[in]	13.2	11.30	11.47	11.27
	eta*	0.59	0.59	0.59	0.59
	M* [kip-in]	326.25	380.33	455.73	503.53
Step 4 SIGMAs	AsN [in2]	0.0561	0.0551	0.0551	0.0550
	FyN [ksi]	7.1152	4.7399	5.1069	4.9342
	Fc'N [ksi]	0.7271	0.6405	0.6489	0.6379
	bN [in]	0.3598	0.3229	0.3232	0.3169
	dN [in]	1.1331	0.9701	0.9846	0.9670
	etaN	0.0295	0.0296	0.0296	0.0296
	MN [kip-in]	55.0679	64.1963	76.9230	84.9914
Means	AsN [in2]	1.56	1.56	1.56	1.56
	FyN [ksi]	47.17	44.33	45.21	44.82
	Fc'N [ksi]	3.42	3.41	3.41	3.41
	bN [in]	7.99	7.95	7.95	7.93
	dN [in]	13.15	13.01	13.04	13.01
	eta-N	0.59	0.59	0.59	0.59
	MN [kip-in]	321.60	316.58	296.92	277.83
Step 5	dg/dAs	480.056	276.409	297.998	281.0260
	dg/dfy	15.70	13.31	13.32	12.9744
	dg/df'c	33.34	20.51	23.20	22.7831
	dg/db	14.58	8.81	10.08	9.9297
	dg/dd	74.41	48.61	52.40	50.5162
	dg/deta	-197.76	-106.73	-122.32	-117.9690
	dg/dM	-1.00	-1.00	-1.00	-1.0000
Step 6	"Downstairs"	154.91	103.66	117.16	119.08
	alpha As	0.1740	0.1468	0.1401	0.1297
	alpha fy	0.7211	0.6085	0.5807	0.5376
	alpha f'c	0.1565	0.1268	0.1285	0.1220
	alpha b	0.7524	0.6756	0.6843	0.6615
	alpha d	0.5443	0.4549	0.4403	0.4102
	alpha eta	-0.0376	-0.0305	-0.0309	-0.0294
	alpha M	-0.3555	-0.6193	-0.6566	-0.7137
Step 7-9	beta estimate	3.000	3.500	4.091	4.0910 ← β
g(X)		539.31	105.74	72.80	-4.40

final values

Table 7.5 p. 218
Expanded

Random Variable	Mean	COV	Standard Deviation
As [in2]	1.56	0.036	0.056
Fy [ksi]	47.7	0.15	7.155
Fc' [ksi]	3.5	0.21	0.74
b [in]	8	0.045	0.36
d [in]	13.2	0.086	1.14
eta [dimensionless]	0.59	0.05	0.030
M [kip-in]	326.25	0.17	55.46

Rand	As	Rand	Fy	rand	F'c	rand	b	rand	d	rand	eta	rand	m	Mc	g(x)
0.51	1.56	0.03	33.99	0.54	3.58	0.06	7.43	0.57	13.40	0.08	0.55	0.60	339.88	653.12	313.24
0.18	1.51	0.70	51.38	0.22	2.92	0.74	8.23	0.26	12.46	0.78	0.61	0.30	297.05	812.64	515.59
0.31	1.53	0.31	44.20	0.32	3.15	0.32	7.83	0.32	12.68	0.33	0.58	0.33	302.11	751.48	449.37
0.53	1.56	0.39	45.78	0.25	3.01	0.11	7.56	0.97	15.38	0.83	0.62	0.69	354.06	962.83	608.77
0.95	1.65	0.79	53.43	0.63	3.74	0.47	7.97	0.31	12.64	0.15	0.56	0.99	461.97	969.22	507.25
0.17	1.51	0.83	54.44	0.48	3.47	0.14	7.61	0.79	14.12	0.45	0.59	0.10	256.02	1009.11	753.09
0.70	1.59	0.91	57.51	0.13	2.66	0.34	7.85	0.55	13.35	0.77	0.61	0.98	437.57	976.15	538.58
0.23	1.52	0.85	55.13	0.47	3.45	0.10	7.54	0.72	13.87	0.35	0.58	0.97	430.71	1004.88	574.17
0.49	1.56	0.62	49.88	0.74	3.98	0.87	8.40	0.99	16.06	0.12	0.56	0.24	287.73	1148.33	860.60
0.12	1.50	0.99	65.21	0.86	4.30	0.73	8.22	0.60	13.48	0.46	0.59	0.33	302.32	1156.13	853.81
0.08	1.48	0.79	53.56	0.50	3.51	0.21	7.71	0.92	14.82	0.63	0.60	0.34	303.71	1036.64	732.93
0.39	1.54	0.95	59.74	0.52	3.53	0.08	7.50	0.65	13.62	0.21	0.57	0.77	367.97	1074.98	707.01
0.28	1.53	0.38	45.55	0.49	3.47	0.59	8.08	0.70	13.78	0.80	0.61	0.90	398.70	852.40	453.70
0.37	1.54	0.44	46.59	0.51	3.52	0.58	8.07	0.65	13.63	0.72	0.61	0.79	370.91	868.65	497.75
0.98	1.68	0.32	44.42	0.66	3.81	0.00	7.02	0.34	12.74	0.68	0.60	0.02	215.44	824.85	609.41
0.54	1.56	0.28	43.55	0.03	2.08	0.77	8.27	0.52	13.25	0.26	0.57	0.01	196.86	749.10	552.24
0.77	1.60	0.54	48.48	0.32	3.16	0.10	7.54	0.88	14.52	0.66	0.60	0.43	316.88	974.47	657.59
0.65	1.58	0.49	47.56	0.34	3.19	0.18	7.68	0.03	11.07	0.88	0.62	0.72	358.89	688.20	329.32
0.77	1.60	0.07	37.14	0.37	3.26	0.68	8.16	0.98	15.50	0.28	0.57	0.58	338.08	845.65	507.57
0.78	1.60	0.02	33.21	0.26	3.03	0.50	8.00	0.75	13.95	0.99	0.66	0.23	284.84	666.30	381.46
0.82	1.61	0.02	33.06	0.22	2.93	0.42	7.92	0.61	13.53	0.81	0.62	0.01	191.85	645.42	453.58
0.15	1.50	0.05	36.19	0.96	4.75	0.86	8.39	0.76	14.00	0.66	0.60	0.56	335.13	716.52	381.40
0.63	1.58	0.55	48.55	0.47	3.44	0.39	7.90	0.31	12.64	0.23	0.57	0.16	269.94	845.82	575.87
0.31	1.53	0.91	57.43	0.51	3.52	0.11	7.56	0.71	13.82	0.31	0.58	0.91	398.98	1049.42	650.45
0.35	1.54	0.71	51.75	0.08	2.48	0.45	7.95	0.82	14.22	0.18	0.56	0.55	333.39	950.75	617.37
0.92	1.64	0.51	47.80	0.09	2.53	0.68	8.17	0.27	12.50	0.86	0.62	0.45	318.76	794.61	475.85
0.52	1.56	0.59	49.40	0.67	3.82	0.74	8.23	0.82	14.23	0.89	0.63	0.97	427.16	979.65	552.49
0.40	1.55	0.09	38.28	0.79	4.08	0.48	7.98	0.17	12.13	0.86	0.62	0.56	334.29	650.69	316.40

0.61	1.58	0.47	47.23	0.34	3.20	0.21	7.71	0.07	11.56	0.94	0.64	0.81	374.52	717.06	342.54
0.79	1.60	0.40	45.85	0.01	1.81	0.62	8.11	0.24	12.38	0.85	0.62	0.46	320.84	681.91	361.07
0.93	1.64	0.85	55.03	0.76	4.03	0.68	8.17	0.59	13.47	0.51	0.59	0.43	315.96	1071.62	755.66
0.87	1.62	0.96	60.01	0.04	2.25	0.13	7.60	0.22	12.32	0.31	0.58	0.39	311.33	881.00	569.67
0.87	1.62	0.69	51.29	0.52	3.53	0.34	7.85	0.17	12.11	0.99	0.66	0.82	376.69	841.93	465.24
0.67	1.59	0.75	52.56	0.83	4.20	0.91	8.47	0.98	15.60	0.06	0.54	0.14	265.59	1194.04	928.46
0.76	1.60	0.27	43.34	0.78	4.08	0.30	7.81	0.81	14.19	0.32	0.58	0.83	380.19	896.83	516.63
0.58	1.57	0.61	49.65	0.63	3.75	0.66	8.15	0.68	13.74	0.71	0.61	0.73	360.99	951.46	590.46
0.39	1.54	0.26	43.17	0.14	2.70	0.01	7.18	0.89	14.56	0.76	0.61	0.63	345.16	830.74	485.59
0.36	1.54	0.31	44.23	0.27	3.05	0.23	7.73	0.19	12.20	0.15	0.56	0.10	256.64	720.64	464.00
0.20	1.51	0.22	42.12	0.24	2.97	0.25	7.76	0.27	12.51	0.29	0.57	0.31	298.11	695.92	397.82
0.83	1.61	0.01	30.42	0.19	2.85	0.37	7.88	0.55	13.34	0.73	0.61	0.91	401.56	589.65	188.09
0.42	1.55	0.12	39.12	0.81	4.16	0.51	8.01	0.21	12.30	0.91	0.63	0.61	342.04	675.39	333.35
0.46	1.55	0.62	49.93	0.78	4.07	0.94	8.56	0.10	11.73	0.26	0.57	0.41	314.34	811.90	497.56
0.98	1.67	0.99	63.60	0.99	5.37	0.00	6.97	0.01	10.55	0.02	0.53	0.03	217.80	964.18	746.38
0.13	1.50	0.42	46.23	0.71	3.91	0.00	6.97	0.29	12.59	0.59	0.60	0.88	390.83	765.50	374.67
0.21	1.52	0.04	34.75	0.86	4.29	0.68	8.17	0.50	13.21	0.33	0.58	0.15	268.26	649.80	381.54
0.96	1.66	0.21	41.98	0.47	3.44	0.72	8.21	0.97	15.38	0.23	0.57	0.48	323.46	972.85	649.39
0.74	1.60	0.35	44.93	0.96	4.80	0.57	8.07	0.19	12.18	0.80	0.61	0.41	313.51	791.90	478.39
0.41	1.55	0.21	41.94	0.01	1.83	0.81	8.32	0.61	13.53	0.42	0.58	0.22	282.82	716.63	433.81
0.78	1.60	0.01	31.35	0.24	2.99	0.47	7.98	0.70	13.81	0.94	0.63	0.17	272.52	626.74	354.22
0.76	1.60	0.82	54.14	0.87	4.34	0.93	8.54	0.99	15.84	0.05	0.54	0.11	257.03	1262.12	1005.09
mean	1.57	0.47	46.92	0.48	3.44	0.44	7.90	0.54	13.41	0.53	0.59	0.49	324.43	859.36	534.93
stdev	0.05	0.31	8.24	0.28	0.74	0.29	0.38	0.30	1.23	0.30	0.03	0.30	61.20	165.70	170.88

unfortunately, no failures! need to increase "n" to 10,000 or so for accuracy.

$\beta = \Phi^{-1}(1-p_f) = \Phi^{-1}(1.0)$ undefined actually, choose 0.99999

$\cong 4.9$