problem 11.2 Rao Rao112.xls12/4/05 1

| | n1 | n2 | n3 | Sum | Constraint |
|---|---|---|---|---|---|
| value | 4 | 5 | 6 | 15 | |
| Cost | 5 | 6 | 8 | 19 | 100 |
| weight | 3 | 1 | 2 | 6 | 50 |
| reliability | 0.9 | 0.7 | 0.8 | 0.994 | 1 |
| nj | 1 | 1 | 1 | | |



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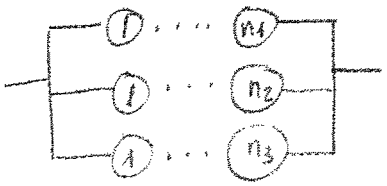
N stages in P with $n_j$ components in series:




Maximize Reliability $R_{system} = 1 - \prod (1 - R_j^{n_j})$

$= 1 - (1 - 0.9^{n_1})(1 - 0.7^{n_2})(1 - 0.8^{n_3})$

Subject to

Cost: $5n_1 + 6n_2 + 8n_3 \leq 100$

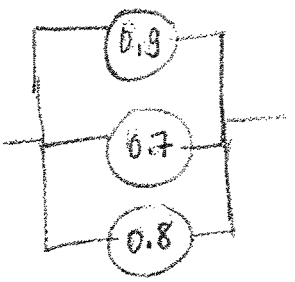
Weight: $3n_1 + n_2 + 2n_3 \leq 50$

$n_1 = n_2 = n_3 = 1$

$R_{system} = 0.9940$

weight = 6

Cost = 19




Note that adding any units in series will reduce the reliability to be combined in parallel. Therefore only 1 unit if each is best.

project:

# Problem 2. Background Calculations

$$g(\ ) = 1.5 A_s b f_y - \frac{0.59 A_s^2 f_y^2}{b f'_c} - M$$

$$\frac{\partial g}{\partial f_y} = 1.5 A_s b - \frac{2(0.59) A_s^2 f_y}{b f'_c}$$

$$\frac{\partial g}{\partial f'_c} = \frac{(-1)(-0.59) A_s^2 f_y^2}{b f'^2_c} = \frac{0.59 A_s^2 f_y^2}{b f'^2_c}$$

$$\frac{\partial g}{\partial M} = -1$$

$f'_c \Rightarrow \lambda_{f'_c} = \ln \mu_{f'_c} - \frac{1}{2}\zeta^2_{f'_c} = 1.609 - 0.002 = 1.609$

$\zeta^2_{f'_c} = \ln(1 + \delta^2_{f'_c}) \Rightarrow COV = 0.15;\ \zeta_{f'_c} = 0.022$

$f_y \Rightarrow \lambda_{f_y} = \ln \mu_{f_y} - \frac{1}{2}\zeta^2_{f_y} = 4.094 - 0.02 = 4.094$

$\zeta^2_{f_y} = \ln(1 + \delta^2_{f_y}) \Rightarrow COV = 0.08;\ \zeta_{f_y} = 0.006$

$$\min A_s = \frac{200 b d}{f_y [psi]}$$

$$\max A_s = 0.75 \rho_b b d = 0.75 \left( \frac{0.85 f'_c}{f_y} \left\{ \frac{87000}{87000 + f_y} \right\} \right)$$

Cost: $0.002 bh + 0.25 A_s$ ← Trying to minimize cost will result in $A_s$ min since $A_s$ is more expensive.

$A_s \leq 0.75 \rho_b b d$

$A_s > \frac{200 b d}{f_y}$

$\beta = 3.09$

I will find b&d for each $A_s$ with $\beta = 3.09$.

$d = 1.5 b$; $h = d + 2''$

concreteoptimizatn.xls As min

$$g(X) = 1.5bA_s f_y - \frac{0.59A_s^2 f_y^2}{bf_c'} - M$$

Cost = 0.002 bh + 0.25As

b = 17.5, d = 26.5, h = 28.5"

∴ Cost = .002 (17.5")(28.5")

+ 0.25(1.53)

= 0.998

+ 0.38

= $1.38/inch.

| | | COV | | | |
|---|---|---|---|---|---|
| Fy mean [ksi] | 60 | | Zeta | 0.006 | |
| Fy stan | 4.8 | 0.08 | Lambda | 4.094 | |
| Fc' mean [ksi] | 5 | | Zeta | 0.022 | |
| Fc' stan | 0.75 | 0.15 | Lambda | 1.609 | |
| M mean [in-kips] | 1800 | 0.1 | | | |
| M stan | 180 | | | | |
| b [inches] | 17.5 | | | | |
| d [inches] | 26.25 | | | | |
| As [min] in2 | 1.53 | 1.53 | 1.53 | 1.53 | 1.53 |
| | | | | | |
| Beta | 3.09 | | | | |
| | | | | | |
| F*y | 60 | 59.90 | 59.90 | 59.90 | 59.90 |
| fc'* | 5 | 4.99 | 4.99 | 4.99 | 4.99 |
| M* | 1800 | 2354.03 | 2354.04 | 2354.04 | 2354.04 |
| | | | | | |
| cdf(f*y) | 0.50127 | 0.401029 | 0.4012686 | 0.401268 | 0.401268 |
| pdf(f*y) | 1.04223 | 1.011631 | 1.011784 | 1.011784 | 1.011784 |
| cdf(f*c') | 0.50444 | 0.456895 | 0.457096 | 0.457096 | 0.457096 |
| pdf(f*c') | 3.58568 | 3.57443 | 3.5745856 | 3.574585 | 3.574585 |
| | | | | | |
| sigNfy | 0.38278 | 0.382157 | 0.3821585 | 0.382158 | 0.382158 |
| meanNfy | 59.9988 | 59.9987 | 59.998703 | 59.9987 | 59.9987 |
| sigNfc' | 0.11125 | 0.110958 | 0.1109592 | 0.110959 | 0.110959 |
| meanNfc' | 4.99876 | 4.998748 | 4.9987481 | 4.998748 | 4.998748 |
| sigNM | 180 | 180 | 180 | 180 | 180 |
| meanNM | 1800 | 1800 | 1800 | 1800 | 1800 |
| | | | | | |
| dg/Fy | 38.2684 | 38.26641 | 38.266427 | 38.26643 | 38.26643 |
| dg/dfc' | 56.8237 | 56.79056 | 56.790374 | 56.79037 | 56.79037 |
| dg/dM | -1 | -1 | -1 | -1 | -1 |
| | | | | | |
| Alpha fy | 0.0811 | 0.0809 | 0.0809 | 0.0809 | 0.0809 |
| Alpha fc' | 0.0350 | 0.0349 | 0.0349 | 0.0349 | 0.0349 |
| Alpha M | -0.99609 | -0.99611 | -0.99611 | -0.99611 | -0.99611 |
| | | | | | |
| g( ) | 552.926 | -4.96831 | -4.966865 | -4.96686 | -4.96686 |

concreteoptimizatn.xls As max

$$g(X) = 1.5bA_s f_y - \frac{0.59A_s^2 f_y^2}{bf_c'} - M$$

Cost = 0.002bh + 0.25 $A_s$

b = 8.5", d = 13", h = 15"

Cost = 0.002(8.5)(15) + 0.25(4.41)

= 0.26 + 1.11 = $1.37/inch

| | | COV | | | |
|---|---|---|---|---|---|
| Fy mean [ksi] | 60 | | Zeta | 0.00638 | |
| Fy stan | 4.8 | 0.08 | Lambda | 4.094324 | |
| Fc' mean [ksi] | 5 | | Zeta | 0.022251 | |
| Fc' stan | 0.75 | 0.15 | Lambda | 1.60919 | |
| M mean [in-kips] | 1800 | 0.1 | | | |
| M stan | 180 | | | | |
| b [inches] | 8.22 | | | | |
| d [inches] | 12.33 | | | | |
| As [max] in2 | 4.58 | 4.41 | 4.41 | 4.41 | 4.41 |
| | | | | | |
| Beta | 3.09 | | | | |
| | | | | | |
| F*y | 60 | 59.96 | 59.96 | 59.96 | 59.96 |
| fc'* | 5 | 4.81 | 4.81 | 4.81 | 4.81 |
| M* | 1800 | 2261.77 | 2262.02 | 2262.01 | 2262.01 |
| | | | | | |
| cdf(f*y) | 0.501273 | 0.455813 | 0.45891 | 0.458847 | 0.4588472 |
| pdf(f*y) | 1.042228 | 1.036588 | 1.0374 | 1.037388 | 1.0373879 |
| cdf(f*c') | 0.504438 | 0.039798 | 0.04296 | 0.042954 | 0.0429539 |
| pdf(f*c') | 3.585678 | 0.802279 | 0.85293 | 0.852783 | 0.8527866 |
| | | | | | |
| sigNfy | 0.382776 | 0.382498 | 0.38252 | 0.382516 | 0.3825164 |
| meanNfy | 59.99878 | 59.99876 | 59.9988 | 59.99877 | 59.998766 |
| sigNfc' | 0.111253 | 0.106971 | 0.10706 | 0.107055 | 0.1070554 |
| meanNfc' | 4.998762 | 4.995057 | 4.9952 | 4.995204 | 4.9952044 |
| sigNM | 180 | 180 | 180 | 180 | 180 |
| meanNM | 1800 | 1800 | 1800 | 1800 | 1800 |
| | | | | | |
| dg/Fy | 20.33687 | 18.91762 | 18.9456 | 18.94556 | 18.945558 |
| dg/dfc' | 1084.036 | 1125.793 | 1125.01 | 1125.01 | 1125.0104 |
| dg/dM | -1 | -1 | -1 | -1 | -1 |
| | | | | | |
| Alpha fy | 0.0359 | 0.0334 | 0.0334 | 0.0334 | 0.0334 |
| Alpha fc' | 0.5563 | 0.5558 | 0.5558 | 0.5558 | 0.5558 |
| Alpha M | -0.83023 | -0.83068 | -0.83065 | -0.83065 | -0.83065 |
| | | | | | |
| g( ) | 504.2481 | -1.74985 | -1.04689 | -1.03554 | -1.035764 |

(Not required)

Here I tried to maximize b for $A_{s,max}$.

$A_s$ upper limit = 4.68 $in^2$

$b = 8.5''$

$d = 13''$

$\beta = 4.82 > 3.09$

Cost = .002(8.5)(13+2) + .25(4.68)

= 0.255 + 1.17

= 1.425 $/in

$$g(X) = 1.5bA_s f_y - \frac{0.59A_s^2 f_y^2}{bf_c'} - M$$

| | | COV | | | |
|---|---|---|---|---|---|
| Fy mean [ksi] | 60 | | Zeta | 0.0064 | |
| Fy stan | 4.8 | 0.08 | Lambda | 4.0943 | |
| Fc' mean [ksi] | 5 | | Zeta | 0.0223 | |
| Fc' stan | 0.75 | 0.15 | Lambda | 1.6092 | |
| M mean [in-kips] | 1800 | 0.1 | | | |
| M stan | 180 | | | | |
| b [inches] | 8.5 | 12.75 | | | |
| d [inches] | 13 | | | | |
| As [max] in2 | 4.99 | 4.67 | 4.68 | 4.68 | 4.68 |
| | | | | | |
| Beta | 4.82 | | | | |
| | | | | | |
| F*y | 60 | 59.93 | 59.94 | 59.94 | 59.94 |
| fc'* | 5 | 4.67 | 4.68 | 4.68 | 4.68 |
| M* | 1800 | #### | 2487.94 | 2487.86 | 2487.87 |
| | | | | | |
| cdf(f*y) | 0.501273 | 0.429 | 0.43781 | 0.4375 | 0.4374965 |
| pdf(f*y) | 1.042228 | 1.027 | 1.03059 | 1.0305 | 1.03047 |
| cdf(f*c') | 0.504438 | 0.001 | 0.00168 | 0.0017 | 0.0016772 |
| pdf(f*c') | 3.585678 | 0.038 | 0.05191 | 0.0518 | 0.0518479 |
| | | | | | |
| sigNfy | 0.382776 | 0.382 | 0.38239 | 0.3824 | 0.3823847 |
| meanNfy | 59.99878 | 60 | 59.9987 | 59.999 | 59.998749 |
| sigNfc' | 0.111253 | 0.104 | 0.1042 | 0.1042 | 0.104198 |
| meanNfc' | 4.998762 | 4.988 | 4.98857 | 4.9886 | 4.9885679 |
| sigNM | 180 | 180 | 180 | 180 | 180 |
| meanNM | 1800 | 1800 | 1800 | 1800 | 1800 |
| | | | | | |
| dg/Fy | 22.14186 | 19.28 | 19.3788 | 19.379 | 19.378631 |
| dg/dfc' | 1244.419 | 1329 | 1325.96 | 1326 | 1325.9576 |
| dg/dM | -1 | -1 | -1 | -1 | -1 |
| | | | | | |
| Alpha fy | 0.0373 | #### | 0.0326 | 0.0326 | 0.0326 |
| Alpha fc' | 0.6092 | #### | 0.6086 | 0.6086 | 0.6086 |
| Alpha M | -0.79211 | -0.79 | -0.79284 | -0.793 | -0.792838 |
| | | | | | |
| g( ) | 772.9309 | -3 | -0.44082 | -0.379 | -0.3812 |