# Advanced FORM

CEE518

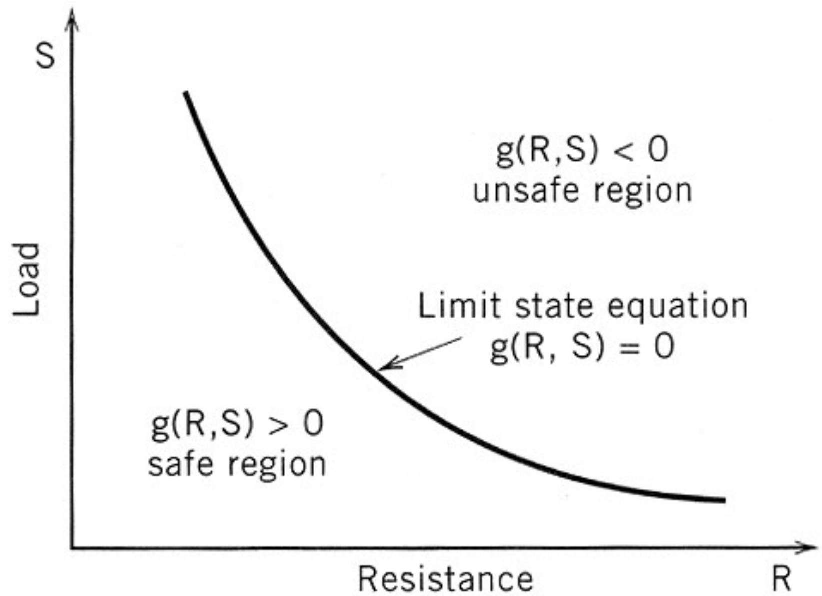



**Figure 7.3** Limit State Concept

Haldar text, Chapter 7

Strength Formulation of Limit State for Steel Beam

$$g(X) = Y = F_y Z - 1140 [lb - in]$$

Nominal $F_y = 36 ksi; \mu_{F_y} = 38 ksi; \sigma_{F_y} = 3.8 ksi$

Nominal Plastic Modulus Z; $\mu_Z = 54 in^3; \sigma_Z = 2.7 in^3$

$$\beta = \frac{38(54) - 1140}{\sqrt{\sigma_Y^2 = (229.2)^2}} = 3.975$$

Stress Formulation of Limit State of Same Beam

$$Y = g(X) = F_y - \frac{1140}{Z} = 0$$

$$\mu_{F_y} = 38\,ksi; \sigma_{F_y} = 3.8\,ksi$$

$$S = \frac{1140}{Z}; \mu_S = \frac{1140}{54} = 21.11\,ksi$$

$$\sigma_S = \sqrt{Var(Z)\left(-\frac{1140}{\mu_Z^2}\right)^2} = 1.056\,ksi \text{ [Taylor series approx.]}$$

$$\beta = \frac{38 - 21.11}{\sqrt{(3.8)^2 + (1.056)^2}} = 4.282$$

- This result is called “lack of invariance.”

AFOSM for Normal Variables: Transform Coordinates to find $\beta$:

$$X_i^{'} = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (i = 1,2,...,n) \quad \text{"reduced coordinate system"}$$

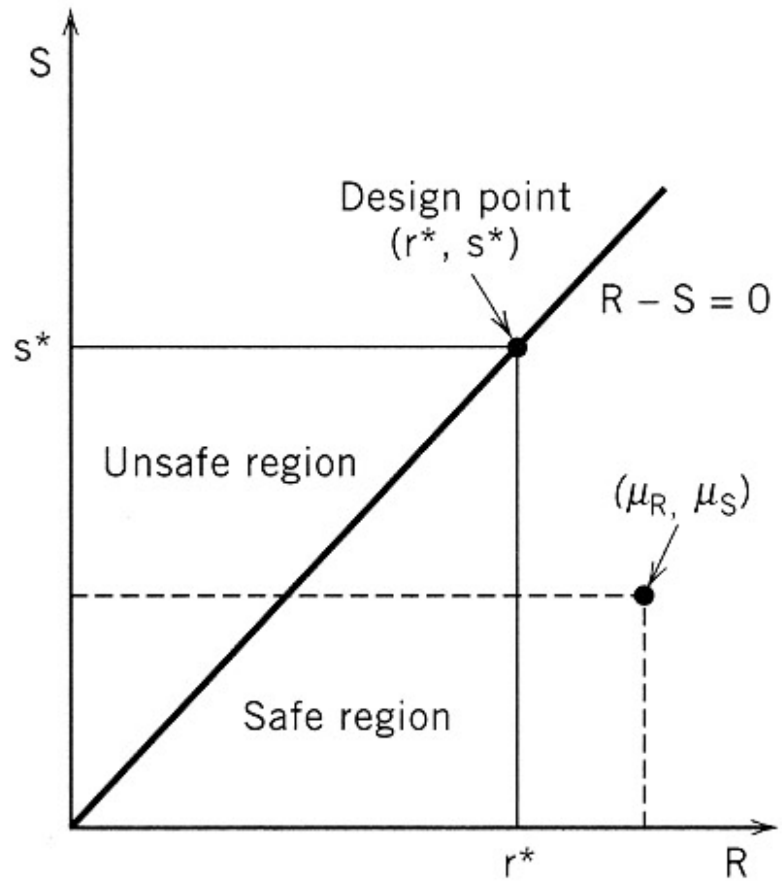
Transform original limit state $g(X) = 0$ into

the reduced limit state $g(X') = 0$.

$\beta$ in the reduced coordinate system can be shown to be

the minimum distance from the origin to the

limit state surface.




($a$) Original coordinates

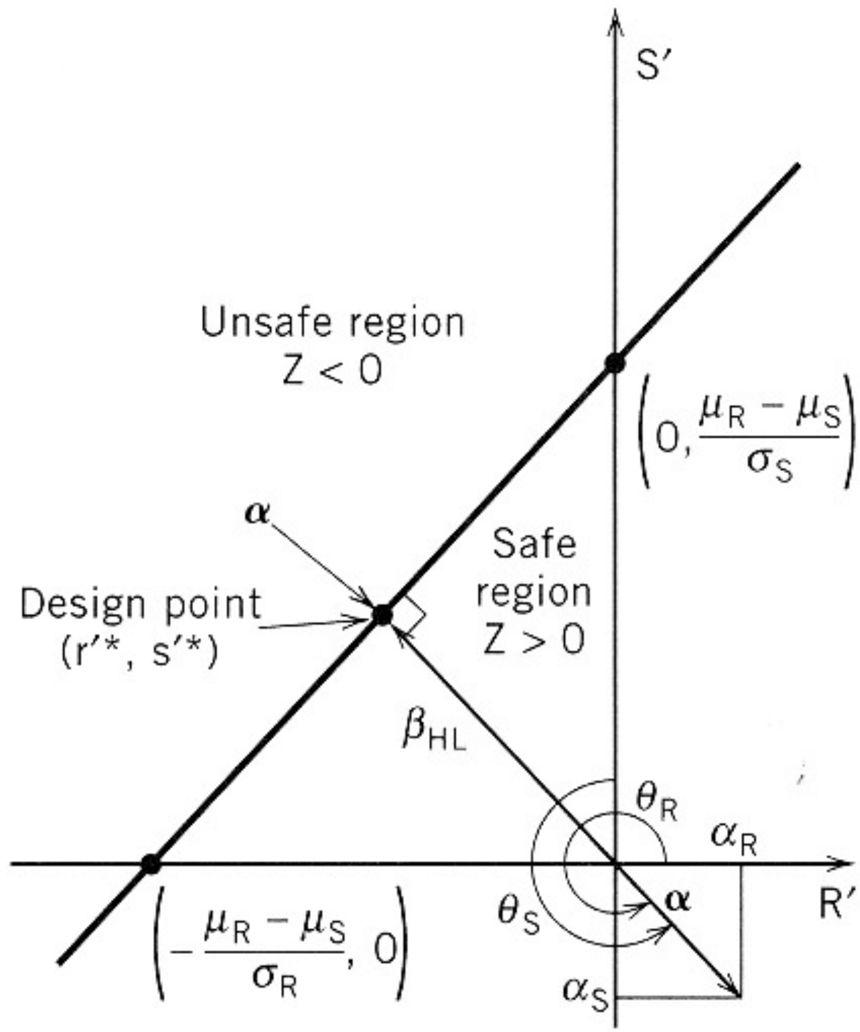
$$Z = R - S = 0$$

$$R' = \frac{R - \mu_R}{\sigma_R}$$

$$S' = \frac{S - \mu_S}{\sigma_S}$$

$$g(X') = \sigma_R R' - \sigma_S S' + \mu_R - \mu_S = 0$$

For linear g(X'), the $\beta$ is $\dfrac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}$




(*b*) Reduced coordinates

For nonlinear g(X), the computation of the minimum distance becomes an optimization exercise…

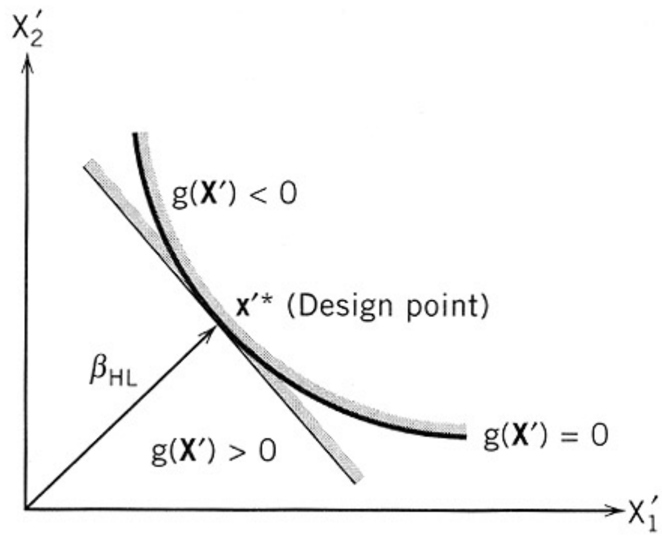



**Figure 7.5** Hasofer–Lind Reliability Index: Nonlinear Performance Function

Minimize $D = \sqrt{x'^T \, x'}$

Subject to the constraint $g(x) = g(x') = 0$

$x' \equiv$ checking point on the limit state equation in reduced coordinates to be estimated

We can obtain the minimum distance using optimization theory as

$$\beta = -\frac{\sum_{i=1}^{n} x_i^{*'} \left( \frac{\partial g}{\partial X_i^{'}} \right)^{*}}{\sqrt{\sum_{i=1}^{n} \left( \frac{\partial g}{\partial X_i^{'}} \right)^{*2}}}$$

$\left( \frac{\partial g}{\partial X_i^{'}} \right)^{*}$ ≡ the ith partial evaluated at the design point

with coordinates $(x_1^{'*}, x_2^{'*}, \ldots, x_n^{'*})$

The design point in reduced coordinates is

$$x_i^{'*} = -\alpha_i \beta \quad (i = 1, 2, ..., n)$$

*for*

$$\alpha_i = \frac{\left(\dfrac{\partial g}{\partial X_i^{'}}\right)^{*}}{\sqrt{\sum_{i=1}^{n}\left(\dfrac{\partial g}{\partial X_i^{'}}\right)^{*2}}} \equiv \text{direction cosines along the } X_i^{'} \text{ axes}$$

We find the design point as

$$x_i^* = \mu_{X_i} - \alpha_i \sigma_{X_i} \beta$$

A 7 - step algorithm has been developed for this process.

However, we are still limited to Normal variables. This is a big problem since most limit states involve non-Normal variables.

In order to get around this problem, we decide to convert all non-Normal distribution values to Normal ones at certain checking points.

# This process is called the Equivalent Normal Transformation

$\mu_{Xi}^N, \sigma_{Xi}^N \equiv$ the parameters of the equivalent Normal distribution

Two unknowns so we need two equations:

$$\Phi\left(\frac{x_i^* - \mu_{Xi}^N}{\sigma_{Xi}^N}\right) = F_{X_i}(x_i^*) \quad \text{[set CDFs = at checking point].}$$

$$\frac{1}{\sigma_{Xi}^N}\phi\left(\frac{x_i^* - \mu_{Xi}^N}{\sigma_{Xi}^N}\right) = f_{X_i}(x_i^*) \quad \text{[set pdfs = at checking point]}$$

Invert to obtain the two unknowns: $\mu_{Xi}^{N}, \sigma_{Xi}^{N}$

$$\frac{x_i^* - \mu_{Xi}^N}{\sigma_{Xi}^N} = \Phi^{-1}[F_{Xi}(x_i^*)]$$

$$\mu_{Xi}^N = x_i^* - \sigma_{Xi}^N \Phi^{-1}[F_{Xi}(x_i^*)]$$

$$\frac{1}{\sigma_{Xi}^N}\phi\left(\frac{x_i^* - \mu_{Xi}^N}{\sigma_{Xi}^N}\right) = f_{Xi}(x_i^*)$$

$$\sigma_{Xi}^N = \frac{\phi\left(\dfrac{x_i^* - \mu_{Xi}^N}{\sigma_{Xi}^N}\right)}{f_{Xi}(x_i^*)} = \frac{\phi\left\{\Phi^{-1}[F_{Xi}(x_i^*)]\right\}}{f_{Xi}(x_i^*)}$$

For highly skewed distributions, these may be inaccurate and methods have been developed to improve them. We will use the original equations for our assignments.

Table 7.2 in the text is a step by step process of solving our original problem for the reliability index. Reproduce this table as you read chapter 7. You will be asked to reproduce the results of Table 7.6 on p. 219 for homework.