YOUR NAME: KEY
CEE518 Autumn 2010 Quiz 1

GRADING	
P.1.	30
P.2.	10
P.3.	60
Sum=100	

1) [30] The drainage from a community during a storm is a normal random variable estimated to have a mean of 1.2 million gallons per day (mgd) and a standard deviation of 0.4 mgd. The storm drain system is designed with a maximum drainage capacity of 1.5 mgd.
   a. What is the underlying probability of flooding that is assumed in the design of the drainage system?
   b. What is the probability that the drainage during a storm will be between 1.0 mgd and 1,6 mgd?
   c. What is the 90th percentile drainage load from the community during a storm?

2) [10] Match the figures and captions.

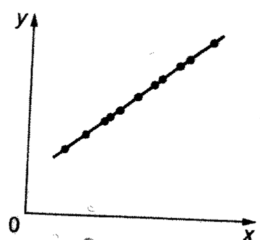
Captions: Figures:

$\rho = 0$

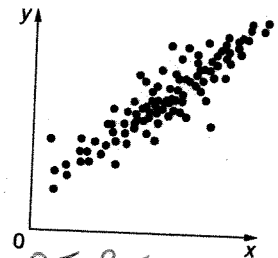
$\rho = 1$

$\rho = -1$

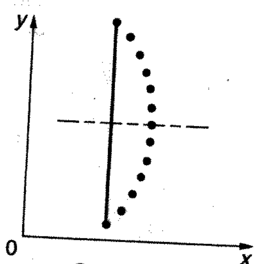
$0 \leq \rho \leq 1$



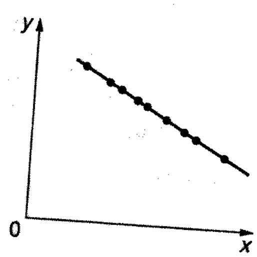
$\rho = +1$



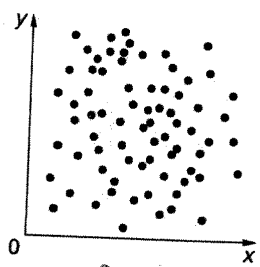
$0 < \rho < 1$



$\rho = 0$



$\rho = -1$



$\rho = 0$

Quiz 1 Problem 1 | Autumn 2010

$X \sim N(1.2, 0.4)$ mgd

1) $P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - \Phi\left(\frac{1.5-1.2}{0.4}\right) = 1 - \Phi(0.75)$

$= 1 - 0.7734 = \underline{\underline{0.227}}$

2) $P(1.0 < X \leq 1.6) = \Phi\left(\frac{1.6-1.2}{0.4}\right) - \Phi\left(\frac{1.0-1.2}{0.4}\right) = \Phi(1.0) - \Phi(-0.5)$

$= 0.8413 - [1 - \Phi(0.5)]$

$= 0.8413 - (1 - 0.6915)$

$= \underline{\underline{0.533}}$

3) $P(X \leq x_{0.90}) = \Phi\left(\frac{x_{0.90} - 1.2}{0.40}\right) = 0.90$

$$\frac{x_{0.90} - 1.2}{0.40} = \underbrace{\Phi^{-1}(0.90)}_{\text{1.28 from Appendix I}}$$

$\underline{\underline{x_{0.90}}} = 1.28(0.40) + 1.2 = \underline{\underline{1.71 \text{ mgd}}}$

3) [60] The joint density function of two random variables is given by

$$f_{XY}(x,y) = \frac{1}{102}\left(x+3x^2\right)y \text{ for } 1 \le x \le 4 \text{ and } 1 \le y \le 2$$

a) Find the marginal density functions of X and Y.
b) Find the mean values of X and Y.
c) Find the correlation coefficient of X and Y.

a) $f_X = \int f_{XY}\,dy = \frac{1}{102}\int_1^2 (x+3x^2)y\,dy = \frac{x+3x^2}{102}\left(\frac{y^2}{2}\Big|_1^2\right)$

$= \frac{x+3x^2}{102}\left(\frac{2^2-1^2}{2}\right) = \underline{\frac{(x+3x^2)}{102}\left(\frac{3}{2}\right)}$

$f_Y = \int f_{XY}\,dx = \frac{1}{102}\int (x+3x^2)y\,dx = \frac{y}{102}\int_1^4 (x+3x^2)dx$

$= \frac{y}{102}\left(\frac{x^2}{2} + \frac{3x^3}{3}\Big|_1^4\right) = \frac{y}{102}\left\{\left(\frac{4^2}{2} + \frac{3(4)^3}{3}\right) - \left(\frac{1}{2}+\frac{3}{3}\right)\right\}$

$= \frac{y}{102}\left\{(8+64)-(1.5)\right\} = \underline{\underline{\frac{70.5}{102}y}}$

b) $\underline{\underline{\mu_X}} = \int_1^4 x f_X(x)dx = \frac{1.5}{102}\int x(x+3x^2)dx = \frac{1.5}{102}\left(\frac{x^3}{3} + \frac{3x^4}{4}\Big|_1^4\right) = \frac{1.5}{102}\left(\left(\frac{4^3}{3} + \frac{3(4)^4}{4}\right)\right.$

$\left.-\left(\frac{1}{3}+\frac{3}{4}\right)\right\} = \frac{1.5}{102}\left(21.3+192-1.083\right) = \frac{1.5}{102}(212) = \underline{\underline{3.12}}$

$\mu_Y = \int_1^2 y f_Y(y)dy = \frac{70.5}{102}\int_1^2 y^2 dy = \frac{70.5}{102}\left(\frac{y^3}{3}\Big|_1^2\right) = \frac{70.5}{102}\left(\frac{2^3}{3} - \frac{1^3}{3}\right)$

$= \frac{70.5}{102}\left(\frac{8}{3} - \frac{1}{3}\right) = \underline{\underline{1.61}}$

c) $\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{E(XY)-E(X)E(Y)}{\sigma_X\sigma_Y} = \frac{4.88-(3.12)(1.61)}{\sigma_X\sigma_Y}$

$E(XY) = \int_1^2\int_1^4 xy f_{XY}\,dx\,dy = \int_1^2\int_1^4 \frac{xy}{102}\left[(x+3x^2)y\right]dx\,dy$

$= \int_1^2\int_1^4 \frac{1}{102}(x^2y+3yx^3)y\,dx\,dy$

$= \int_1^2\int_1^4 \frac{1}{102}\left(x^2y^2+3y^2x^3\right)dx\,dy$

$= \int_1^2 \frac{1}{102}\left\{\left(y^2\frac{x^3}{3} + 3y^2\frac{x^4}{4}\Big|_1^4\right\}dy\right.$

$= \frac{1}{102}\int_1^2\left\{\left(y^2\left(\frac{4^3}{3}\right) + 3y^2\left(\frac{4^4}{4}\right)\right) - \left(y^2\left(\frac{1}{3}\right) + 3y^2\left(\frac{1}{4}\right)\right)\right\}$

$= \frac{1}{102}\int_1^2\left(21.3y^2 + 192y^2 - \frac{y^2}{3} - 0.75y^2\right)dy$

$= 2.095\int_1^2 y^2 dy = 2.095\left(\frac{y^3}{3}\Big|_1^2\right) = 2.095\left(\frac{8}{3}-\frac{1}{3}\right) = \underline{4.88}$

σ_x :

$$Var(X) = \int_1^4 (x-3.12)^2 \left\{ \underset{\overset{|||}{0.0147}}{\left(\frac{1.5}{102}\right)} (x+3x^2) \right\}$$

$(x-3.12)^2 = x^2 - 6.24x + 9.73$

$(\quad)^2(x+3x^2) = x^3 - 6.24x^2 + 9.73x$
$+3x^4 - 18.72x^3 + 87.57x^2$

$3x^4 + x^3 - 18.72x^3 - 6.24x^2 + 87.57x^2 + 9.73x$

$3x^4 - 17.72x^3 + 81.33x^2 + 9.73x$

$$Var(X) = 0.0147 \int_1^4 (3x^4 - 17.72x^3 + 81.33x^2 + 9.73x)\,dx$$

$$= 0.0147 \left(3\frac{x^5}{5} - 17.72\frac{x^4}{4} + 81.33\frac{x^3}{3} + 9.73\frac{x^2}{2} \right|_1^4$$

$$= 0.0147 \left\{ \frac{3}{5}(4)^5 \overset{1024}{} - \frac{17.72(4)^4}{4} \overset{256}{} + \frac{81.33}{3}(4)^3 \overset{64}{} + \frac{9.73}{2}(4)^2 \overset{16}{} \right\}$$

$$- .0147 \left\{ \underset{0.6}{\frac{3}{5}} - \underset{4.43}{\frac{17.72}{4}} + \underset{27.11}{\frac{81.33}{3}} + \underset{4.87}{\frac{9.73}{2}} \right\}$$

$$= 0.0147 \{(614.4 - 1134.1 + 1735 + 77.92) - (+28.15)\}$$

$$= (1265)(0.0147) = 18.59$$

$\sqrt{} = 4.31 = \sigma_x$

$$Var(Y) = \int_1^2 \left(\frac{70.5}{102}\right)(y-1.61)^2\, y\,dy = 0.691 \int_1^2 (y-1.61)^2 y\,dy$$

$$= 0.691 \int_1^2 (y^3 - 3.22y^2 + 2.59y)\,dy$$

$$= 0.691 \left(\frac{y^4}{4} - 3.22\frac{y^3}{3} + 2.59\frac{y^2}{2} \right|_1^2$$

$$= 0.691 \left(\left(\frac{2^4}{4} - 3.22\frac{(2)^3}{3} + 2.59\frac{(2)^2}{2}\right) - \left(\frac{1}{4} - \overset{1.07}{\frac{3.22}{3}} + \overset{1.29}{\frac{2.59}{2}}\right)\right)$$

$$= 0.691 \{(4 - 8.59 + 5.18) - (0.47)\}$$

$$= 0.083$$

$\sigma_y = \sqrt{} = 0.288$

$$\therefore \quad \rho_{xy} = \frac{4.88 - 3.12(1.61)}{4.31(.288)} = \frac{4.88 - 5.02}{1.24} = \underline{\underline{-0.113}}$$

project:

checking to see if $f_x f_y = f_{xy}$

$$\frac{(x+3x^2)}{102}\left(\frac{3}{2}\right)\left\{\frac{70.5}{102}\right\} y \stackrel{?}{=} \frac{1}{102}(x+3x^2)y$$

$$\frac{(x+3x^2)}{102}\,\frac{105}{102}\,y \approx \frac{1}{102}(x+3x^2)y$$

$$\frac{1.04}{102}(x+3x^2)y \approx \frac{1}{102}(x+3x^2)y \qquad \text{OK}$$

so basically $\rho_{xy} = 0$.