C518 tsepac.doc | Practice Problems

1) $Y = g(X) = F_b - \frac{6M}{bd^2}$

	μ	cov	σ
M	100,000	0.12	12,000
F_b	1600	0.32	512
b	5.6	0.04	0.224
d	11.4	0.03	0.342

$\beta = \frac{\mu_Y}{\sigma_Y}$

Use TSE: $\mu_Y = g(\mu_{x_i}) = \mu_{F_b} - \frac{6\mu_M}{\mu_b \mu_d^2} = 1600 - \frac{6(100,000)}{5.6(11.4)^2}$

$= 776$ psi

$Var(Y) = \sigma_Y^2 = \left(\frac{\partial g}{\partial x_i}\right)^2 Var(x_i) \Rightarrow$

$\frac{\partial g}{\partial F_b} = 1$

$\frac{\partial g}{\partial M} = -\frac{6}{bd^2}$

$\frac{\partial g}{\partial b} = \frac{(+)6M}{b^2 d^2}$

$\frac{\partial g}{\partial d} = \frac{-6M(-2)}{b d^3} = \frac{12M}{bd^3}$

$\therefore Var(Y) = (1)^2(512)^2 + \left(\frac{-6}{(5.6)(11.4)^2}\right)(12,000)^2$

$+ \left(\frac{6(100,000)}{(5.6)^2(11.4)^2}\right)(0.224)^2 + \left(\frac{12(100,000)}{(5.6)(11.4)^3}\right)(0.342)^2$

$= 262144 + [.0082(12,000)^2] + [147.2(.224)^2] + [144.6(.342)^2]$

$= 262144 + 1180800 + 7.4 + 16.9$

$= 1442968$

$\sigma_Y = 1201$ psi

$\therefore \beta = \frac{\mu_Y}{\sigma_Y} = \frac{776 \text{ psi}}{1201 \text{ psi}} = 0.646$ not so great

2) $C = \frac{WF}{\sqrt{E}} = g(x)$

$P(C > \$35{,}000) = 1 - P(C \leq 35{,}000) = 1 - F_C(35{,}000)$

$$\frac{\partial C}{\partial W} = \frac{F}{\sqrt{E}}$$

$$\frac{\partial C}{\partial F} = \frac{W}{\sqrt{E}}$$

$$\frac{\partial C}{\partial E} = \frac{-1/2\, WF}{E^{3/2}}$$

$$\underline{\underline{\mu_C}} = \frac{\mu_W \mu_F}{\sqrt{\mu_E}} = \frac{2000(20)}{\sqrt{1.6}} = \underline{\underline{\$31622.8}}$$

$$Var(c) = \left(\frac{\partial g}{\partial X_i}\right)^2 Var(X) = \left(\frac{F}{\sqrt{E}}\right)^2 Var(W) + \left(\frac{W}{\sqrt{E}}\right)^2 Var(F) + \left(\frac{-\frac{1}{2}WF}{E^{3/2}}\right)^2 Var(E)$$

$$= \left(\frac{\mu_F}{\sqrt{\mu_E}}\right)^2 (0.2(2000))^2 + \left(\frac{\mu_W}{\sqrt{\mu_E}}\right)^2 (20(0.15))^2 + \left(\frac{\mu_W \mu_F}{2\mu_E^{3/2}}\right)^2 (1.6(0.125))^2$$

$$= \left(\frac{20}{\sqrt{1.6}}\right)^2 (160{,}000) + \left(\frac{2000}{\sqrt{1.6}}\right)^2 (9) + \left(\frac{2000(20)}{2(1.6)^{3/2}}\right)^2 (0.04)$$

$$= 40000000 + 22500000 + 3906250$$

$$= 66406250$$

$$\underline{\sigma_C = \$8149}$$

$\therefore$ Assume $C \sim N(31622.8, 8149)$

$$\boxed{\Phi} \left(\frac{35{,}000 - 31622.8}{8149}\right) = \Phi(0.414) = \overline{0.65910} = \underline{\underline{65.9\%}}$$

$$\therefore \quad 1 - F_C(35{,}000) = \boxed{34.1\%}$$

3) $V = \frac{1.49}{n} R^{2/3} S^{1/2} = g(x)$

$$\underline{\underline{\mu_V}} = \frac{1.49}{\mu_n} (\mu_R)^{2/3} (\mu_S)^{1/2} \text{ fps} = \underline{\underline{182.2 \text{ fps}}}$$

$$Var(v) = \left(\frac{\partial g}{\partial x_i}\right)^2 Var(x_i)$$

$$\frac{\partial v}{\partial S} = \left(\frac{1.49}{n} R^{2/3}\right)\left(\frac{1}{2} S^{-1/2}\right) = \frac{0.745}{n} R^{2/3} S^{-1/2}$$

$$\frac{\partial v}{\partial R} = \left(\frac{1.49}{n} S^{1/2}\right) \frac{2}{3} R^{-1/3} = \frac{0.993}{n} R^{-1/3} S^{1/2}$$

$$\frac{\partial v}{\partial n} = 1.49 S^{1/2} R^{2/3} \left(\frac{-1}{n^2}\right) = \frac{-1.49}{n^2} R^{2/3} S^{1/2}$$

$$Var = \left(\frac{0.745}{(0.013)} \underbrace{(2)^{2/3}}_{1.59} (1)^{-1/2}\right)^2 \left(1(0.10)\right)^2$$

$$+ \left(\frac{0.993}{(0.013)} \underbrace{(2)^{-1/3}}_{0.796} (1)^{1/2}\right)^2 \left(2(0.05)\right)^2$$

$$+ \left(\frac{-1.49}{(0.013)^2} \underbrace{(2)^{2/3}}_{1.59} (1)^{1/2}\right)^2 \left(0.013(0.3)\right)^2$$

$$= 8303(0.01) + 3696.9(0.01) + (19651945.9)(0.0000152)$$

$$= 83.03 + 36.97 + 2989$$

$$= 3109$$

$$\underline{\underline{\sigma_V = 55.8 \text{ fps}}}$$

Practice Problems

4) $\mu_{F_y} = 47.9$ ksi $\quad \sigma_{F_y} = 3.3$ ksi

$\mu_F = 36.0$ ksi $\quad \sigma_F = 7.2$ ksi

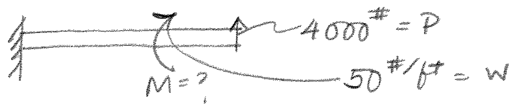
a) $\Phi\left(\dfrac{-(47.9-36.0)}{\sqrt{(3.3)^2+(7.2)^2}}\right) = \Phi\left(\dfrac{-11.9}{7.9}\right) = \Phi(-1.50)$

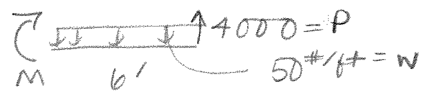
$p_f = 1 - 0.84134 = \underline{15.9\%}$

b) $\sigma_F = 3.6$ ksi vs. 7.2 ksi

$\Phi\left(\dfrac{-11.9}{4.9}\right) = \Phi(-2.44)$

$p_f = 1 - 0.99266 = 0.00734 = \underline{0.73\%}$ wow → big difference

5) 

M = 

$M - 6P + w(6)(3) = 0$

$M = 6P + 18w = 6P + 900$ ft-#

$\mu_M = 6\mu_P + 900 = 6(4000^{\#}) + 900 = \underline{24,900 \text{ \#-ft}}$

$Var(M) = \left(\dfrac{\partial g}{\partial X}\right) Var(X) \Rightarrow \dfrac{\partial g}{\partial P} = 6$

$Var(M) = (6)^2(0.15(4000)) = 21,600 (\text{ft-\#})^2$

$\underline{\sigma_M = 146.9 \text{ ft-\#}}$

Mcapacity − Mdemand = 0 ≡ Failure

Mc − Md

$\beta = \Phi^{-1}(1-p_f) = \dfrac{\mu_{Mc}-\mu_{Md}}{\sqrt{\sigma_{Mc}^2+\sigma_{Md}^2}} = \dfrac{\mu_{Mc}-\mu_{Md}}{\sqrt{\delta_{Mc}\mu_{Mc}+\sigma_{Md}^2}} = \Phi^{-1}(1-.005)$

$\dfrac{\mu_{Mc} - 24,900}{146.9} = 2.58 \quad \therefore \quad \mu_{Mc} = 25279$ #-ft $\quad (\sigma_{Mc} = 0\%)$

Practice Problems

6) $g(X) = R/S = Y$ $\quad\quad Y = LN(\lambda_Y, \zeta_Y)$

$\lambda_Y = \lambda_R - \lambda_S$

$\zeta_Y^2 = \zeta_R^2 + \zeta_S^2$

$\lambda_R = \ln(\mu_R) - \frac{1}{2}\zeta_R^2 = \underline{6.7359}$

$\zeta_R^2 = \ln(1 + COV_R^2) = \underline{0.0275}$

$\lambda_S = \ln(326.4) - \frac{1}{2}\zeta_S^2 = \underline{5.7596}$

$\zeta_S^2 = \ln(1 + COV_S^2) = \underline{0.0285}$

$\underline{\zeta_Y} = \sqrt{0.0275 + 0.0285} = \underline{0.237}$

$\underline{\lambda_Y} = 6.7359 - 5.7596 = \underline{0.9763}$

$P(Y \leq 1) = \Phi\left(\frac{\ln(1) - 0.9763}{0.237}\right) = \Phi(-4.12) \approx \underline{2.0669 \times 10^{-5}}$

very small p_f.

(The higher β, the smaller p_f.)

$\beta \downarrow 4.12$ $\quad\quad p_f \downarrow \sim 10^{-5}$