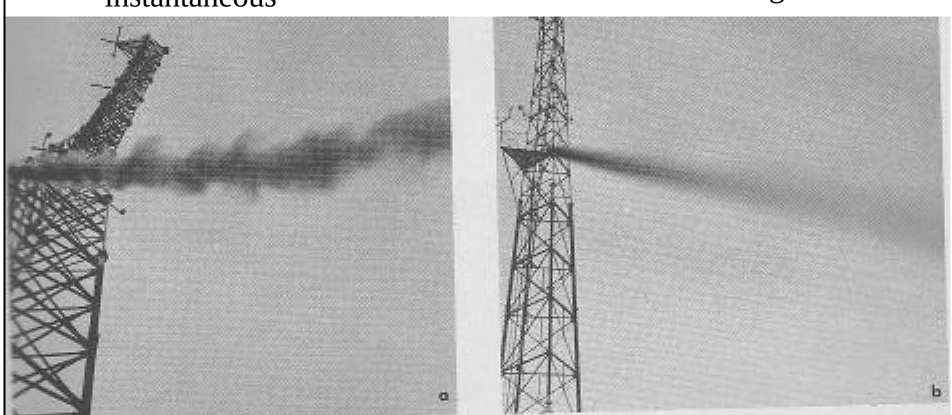
Gaussian Plumes from "Point" Sources

- Time averaged vs instantaneous plumes
- Simplified steady-state plume model
- "Eddy" diffusion, advection/diffusion equation
- Gaussian point source plume model
- Plume sigma values vs stability and distance
- Plume reflection
- Non-gaussian plumes
- Plume Rise; plume trajectories
- Buoyancy-induced dispersion
- Stack downwash

View from below of "Coning" plume
under neutral atmospheric conditions

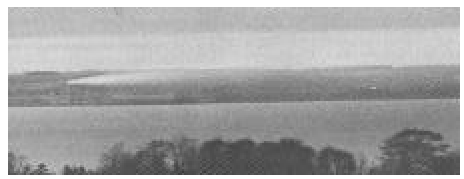
instantaneous | time-averaged



Source: Slade et al "Meteorology and Atomic Energy, 1968"

Instantaneous
Plume Shape

Time-averaged
Plume Shape



Describing Plume Concentrations

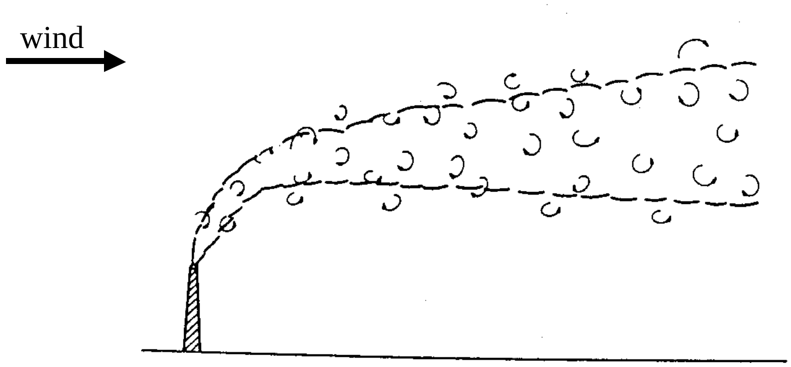
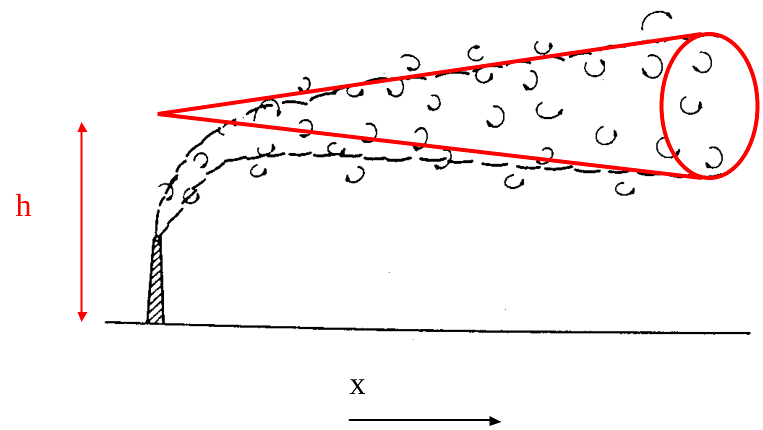


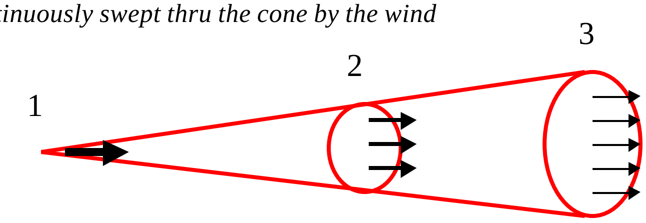
Fig 4-3, p.44 in Martin et al

Tim’s Simple Plume Model



Simplified Steady-State Plume Model

**Pollutant is well mixed and confined within the cone*
**Pollutant is continuously swept thru the cone by the wind*



mass/time passing point 1	=	mass/time passing thru disk area 2	=	mass/time passing thru disk area 3

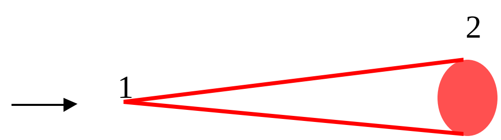
$C_1 > C_2 > C_3$

Concentration vs. distance downwind depends upon cone shape

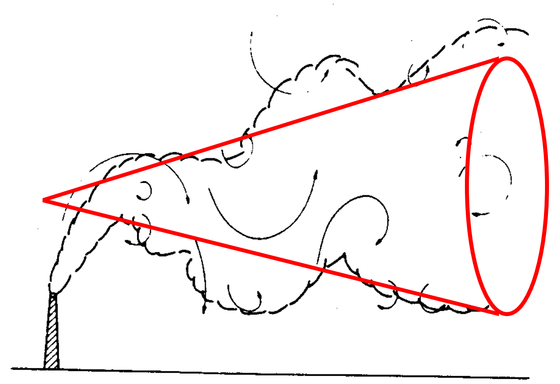
Simple Model #1:

$$\text{Concentration of air at 2} = \frac{\text{Mass emission rate}}{(\text{wind speed})(\text{area of disk 2})}$$

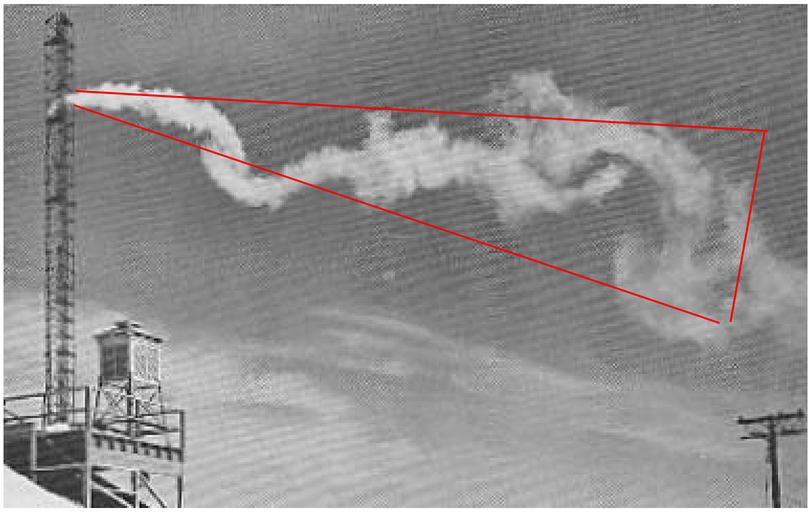
$$\frac{\mu g}{m^3} = \frac{\mu g/\text{sec}}{(m/\text{sec})(m^2)}$$



Disk shape depends upon stability category



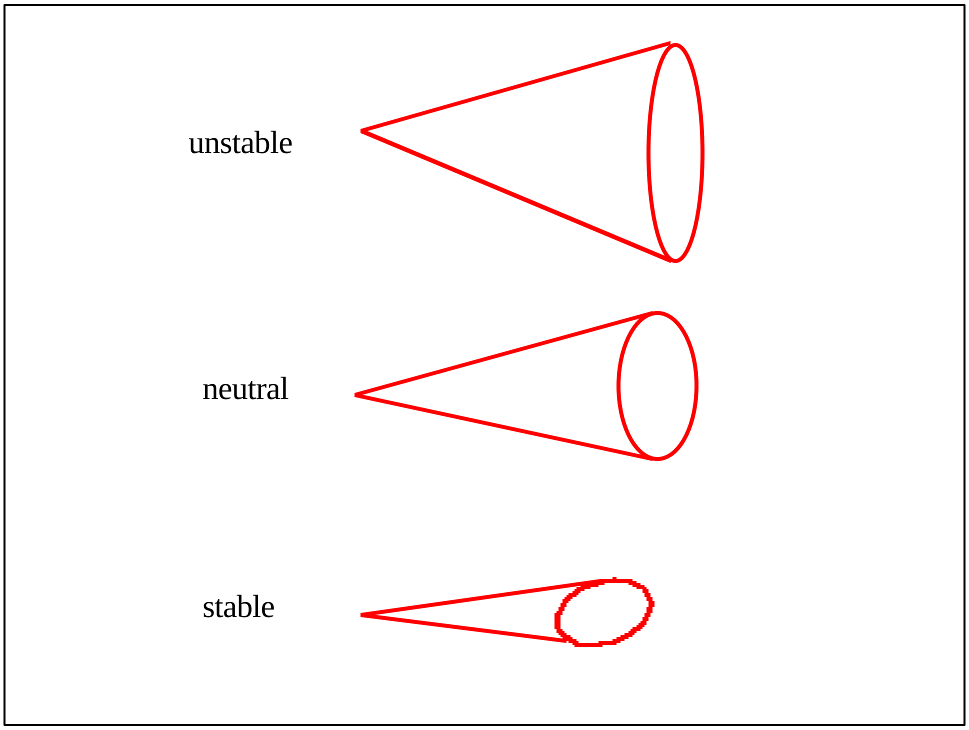
More unstable and thus more pronounced vertical spreading



Perspective View of "Fanning" Plume in Very Stable Air

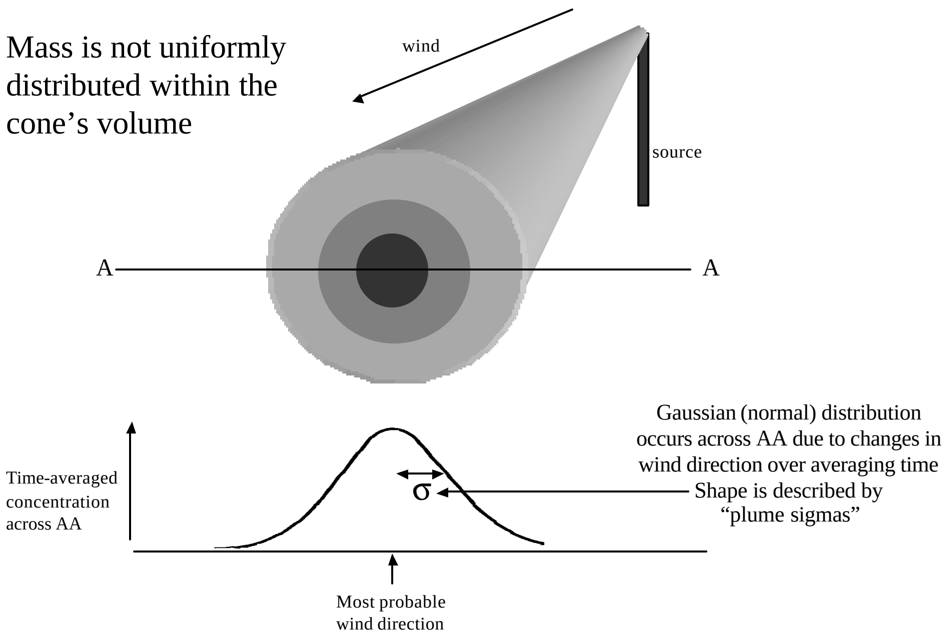
Less vertical motion

Source: Slade et al "Meteorology and Atomic Energy, 1968"



More Detailed Plume Model

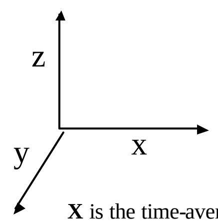
Mass is not uniformly distributed within the cone's volume

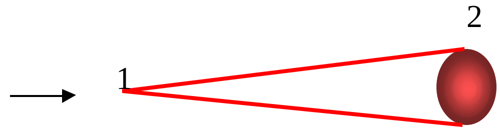


Simple Model #2:

$$\text{Conc at 2} = \frac{\text{Mass emission rate}}{(\text{wind speed})(\text{area of disk 2})}[\text{Gaussian distribution function}]$$

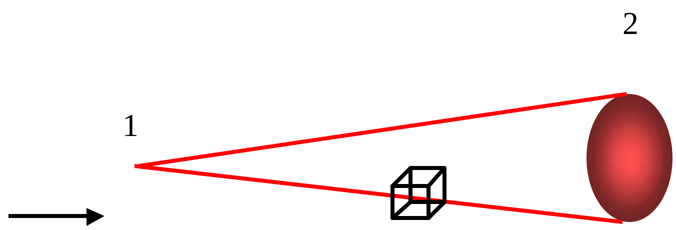
$$\frac{\mu g}{m^3} = \frac{\mu g/\text{sec}}{(\text{m/sec})(m^2)}[-]$$



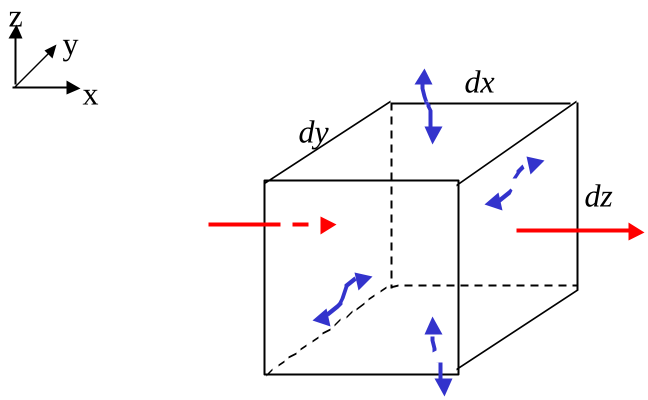


X is the time-averaged wind direction,
Y is the cross-wind direction,
Z is the vertical dimension

Gaussian Plume Model



In order to derive an equation describing the distribution of mass within the plume, we must first consider the transport of mass within a small control volume

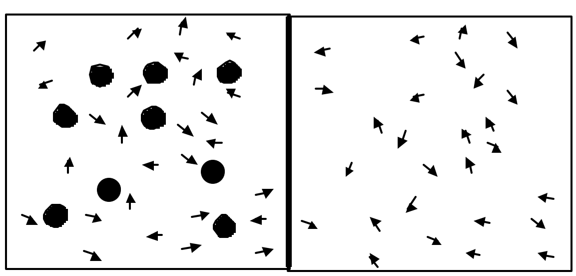


*Transport of mass in x direction depends on the average horizontal wind

*Transport of mass in the y and z directions depends on turbulent motions

“Eddy” Diffusion

Consider two enclosed air volumes separated by a wall
(arrows represent eddy motions; balls represent pollutant molecules)



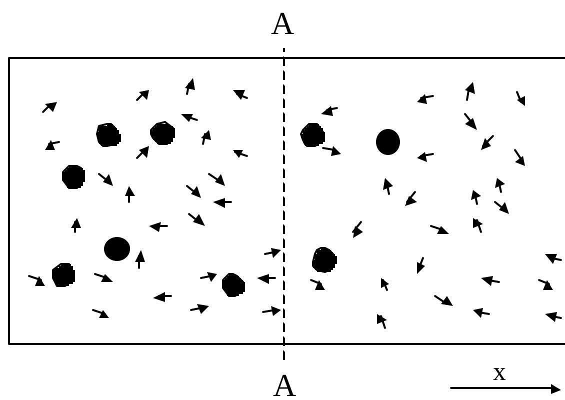
Size of eddies > size of pollutant molecules

Pollutant molecules are moving around in random directions due to random eddy motions

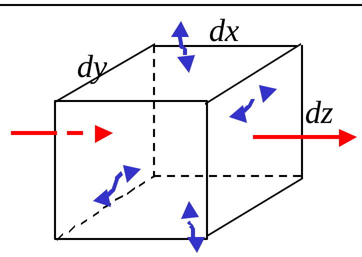
Random eddy motions in pollution-free air

Now remove the wall between the enclosures

The rate of pollutant molecules crossing plane AA depends upon the concentration difference between the two sides. Specifically, this rate $= K\, \delta c/\delta x$, where K is termed an "eddy diffusivity" with units of m^2/sec. The magnitude of K depends upon the magnitude of the eddy motions.



Eventually in this case, the rate of net rate of pollution crossing AA is zero (an equal number of molecules cross in both directions). However the value of K remains constant throughout the "experiment"



V =Volume = $dxdydz$

$A_{xy} = dxdy$

$A_{yz} = dydz$

$A_{xz} = dxdz$

Changes in x Direction:

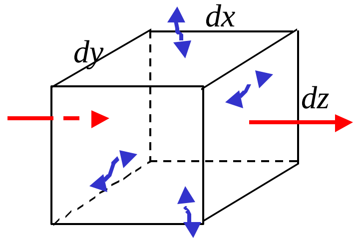
Net rate of change of mass flow =

(Mass flow rate in) - (Mass flow rate out)

Mass Flow Rate In = $C\ u\ A_{yz}$ { [µg/m³] [m/s] [m²]}

$$\text{Mass Flow Rate Out} = C\,u\,A_{yz} + \frac{\partial}{\partial x}\left(C\,u\,A_{yz}\right)dx$$

$$\text{Net Rate of Change} = -\frac{\partial}{\partial x}\left(C\,u\,A_{yz}\right)dx = -\frac{\partial}{\partial x}\left(C\,u\right)V$$



V = Volume = *dxdydz*
A_{xy} = *dxdy*
A_{yz} = *dydz*
A_{xz} = *dxdz*

Changes in z Direction via " Turbulent Diffusion":

$$\text{Mass Flow Rate In} = -A_{xy}\frac{\partial}{\partial z}\{K_z C\} \qquad [m^2][m^{-1}][mg\ m^{-3}][m^2\ sec^{-1}]$$

$$\text{Net Rate of Change} = \frac{\partial}{\partial z}\left\{\frac{\partial}{\partial z}(K_z C)\right\}V$$

A similar result is obtained in the y direction. Given that the net rate of change in the volume [= V(δc/δt)] is the change in all three directions , we obtain an overall expression in terms of x, y and z.

"Advection-Diffusion" Equation

$$\frac{\partial C}{\partial t} = -u\frac{\partial C}{\partial x} + \frac{\partial}{\partial y}\left\{K_y\frac{\partial C}{\partial y}\right\} + \frac{\partial}{\partial z}\left\{K_z\frac{\partial C}{\partial z}\right\}$$

+ other losses due to deposition and chemical reactions

= 0 for steady-state models

"Advection", i.e., transport by the mean wind, u

Effect of turbulent "diffusion", i.e., exchange of polluted air parcel with surrounding air parcels. If the surrounding air is cleaner, δC/δz & δC/δy are negative. K is the "eddy diffusivity" and represents the intensity of turbulent motions and varies with stability

The **Gaussian plume equation** is a particular solution to this more general equation under the following assumptions:

* Steady state conditions $\frac{\partial C}{\partial t} = 0$

* Constant wind speed with height (u does not depend on z)

* Constant eddy diffusivity (K does not depend on y or z)

Define: $\sigma_z^2 = \frac{2\,K_z\,x}{u}$ $\quad \sigma_y^2 = \frac{2\,K_y\,x}{u}$

* Mass is conserved

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C\,dy\,dz = Q \qquad [\text{for } x > 0] \qquad \{\mu g/\text{sec}\}$$

Gaussian "Point" Source Plume Model:

$$C(x, y, z) = \frac{Q}{2\pi u\,\sigma_y \sigma_z}\left\{\exp\left(\frac{-(z-h)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+h)^2}{2\sigma_z^2}\right)\right\}\left\{\exp\left(\frac{-(y)^2}{2\sigma_y^2}\right)\right\}$$

Pollutant concentration as a function of downwind position (x,y,z)

Mass emission rate

"Effective" stack height, including rise of the hot plume near the source

Wind speed evaluated at "effective" release height

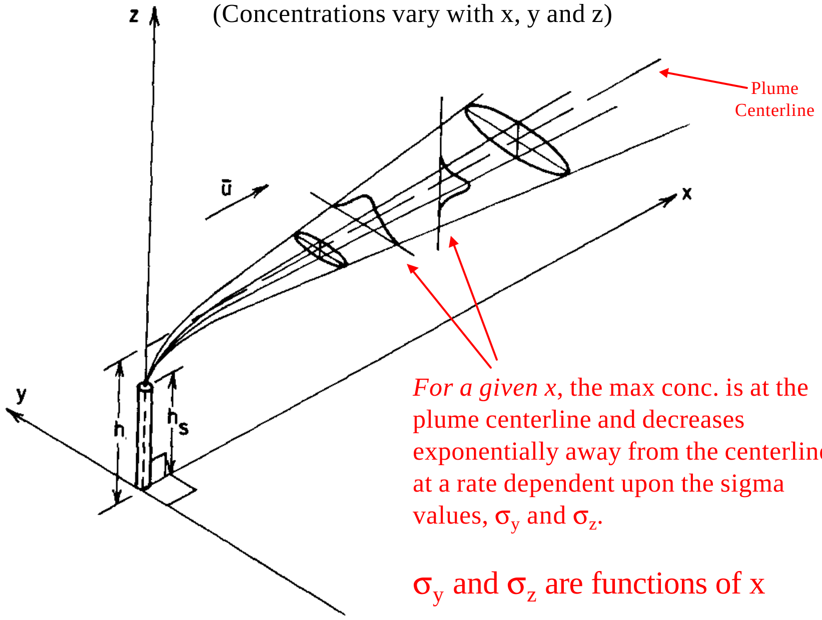
Corresponds to disk area in simple model (values depend upon downwind distance, x)

Distribution of mass in vertical dimension (z) at a given downwind distance, x (includes the effect of surface reflection)

Distribution of mass in cross-wind dimension (y) at a given downwind distance, x

Gaussian Plume

(Concentrations vary with x, y and z)



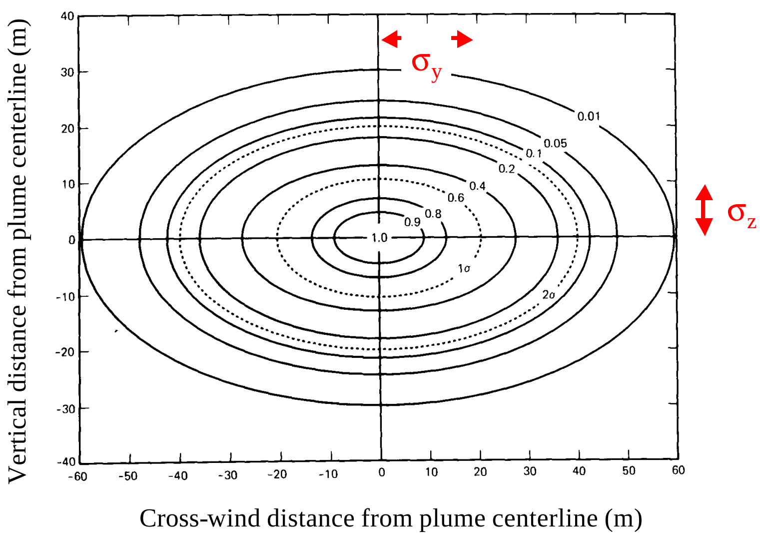
For a given x, the max conc. is at the plume centerline and decreases exponentially away from the centerline at a rate dependent upon the sigma values, σ_y and σ_z.

σ_y and σ_z are functions of x

Concentration distribution in a Gaussian plume

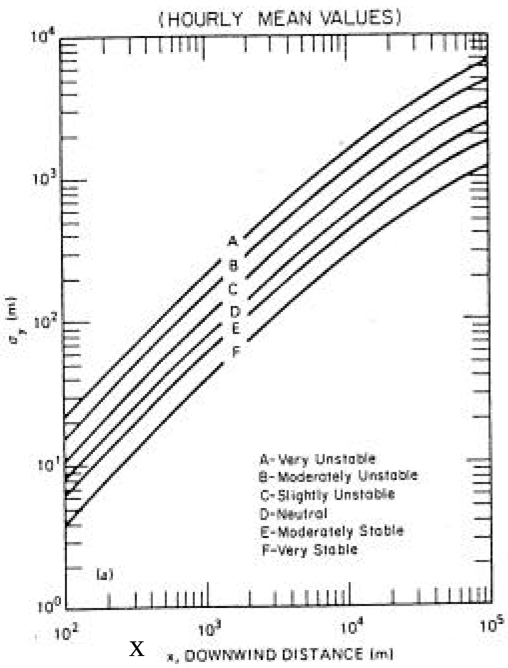
(σ_y = 20 m; σ_z = 10m; centerline concentration = 1.0)

Note: theoretical plume has infinite extent in all directions!

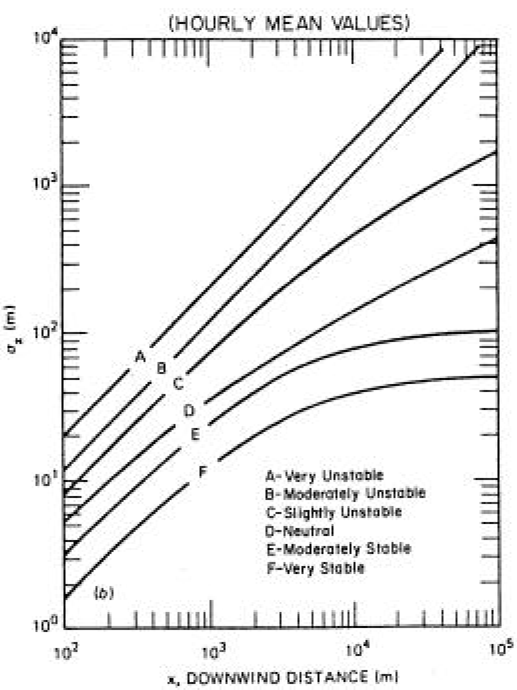


Source: Hanna et al, 1981

Sigma-y



Sigma-z



Plume sigma formulas from EPA's ISC Model

Vertical distribution: $$s_z = ax^b$$

x is in *kilometers*
σ_z is in *meters*
a, *b* depend on *x*

Cross-wind distribution: $$s_y = 465.11628x(\tan \Theta)$$

$$\Theta = 0.017453293(c - d \ln(x))$$

x is in *kilometers*
σ_y is in *meters*
Θ is in *radians*

$$s_z = ax^b$$

Pasquill Stability Category	x (km)	a	b
A*	<.10	122.800	0.94470
	0.10 - 0.15	158.080	1.05420
	0.16 - 0.20	170.220	1.09320
	0.21 - 0.25	179.520	1.12620
	0.26 - 0.30	217.410	1.26440
	0.31 - 0.40	258.890	1.40940
	0.41 - 0.50	346.750	1.72830
	0.51 - 3.11	453.850	2.11660
	>3.11	**	**

* If the calculated value of óz exceed 5000 m. óz is set to 5000 m.

$$s_z = ax^b$$

Pasquill Stability Category	x (km)	a	b
B*	<.20	90.673	0.93198
	0.21 - 0.40	98.483	0.98332
	>0.40	109.300	1.09710
C*	All	61.141	0.91465
D	<.30	34.459	0.86974
	0.31 - 1.00	32.093	0.81066
	1.01 - 3.00	32.093	0.64403
	3.01 - 10.00	33.504	0.60486
	10.01 - 30.00	36.650	0.56589
	>30.00	44.053	0.51179

* If the calculated value of $\acute{o}_z$ exceed 5000 m, $\acute{o}_z$ is set to 5000 m.

** $\acute{o}_z$ is equal to 5000 m.

$$s_z = ax^b$$

Pasquill Stability Category	x (km)	a	b
E	<.10	24.260	0.83660
	0.10 - 0.30	23.331	0.81956
	0.31 - 1.00	21.628	0.75660
	1.01 - 2.00	21.628	0.63077
	2.01 - 4.00	22.534	0.57154
	4.01 - 10.00	24.703	0.50527
	10.01 - 20.00	26.970	0.46713
	20.01 - 40.00	35.420	0.37615
	>40.00	47.618	0.29592
F	<.20	15.209	0.81558
	0.21 - 0.70	14.457	0.78407
	0.71 - 1.00	13.953	0.68465
	1.01 - 2.00	13.953	0.63227
	2.01 - 3.00	14.823	0.54503
	3.01 - 7.00	16.187	0.46490
	7.01 - 15.00	17.836	0.41507
	15.01 - 30.00	22.651	0.32681
	30.01 - 60.00	27.074	0.27436
	>60.00	34.219	0.21716

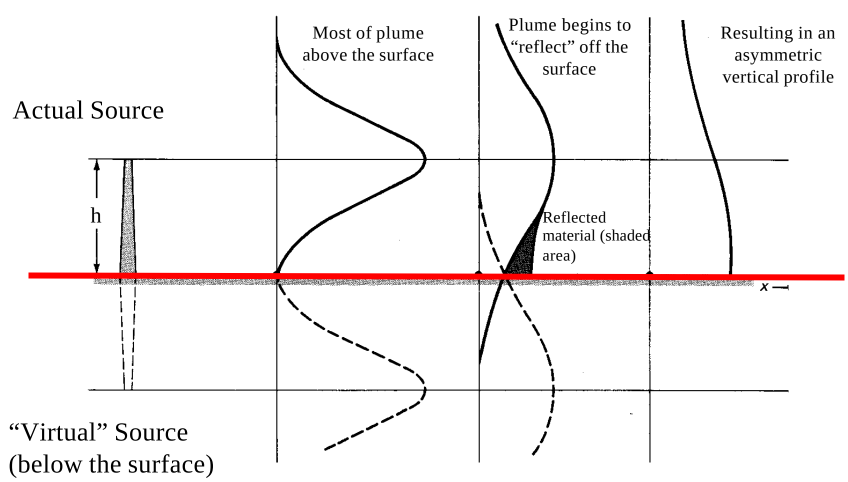
$$\Theta = 0.017453293(c - d \ln(x))$$

Pasquill Stability Category	c	d
A	24.1670	2.5334
B	18.3330	1.8096
C	12.5000	1.0857
D	8.3330	0.72382
E	6.2500	0.54287
F	4.1667	0.36191

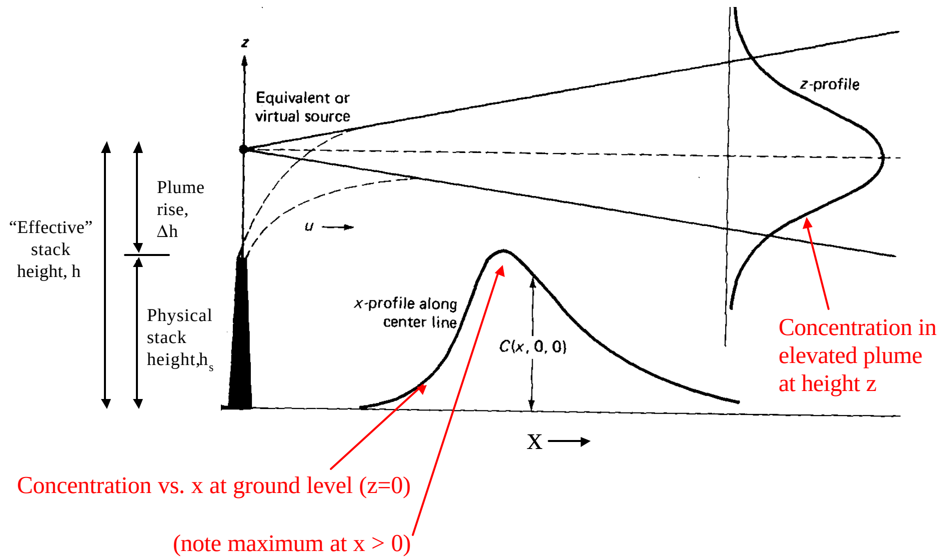
Plume "Reflection" off of the Ground

(pollutant cannot penetrate the ground)

Reflection is modeled by adding a "virtual" source contribution to the "real" one



A virtual" source can also be used to model the effect of hot, rising plumes



Example Calculation

Given:

Q = 10 grams/sec; h = 50m; x = 500 m = 0.5 km; u_{100} = 6 m/s; Stability Class "D"

Compute:

C(500, 0, 0) ,i.e., the ground level concentration at plume centerline, 500 meters downwind.

$$\sigma_z = ax^b = 32.093(0.5)^{0.81066} = 18.3\text{m}$$

$$\Theta = 0.017453293\,(8.3330 - 0.72382\ln[0.5]) = 0.1542\,\text{radians}$$

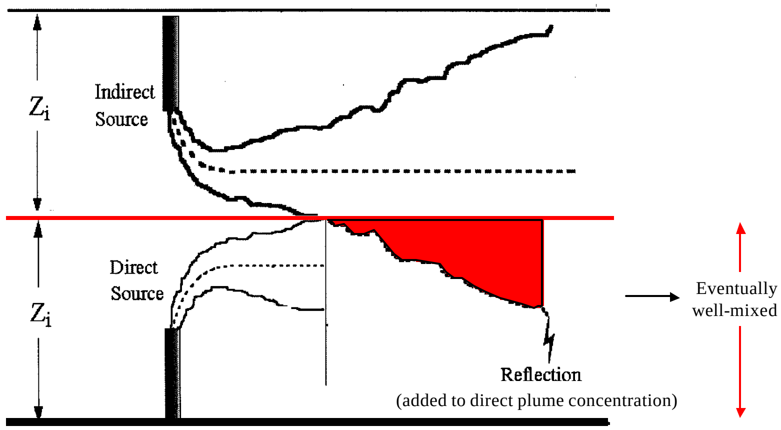
$$\sigma_y = 465.11628\,x(\tan\Theta) = 465.11628(0.5)[\tan(0.1542)] = 36.1\text{m}$$

$$C(x,\ y,z) = \frac{Q}{2\pi\ u\ \sigma_y \sigma_z}\left\{\exp\left(\frac{-(z-h)^2}{2\sigma_z^2}\right) + \exp\left(\frac{-(z+h)^2}{2\sigma_z^2}\right)\right\}\left\{\exp\left(\frac{-(y)^2}{2\sigma_y^2}\right)\right\}$$

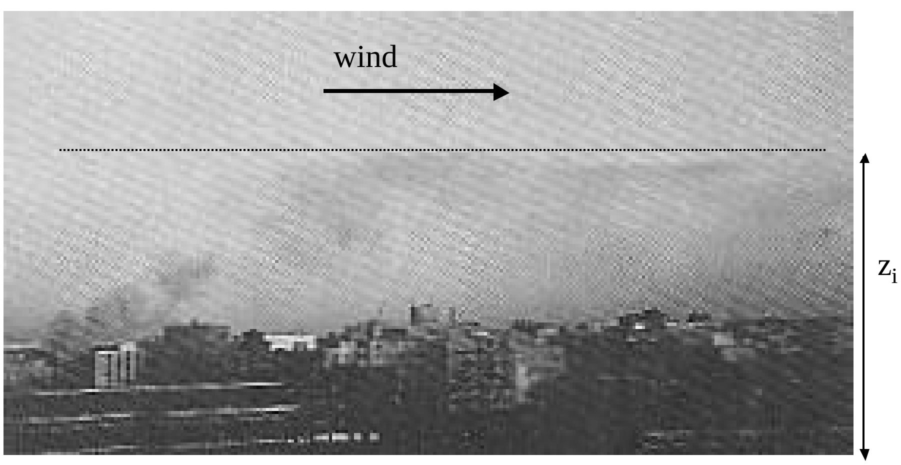
$$C(500,0,0) = \frac{10}{2\pi\ (6)(36.1)(18.3)}\left\{\exp\left(\frac{-(0-50)^2}{2(18.3)^2}\right) + \exp\left(\frac{-(0+50)^2}{2(18.3)^2}\right)\right\}\left\{\exp\left(\frac{-(0)^2}{2(36.1)^2}\right)\right\}$$

$$C(500,0,0) = \frac{10}{2\pi\ (6)(36.1)(18.3)}\{0.0479\}\{1\} = 1.92\text{x}10^{-5}\,\text{g/m}^3 = 19.2\,\mu\text{g/m}^3$$

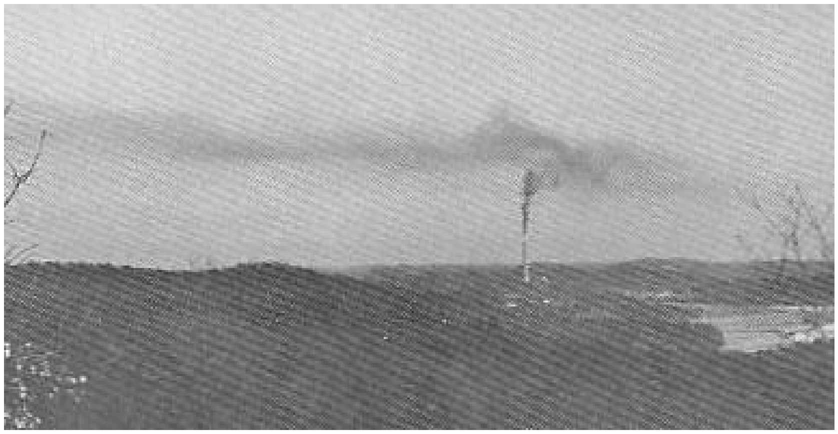
Virtual Source Also Used to Model Reflection Off of the Top of the Mixed Layer



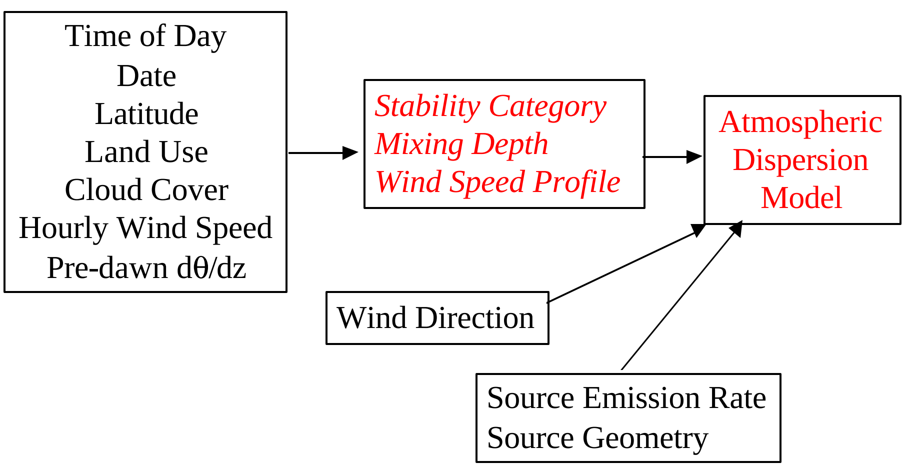
Plume Reflection off of the top of the mixed layer



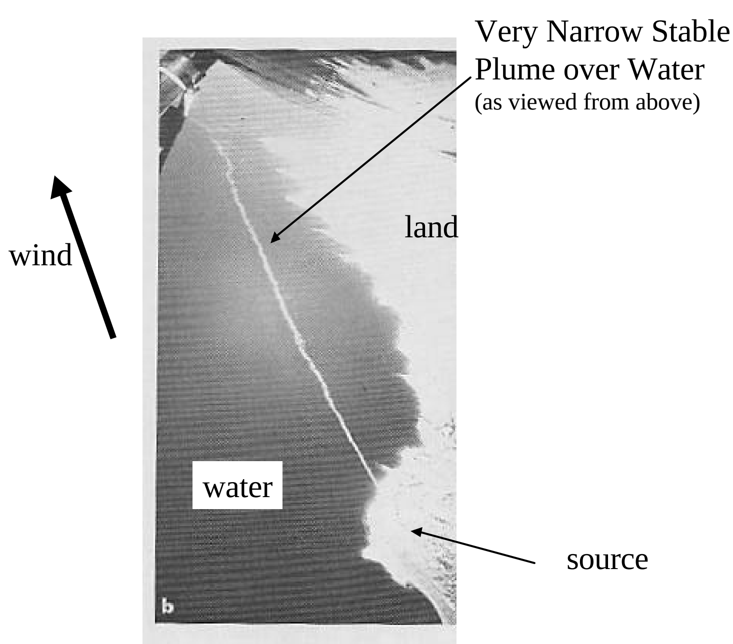
Plume “Trapped” in Stable Layer Above Mixed Layer



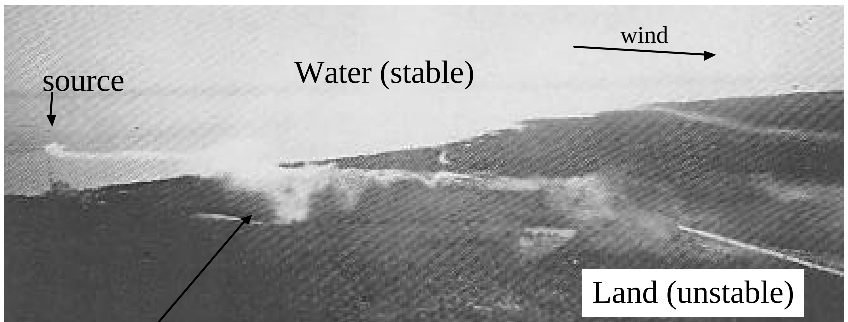
Minimum Necessary Information Needed to Implement a Simple Atmospheric Dispersion Model



Non-Gaussian Plumes



Plume Fumigation During On-shore Flow

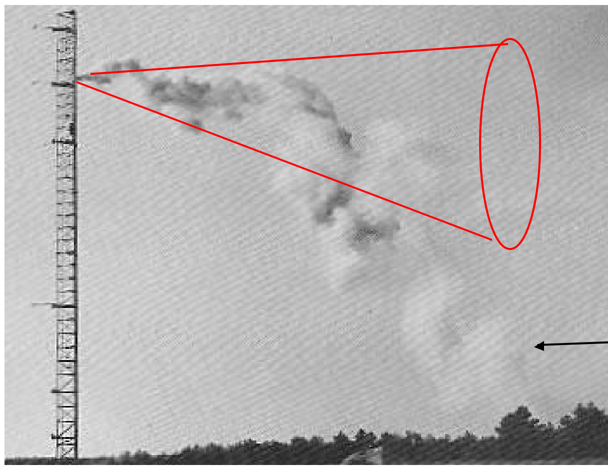


‘Fumigation’

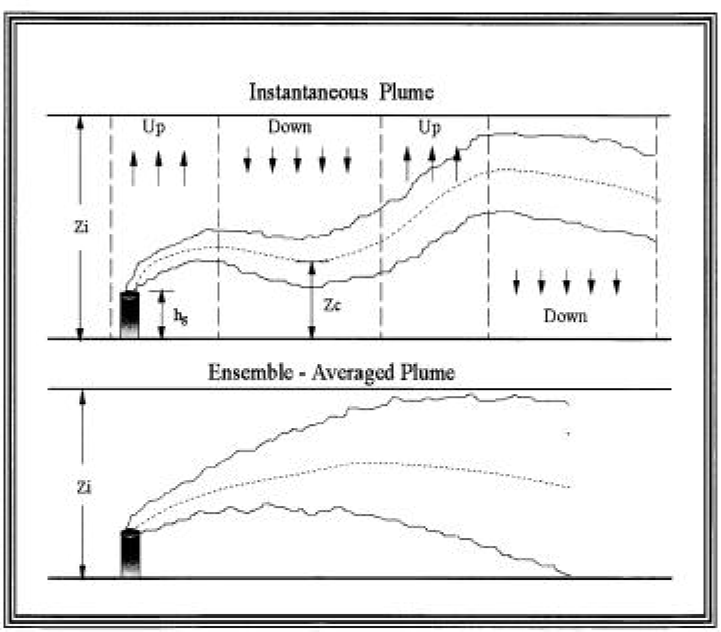
Plume “Trapped” in Building Wake

Plume “Looping” During Unstable Conditions

(large-scale vertical motions)



Extreme Departure From Gaussian

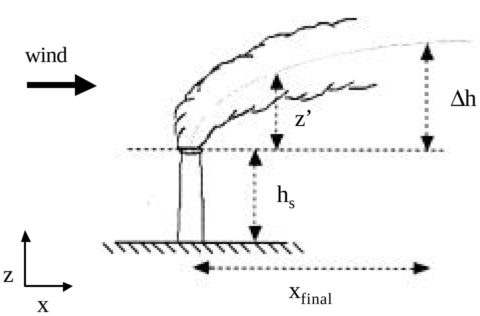


Instantaneous and corresponding ensemble-averaged plume in the CBL

Plume Rise in Neutral or Stable Air

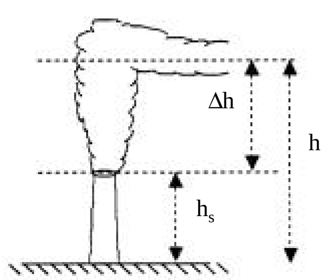
Many tall industrial stacks release hot, effluents into the air. Hot air rises and cools. There are two well established results from the science of plume rise: the immediate downwind trajectories of "bent over" plumes and the maximum attained height of "vertical" plumes.

‘Bent over’ plume trajectory, z’
(z’ is a function of x)



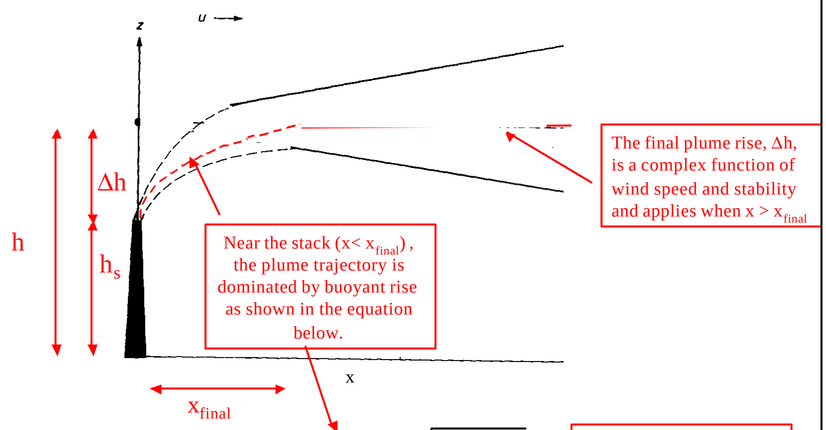
Higher wind speeds in stable/neutral air

Vertical Plume Rise, Δh



Low wind speeds in stable air

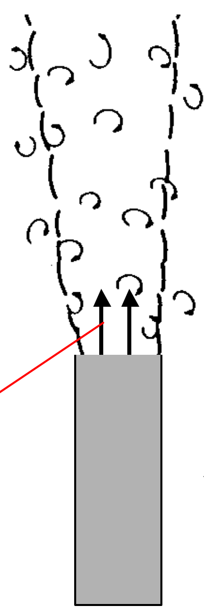
‘Bent over’ plume trajectory under stable and neutral conditions



$$z' = \frac{1.6\ \sqrt[3]{F_0\ x^2}}{u}$$

F_0 is the initial buoyancy flux at the stack exit [$m^4\ s^{-3}$]

Buoyancy Flux, F_0



There is also some rise due to momentum (stack gas velocity), but this is usually small compared with the effect of buoyancy.

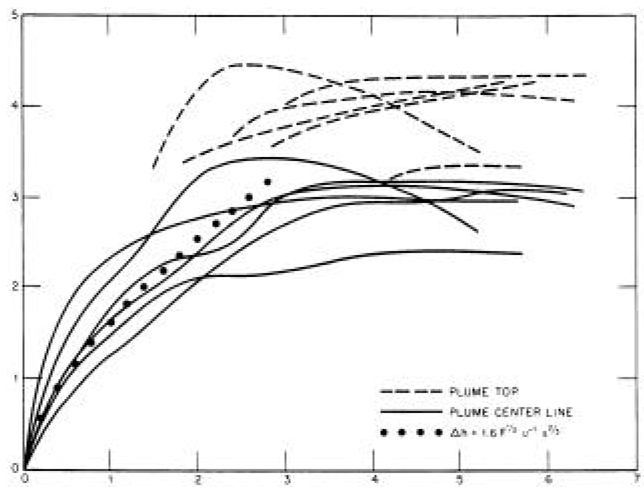
The larger the stack radius, the more time it takes for the gas to cool by mixing with surrounding, cooler air

$$F_0 = \frac{g}{T}(T_0 - T)\, w_0\, (R_0)^2$$

T_0 = Stack gas exit temperature (K)
w_0 = Stack gas exit velocity (m/s)
R_0 = Stack radius at exit (m)

Observed Plume Trajectories

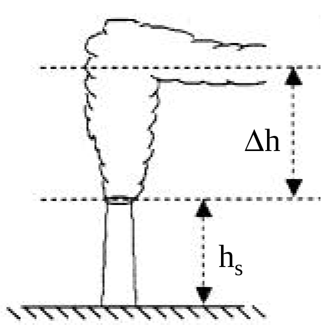
Dimensionless plume rise parameter (Δh $[u\ s]^{0.333}$ $F_0^{-0.333}$)



Dimensionless downwind parameter ($x\ u^{-1}\ s^{0.5}$)

Source: Simon and Proudfit (1967); data from Ravenswood power plant, New York

Vertical Plume Rise Under Stable Conditions

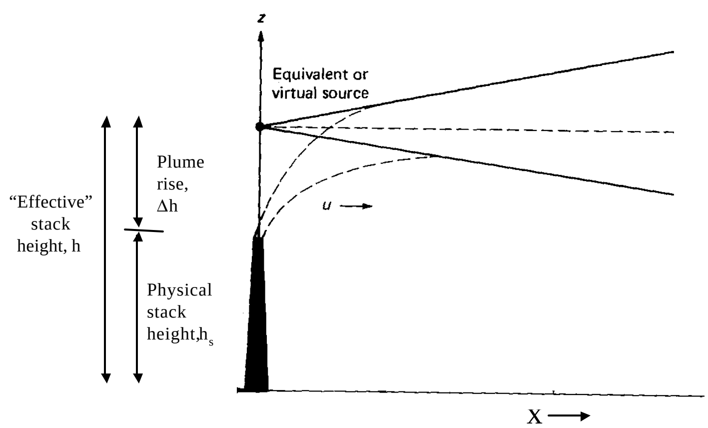


$$\Delta h = \frac{4(F_0)^{0.25}}{s^{0.375}}$$

Where s is the stability parameter, a continuous descriptor of the strength of the atmospheric restoring force under stable conditions

$$s = \frac{g}{T}\left(0.0098 - \frac{d\Theta}{dz}\right)$$

A virtual” source can be used to approximate the effect of plume rise on downwind concentrations



Buoyancy Induced Dispersion

Near the source, the rising plume entrains surrounding air as it rises. This dilution of the plume is not accounted for in the classic plume dispersion equations. To account for this extra dilution, an additional term is added to the “plume sigmas” (plume spreading parameters) that is a function of plume trajectory as follows:

$$\left(s_z^2\right)_{effective} = \left(\frac{z'}{3.5}\right)^2 + s_z^2$$

$$\left(s_y^2\right)_{effective} = \left(\frac{z'}{3.5}\right)^2 + s_y^2$$

Stack-Induced Plume Downwash

Wind flowing past a stack can create a region of lower pressure immediately downwind of the stack. If the vertical momentum of the stack gas is not sufficient, the plume will be drawn downward on the downwind side of the stack, lowering the effective stack height, h.

For $w_0 < 1.5$ u

$$h_s' = h_s + 2D_0\left(\frac{w_0}{u} - 1.5\right)$$

For $w_0 >= 1.5$ u $\quad h_s' = h_s$

Where h_s' is the adjusted physical stack height (not including plume rise)