Cooper Alley 3rd Edition, Prob 3.3 Page 120

Given

A cascade impactor was used to sample the gases from a stack and the resulting data is shown in the table below. The particle mass is obtained by weighing the cascade impactor particle collection impaction plates (located downstream of each impactor jet stage) before and after sampling the stack gases. The % Mass and Cumulative Wt % values were calculated with MathCad.

Jet Stage	Jet Stage da_{50} (micrometers)	Particle Mass (mg)	% Mass in Increment	Cumulative % Wt Less than da_{50}
1	50	5	1.389	98.611
2	30	30	8.333	90.278
3	16	75	20.833	69.444
4	8	100	27.778	41.667
5	4	125	34.722	6.944
Filter		25	6.944	
		360		

$i := 0..5$

Find

$\mu m := 10^{-6} \cdot m$

a) The % particle mass in each particle diameter increment, the cumulative particle mass less than the impactor jet stage cut diameter da_{50}, and plot the particle size distribution on log-probability graph paper.

b) The mass median diameter of the particle size distribution and the geometric standard deviation σ_g

$$Wt := \begin{pmatrix} 5 \\ 30 \\ 75 \\ 100 \\ 125 \\ 25 \end{pmatrix} \cdot mg \qquad da50 := \begin{pmatrix} 50 \\ 30 \\ 16 \\ 8 \\ 4 \\ 0.05 \end{pmatrix}$$

$$Wt\%_i := \frac{100 \cdot Wt_i}{360 \cdot mg} \qquad TotalWt := \sum_i Wt_i \qquad TotalWt = 360\, mg$$

$Cum1 := 100 - Wt\%_0 \quad Cum1 = 98.611$

$Cum2 := Cum1 - Wt\%_1 \quad Cum2 = 90.278$

$Cum3 := Cum2 - Wt\%_2 \quad Cum3 = 69.444$

$Cum4 := Cum3 - Wt\%_3 \quad Cum4 = 41.667$

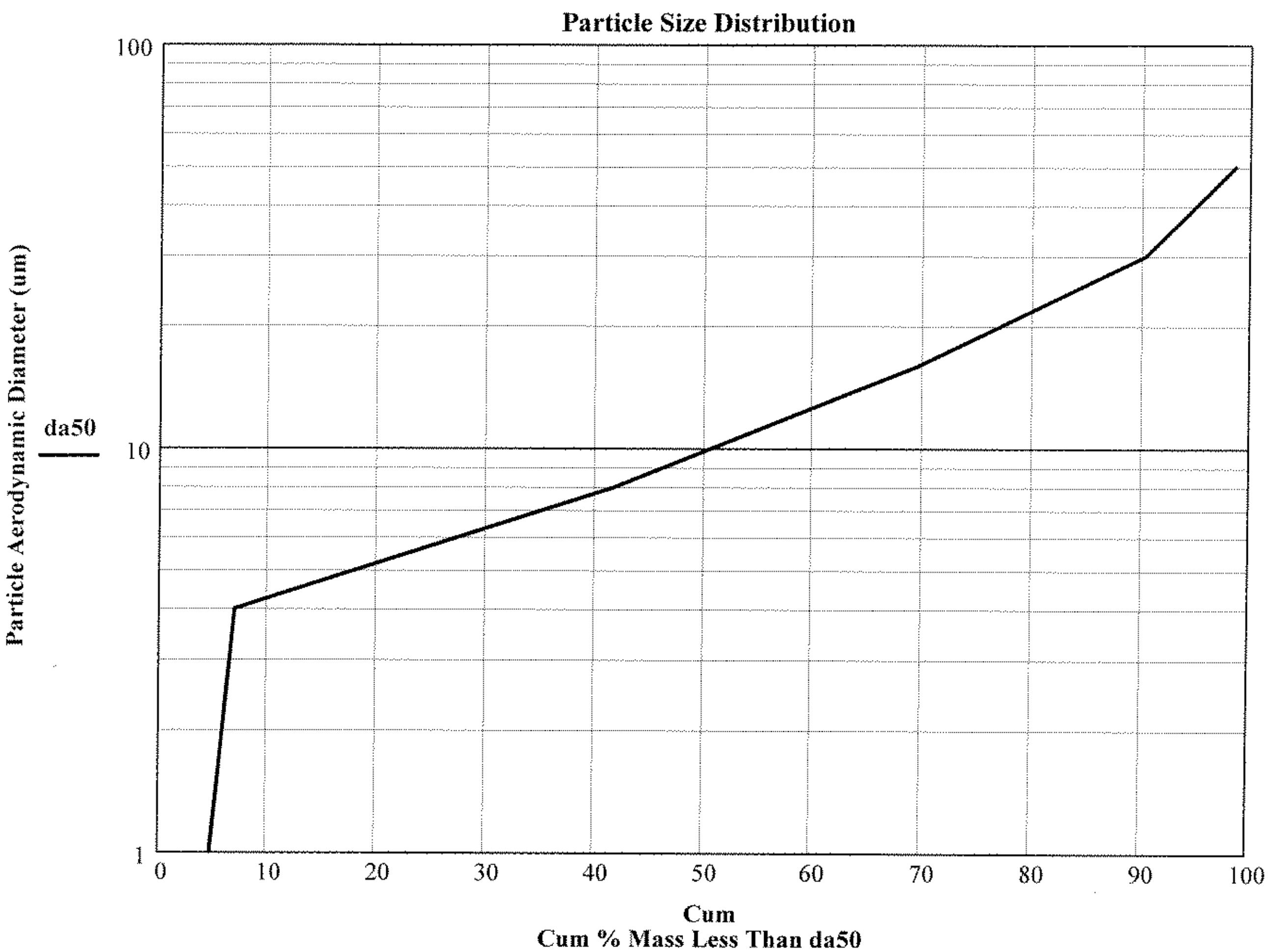
$Cum5 := Cum4 - Wt\%_4 \quad Cum5 = 6.944$

$Cum6 := Cum5 - Wt\%_5 \quad Cum6 = 1.332 \times 10^{-14}$

$$Cum := \begin{pmatrix} 98.611 \\ 90.278 \\ 69.444 \\ 41.667 \\ 6.944 \\ 0.01 \end{pmatrix} \qquad Wt\% = \begin{pmatrix} 1.389 \\ 8.333 \\ 20.833 \\ 27.778 \\ 34.722 \\ 6.944 \end{pmatrix}$$

$$TotalWt\% := \sum_i Wt\%_i$$

$TotalWt\% = 100$



B. Impactor Particle Size Distribution

From graph mass median diameter = about 10 microns $MMD := 10\cdot\mu m$

From graph d_{84} = about 25.5 microns $d_{84} := 25.5\cdot\mu m$

From graph $d_{15.9}$ = about 5.5 microns $d_{15.9} := 5.5\cdot\mu m$

Equation 3.5 page 104 assumes the particle size distribution is log normal (straight line on log-probability graph paper)

$$\sigma g := \left(\frac{d_{84}}{d_{15.9}}\right)^{0.5} \qquad \sigma glarge := \left(\frac{d_{84}}{MMD}\right) \qquad \sigma gsmall := \frac{MMD}{d_{15.9}}$$

$$\sigma g := \left(\frac{25.5\cdot\mu m}{5.5\cdot\mu m}\right)^{0.5} \qquad \sigma gsmall := \frac{10\cdot\mu m}{5.5\cdot\mu m}$$

$$\sigma g = 2.153 \qquad \sigma glarge = 2.55 \qquad \sigma gsmall = 1.818$$

These particle size geometric standard deviation σ_g are not all the same magnitude, probably because of difficulty obtaining the d_{84} and $d_{15.9}$ from the graph and also the distribution might not be log normal.

Where can you get log-probability graph paper to plot this particle size distribution?

Calculation of Overall Particle Collection of Electrostatic Precipitator

Problem 3.5 Cooper & Alley 3rd Ed pp 120

$i := 0..6 \qquad \mu m := 10^{-6} \cdot m$

Given:

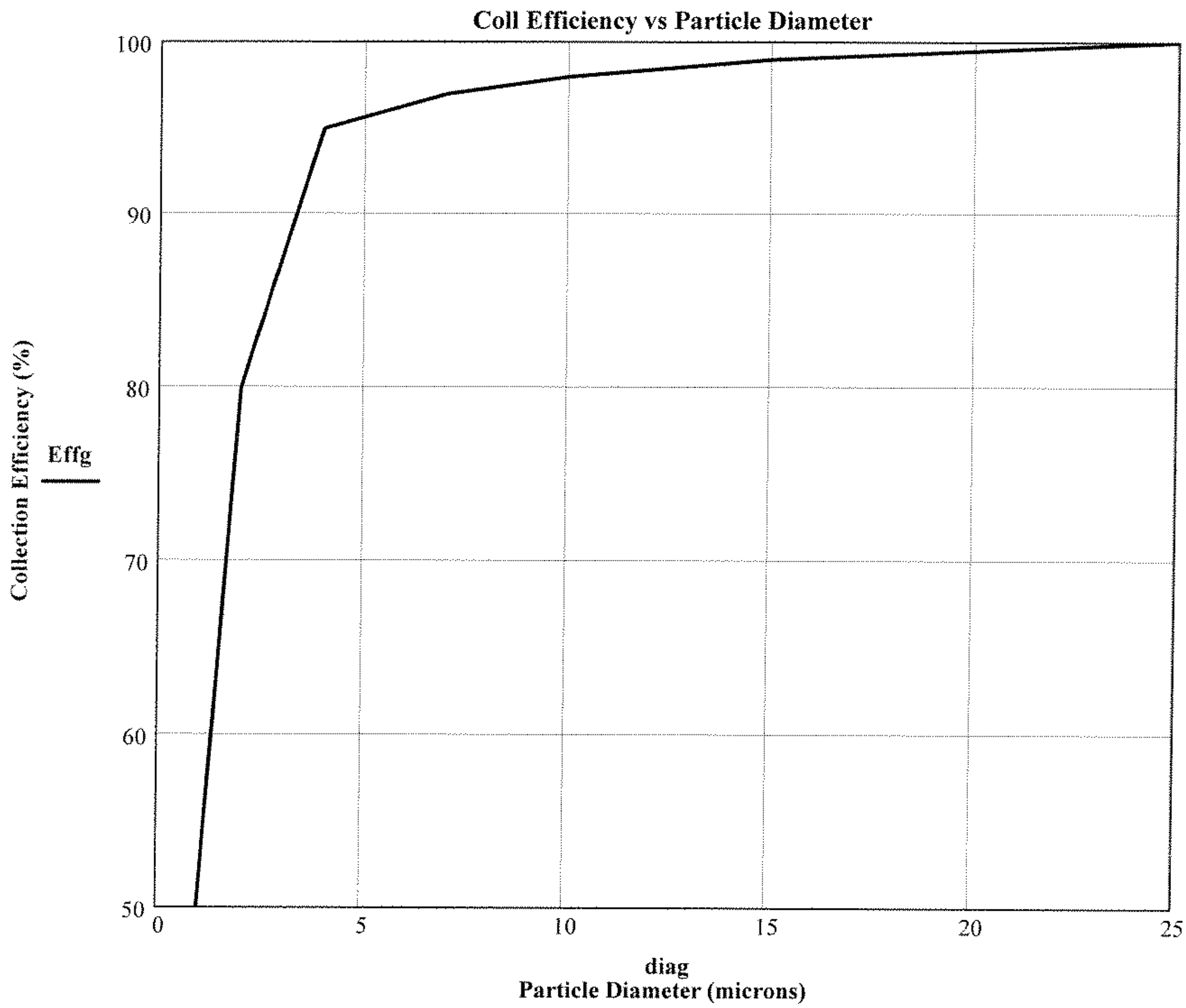
a. Particle size distribution at inlet to electrostatic precipitator (from Cooper 3.3 and this was plotted to get the particle mass fraction in each particle size range corresponding to the 7 particle diameters)

b. Particle collection efficiency as function of particle diameter for the 7 particle diameters (25, 15, 10, 7, 4, 2, and 1) Shown as Eff in matrix at right.

$$dia := \begin{pmatrix} 25 \\ 15 \\ 10 \\ 7 \\ 4 \\ 2 \\ 1 \end{pmatrix} \cdot \mu m \qquad Eff := \begin{pmatrix} 1.00 \\ .99 \\ .98 \\ .97 \\ .95 \\ .80 \\ .50 \end{pmatrix}$$

Shown below is a graph of the electrostatic precipitator particle collection efficiency % as a function of particle diameter in microns.

$diag := 10^{6} \cdot dia \qquad Effg := 100 \cdot Eff$



Up = particle mass fraction at upper particle diameter in size range at ESP inlet

Down = particle mass fraction at lower particle diameter in size range at ESP inlet

$$\mathbf{Up} := \begin{pmatrix} 1.00 \\ .98611 \\ 0.830 \\ 0.480 \\ 0.370 \\ 0.06944 \\ 0.030 \end{pmatrix} \qquad \mathbf{Down} := \begin{pmatrix} 0.98611 \\ 0.830 \\ 0.480 \\ 0.370 \\ 0.06944 \\ 0.030 \\ 0 \end{pmatrix}$$

$$\mathbf{MassFraction_i := Up_i - Down_i}$$

$$\mathbf{MassFraction} = \begin{pmatrix} 0.01389 \\ 0.15611 \\ 0.35 \\ 0.11 \\ 0.30056 \\ 0.03944 \\ 0.03 \end{pmatrix}$$

MassFraction is the particle mass fraction in the particle diameter size range

$$\mathbf{Eff} := \begin{pmatrix} 1.00 \\ .99 \\ .98 \\ .97 \\ .95 \\ .80 \\ .50 \end{pmatrix}$$

MassFractionCollected is the particle mass collected by Electrostatic precipitator for that particle size diameter

$$\mathbf{MassFractionCollected_i := MassFraction_i \cdot Eff_i}$$

$$\mathbf{MassSum} := \sum_i \mathbf{MassFraction_i}$$

$$\mathbf{MassSum} = 1$$

$$\mathbf{MassFractionCollected} = \begin{pmatrix} 0.01389 \\ 0.154549 \\ 0.343 \\ 0.1067 \\ 0.285532 \\ 0.031552 \\ 0.015 \end{pmatrix}$$

$$\mathbf{FractCollSum} := \sum_i \mathbf{MassFractionCollected_i}$$

$$\mathbf{FractCollSum} = 0.9502$$

ESP Overall Particle Collection Efficiency = 95.02%

$$\mathbf{MassFractionEmitted_i := MassFraction_i - MassFractionCollected_i}$$

$$\mathbf{FractEmittedSum} := \sum_i \mathbf{MassFractionEmitted_i}$$

$$\mathbf{MassFractionEmitted} = \begin{pmatrix} 0 \\ 0.0015611 \\ 0.007 \\ 0.0033 \\ 0.015028 \\ 0.007888 \\ 0.015 \end{pmatrix}$$

$$\mathbf{FractEmittedSum} = 0.0498$$

Overall Penetration of Particles Through the ESP = 4.98%

$$\mathbf{EmittedWt\%_i} := 100 \cdot \left(\frac{\mathbf{MassFractionEmitted_i}}{\mathbf{FractEmittedSum}} \right)$$

These equations are to calculate the size distribution of particles downstream of the ESP (mass conc and size distribution can be used to calculate the plume opacity)

$$\mathbf{SumEmittedWt\%} := \sum_i \mathbf{EmittedWt\%_i}$$

$$\mathbf{SumEmittedWt\%} = 100$$

$$\mathbf{EmittedWt\%} = \begin{pmatrix} 0 \\ 3.13618 \\ 14.06269 \\ 6.62955 \\ 30.19059 \\ 15.84664 \\ 30.13434 \end{pmatrix} \qquad \mathbf{dia} = \begin{pmatrix} 25 \\ 15 \\ 10 \\ 7 \\ 4 \\ 2 \\ 1 \end{pmatrix} \mu\mathrm{m}$$

$\mathbf{EmittedCum\%1} := 100 - \mathbf{EmittedWt\%}_1$

These equations are to calculate the cumulative wt% of particles less than the stated particle diameter, shown as EmitCum below

$\mathbf{EmittedCum\%1} = 96.864$

$\mathbf{EmittedCum\%2} := \mathbf{EmittedCum\%1} - \mathbf{EmittedWt\%}_2$

$\mathbf{EmittedCum\%3} := \mathbf{EmittedCum\%2} - \mathbf{EmittedWt\%}_3$

$\mathbf{EmittedCum\%4} := \mathbf{EmittedCum\%3} - \mathbf{EmittedWt\%}_4$

$\mathbf{EmittedCum\%5} := \mathbf{EmittedCum\%4} - \mathbf{EmittedWt\%}_5$

$\mathbf{EmittedCum\%6} := \mathbf{EmittedCum\%5} - \mathbf{EmittedWt\%}_6$

$\mathbf{EmittedCum\%2} = 82.801$

$\mathbf{EmittedCum\%3} = 76.172$

$\mathbf{EmittedCum\%4} = 45.981$

$\mathbf{EmittedCum\%5} = 30.134$

$$\mathbf{EmitCum} := \begin{pmatrix} 100 \\ 96.864 \\ 82.801 \\ 76.172 \\ 45.981 \\ 30.134 \\ 0 \end{pmatrix}$$

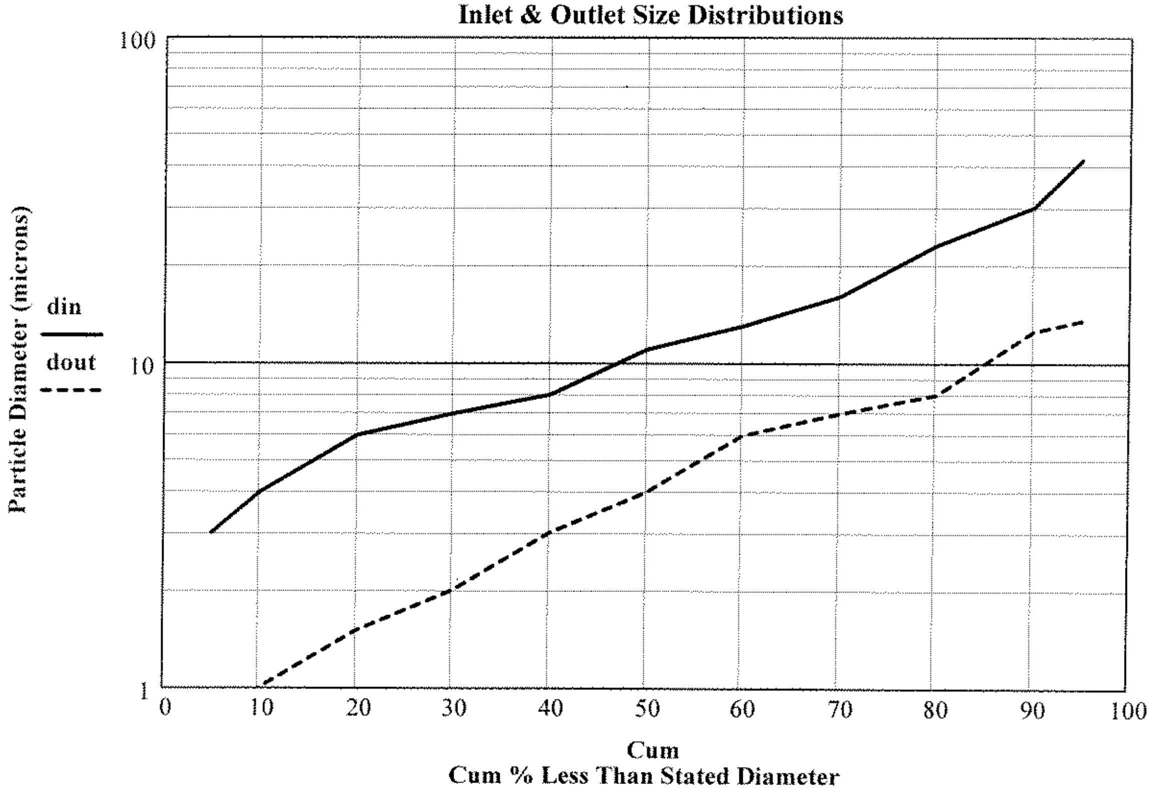
$$\mathbf{Cum} := \begin{pmatrix} 95 \\ 90 \\ 80 \\ 70 \\ 60 \\ 50 \\ 40 \\ 30 \\ 20 \\ 10 \\ 5 \end{pmatrix}$$

$j := 0 .. 10$

Note that the particle diameter units of μm were not used on the din and dout matrices on the right because the vertical axis on the graph would then have units and this would then require some adjustments to the vertical axis of the use of a dummy variable for the diameters.

$$\mathbf{din} := \begin{pmatrix} 42 \\ 30 \\ 23 \\ 16 \\ 13 \\ 11 \\ 8 \\ 7 \\ 6 \\ 4 \\ 3 \end{pmatrix} \qquad \mathbf{dout} := \begin{pmatrix} 13.5 \\ 12.5 \\ 8 \\ 7 \\ 6 \\ 4 \\ 3 \\ 2 \\ 1.5 \\ 1.0 \\ 0.5 \end{pmatrix}$$

Inlet & Outlet Size Distributions



Cooper & Alley 3rd Ed, Problem 3.10 page 120

Given
Particle of 0.03 micron diameter in air
at 150°C or 302°F and 1 atm pressure
Find:
Cunningham Slip Correction Factor

$T := (460 + 302) \cdot R$

$T = 423.333\ K$

$T = 762\ R$

$\mu m := \frac{m}{10^6}$

The equations for the Cunningham Slip Correction factor are given on pages 110 and 111 Cooper & Alley

T=Temp. Deg. K

$M := 29 \cdot \frac{gm}{mole}$

Values for air viscosity μg and density ρg from page 680 Cooper

$\mu g := 0.058 \cdot \frac{lb}{hr \cdot ft}$ $\rho g := 0.0521 \cdot \frac{lb}{ft^3}$

$\mu g = 2.398 \times 10^{-4} \frac{gm}{cm \cdot sec}$ $\rho g = 8.346 \times 10^{-4} \frac{gm}{cm^3}$

Gas Kinematic Viscosity = μg / ρg = νg

$\nu g := \frac{\mu g}{\rho g}$ $\nu g = 0.2873 \frac{cm^2}{sec}$

Universal Gas Constant = RG

$RG := 0.082054 \cdot \frac{liter \cdot atm}{mole \cdot K}$

Mean velocity of air molecule of MW = 29 = U in cm/sec

$U := \left(\frac{8 \cdot RG}{\pi \cdot M} \cdot T \right)^{\frac{1}{2}}$ $U = 5.559 \times 10^4 \frac{cm}{sec}$

Gas (air) molecule mean free path = λ in cm

$\lambda := \frac{\nu g}{(0.499) \cdot (U)}$ $\lambda = 0.1036\ \mu m$

dia=particle diameter

$dia := 0.03 \cdot \mu m$

Cunningham Equation
Eq. 3.12 page 110 Cooper

$$C := 1 + 2\left(\frac{\lambda}{dia}\right) \cdot \left[1.257 + 0.40 \cdot e^{-(.55) \cdot \frac{dia}{\lambda}}\right]$$

Cunningham Slip Correction Factor = C

$C = 12.033$

Knudsen Number = Kn = λ/particle diameter

$Kn := \frac{\lambda}{dia}$

Continuum fluid regime for Kn less than 0.1
Transition 0.1 to 10
Free Molecular Greater 10

$Kn = 3.452$

So for particles of 0.03 micron diameter (30 nanometers diameter) in air, the Cunningham Slip correction factor is 12.033 and the Knudsen number is 3.452 at 150°C or 302°F and 1 atm pressure.

This implies that these small particles ("nanoaerosol particles") behave like large gas molecules.