A Theoretically Based Mathematical Model for Calculation of Electrostatic Precipitator Performance

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A mathematical model is described which relates collection efficiency to precipitator size and operating parameters. Procedures are given for calculating particle charging rates, electric field as a function of position in wire-plate geometry, and the theoretically expected collection efficiencies for various particle sizes and precipitator operating conditions. Methods are proposed for empirically representing the losses in collection efficiency caused by non-uniform gas velocity distributions, gas by-passage of the electrified regions, and particle reentrainment due to rapping of the collection electrodes. Incorporation of these proposed techniques into a mathematical model of precipitator performance results in reduction of the theoretically calculated overall collection efficiencies. The reduced efficiencies are compared with those obtained from measurements on precipitators treating flue gas from coal-fired generating stations. The effects of changes in particle size distributions on calculated collection efficiencies obtained from the mathematical model are also presented.

This paper summarizes work conducted at Southern Research Institute concerning the development of a mathematical model for simulating electrostatic precipitator performance. The approach which has been taken is to define the collection efficiency under ideal conditions in terms of precipitator geometry and operating parameters for dusts of various sizes and properties. Empirical corrections to the theoretical performance are then made to account for non-uniformity of gas flow and reentrainment of dust laden gas through non-electrified regions above and below the collection electrodes.

The incentive for using a theoretical approach lies in the ability to account for the variations in both dust properties and precipitator operating parameters in a logical and orderly fashion. Methods which are exclusively empirical can lead to serious miscalculation in collection electrode area requirements for a specific installation. A theoretical approach offers the potential for increased confidence in design and in cost savings by preventing under sizing on the one hand and over sizing on the other. The accuracy of predictions obtained from such an approach are subject to the accuracy with which the properties comprising the independent variables are measured, the degree to which the theoretical relationships describe precipitator operation, and the precision with which the factors that correct for non-ideal conditions can be modelled and determined. At present, it is necessary to use assumed values for parameters describing non-ideal conditions. Comparisons between predicted and measured performance using the relationships described in this paper and the limited amount of applicable test data indicate that the model in its present stage of development is useful in developing performance curves that relate efficiency to specific collecting area under various conditions.

Theoretical Background

The fundamental steps in the precipitation process are particle charging, particle collection, and the removal and disposal of the collected material. Particle charging is accomplished by a source of charge carriers in the presence of an electric field which drives the charge carriers to the particulate. Collection of the charged particulate occurs as the electric field drives the particles to a collecting electrode, where they are held by mechanical and electrical forces. Removal of the collected material is accomplished by the application of a force to the collecting electrode in such a manner that the collected ash is dislodged, and falls into a receiving hopper for subsequent transport to a disposal system.

In order to calculate the theoretically expected performance of an electrostatic precipitator, it is necessary to calculate particle charge as a function of particle size, residence time, and precipitator operating conditions. Field charging theory adequately describes experimentally observed particle charge values for particles larger than 2.0 μm with moderate to high values of applied electric field. Calculation of particle charge for diameters exceeding 2.0 μm is therefore relatively straightforward if the precipitator operating conditions are adequately defined. For particle diameters less than 2.0 μm, the calculation of particle charge is complex, and has been the subject of considerable effort by a number of investigators. A more detailed discussion of particle charging theories is given in a paper by Smith and McDonald.[1]

The expression currently used in the model for particle charging calculations is given below in differential form:

$$\frac{dq}{dt} = \frac{N_0 e b q_s}{4\epsilon_0}\left(1 - \frac{q}{q_s}\right)^2 + \pi r^2 e \bar{v} N_0 \exp\left(-\frac{qe}{4\pi\epsilon_0 r k T}\right) \quad (1)$$

where q_s is a modified saturation charge given by

$$q_s = 4\pi\epsilon_0 (r + \lambda)^2 E_0 \left(1 + 2\frac{K-1}{K+2}\frac{(r)^3}{(r+\lambda)^3}\right) \quad (2)$$

in which

q = charge, coul
N_0 = free ion density, no/m^3
e = electronic charge, coul
ϵ_0 = permittivity of free space, coul^2/(N-m^2)
E_0 = electric field, volt/m
b = ion mobility, m^2/(volt-sec)
$\bar{v}$ = mean thermal speed of ions, m/sec
r = particle radius, m
k = Boltzmann's constant, J/°K
T = temperature, °K
t = time, sec
K = dielectric constant
λ = an adjustable parameter

This expression is similar to one developed by Cochet.[2] Although the above expression represents the charging of submicron particles with sufficient accuracy to be useful in calculating precipitator performance, there are indications that the relationships developed by Smith and McDonald describe the charging process more accurately. Therefore, it may be desirable to incorporate their calculation procedure into the model at a later date.

The value of electric field used for the particle charging calculations is simply the average value between the discharge and collecting electrodes. In order to calculate the velocity of charged particles near the collecting electrode, however, it is necessary to compute the local electric field values in this region of space. A review of previous work on this problem indicated that the most promising method of calculation was a numerical technique introduced by Leutert and Böhlen.[3] The equations which must be solved are given in two dimensions, written in discrete form:

$$\frac{\Delta^2 V}{\Delta X^2} + \frac{\Delta^2 V}{\Delta Y^2} = -\frac{\rho}{\epsilon_0} \qquad (3)$$

and

$$\rho^2 = \epsilon_0\left(\frac{\Delta V}{\Delta X}\frac{\Delta \rho}{\Delta X} + \frac{\Delta V}{\Delta Y}\frac{\Delta \rho}{\Delta Y}\right) \qquad (4)$$

where

V = potential, volts
ρ = space charge, coul/m^3
X = distance perpendicular to gas flow from wire to plate
Y = distance parallel to gas flow from wire to wire

A computer program based on these equations was written and incorporated into the model as a subroutine. In order to check the accuracy of the calculation procedure, the computer program has been used to calculate potential profiles and electric fields based on the geometry and operating conditions given for electric field measurements reported in the literature.

Figure 1 shows calculations based on the geometry and operating conditions, reported by Penney and Matick,[4] and their experimental results. Fairly good agreement is found for the potential profiles from the wire to the plate and from a point midway between wires to the plate. Also the field near the plate (the slope of the potential curve) is in excellent agreement. As a result of these and other comparisons between computed and measured results it was concluded that Leutert and Böhlen's technique provides a basis for computing electric fields in the region of interest adjacent to the collecting electrode.

If the particle charge and the electric field adjacent to the collecting electrode have been calculated, the next step in calculating theoretical collection efficiency is the calculation of electrical drift velocity, or migration velocity resulting from the coulomb and viscous drag forces acting upon a suspended particle. For particle sizes in the size range of interest, the acceleration time is negligible and the migration velocity is given by

$$w = \frac{qE_pC}{6\pi r\mu} \qquad (5)$$

where

w = migration velocity of a particle of radius r, m/sec
E_p = electric field near collecting electrode, volt/m
C = Cunningham correction factor
μ = gas viscosity (kg/(m-sec))

Gas flow velocities in most cases of practical interest are between 0.60 and 1.8 m/sec, while theoretical migration velocities for particles smaller than 6.0 μm are usually less than 0.3 m/sec. The path of these smaller particles therefore tends to be dominated by the turbulent motion of the gas stream in the inter-electrode region. The classical equation for describing particle collection in electrostatic precipitators under turbulent flow conditions was derived by Deutsch and gives collection efficiency as a function of gas volume flow, collection area, and migration velocity:

$$\eta = 100\,[1 - \exp(-A_p w/Q)] \qquad (6)$$

where

η = collection efficiency of a particle of radius r, %
A_p = collecting area, m^2
Q = gas volume flow, m^3/sec

The assumptions on which the derivation of this equation is based are:

a) Gas turbulence provides sufficient mixing to establish a uniform particle concentration at any cross section of the precipitator.
b) The gas velocity through and across the precipitator is uniform except for a boundary layer near the wall.
c) The particle migration velocity near the collecting surface is constant for all particles and small compared with the average gas velocity. This implies that the equation is strictly applicable only to a monodisperse particulate with a diameter less than about 6 to 10 μm.

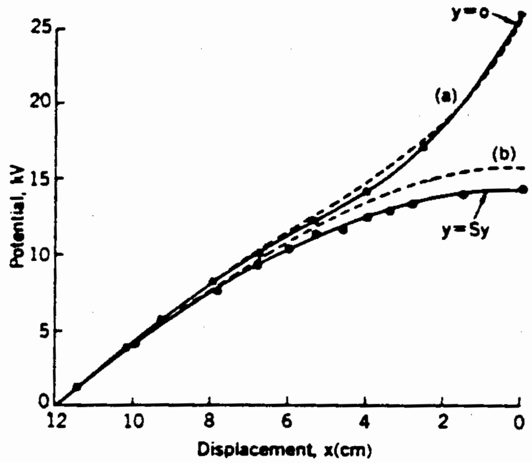

Figure 1. Potential profiles in a wire plate precipitator. (a) x = 0 corresponds to the wire, x = 12 cm the plate. Solid line, Penney and Matick (experimental). Dashed line, SRI (calculated). (b) x = 0 corresponds to a point midway between wires, x = 12 cm the plate. Solid line Penney and Matick (experimental). Dashed line, SRI (calculated).

d) There are no disturbing effects, such as reentrainment, back corona, etc.[5]

White[5] has reported a series of experiments using oil fumes under experimental conditions that were consistent with all of the above assumptions. The results indicate that the Deutsch equation adequately describes the collection mechanism for particulate in an electrostatic precipitator under idealized conditions. It has been common practice, however, to attempt to use the Deutsch equation to correlate field data obtained under conditions which violate most of the assumptions on which the original derivation was based. The correlation procedure usually involves solving the Deutsch equation as written below:

$$w_p = \frac{Q}{A}\ln\left(\frac{100}{100 - \eta_0}\right) \qquad (7)$$

where

w_p = precipitation rate parameter, m/sec
η_0 = overall mass collection efficiency, %

The quantity w_p can vary widely due to variations in the particle size distribution of the particulate and the operating conditions of the precipitator.

The approach which has been used in our work for calculating theoretical particle collection efficiencies consists of the following steps:

1. Migration velocities are calculated for representative particle sizes as a function of electrical conditions for length increments in the direction of gas flow through the precipitator.
2. The Deutsch equation is used to calculate the number of particles in each size band (the particle size distribution is represented by a histogram) which are collected in each incremental length.
3. The collection efficiency of each representative particle size is determined by summing over the length of the precipitator. The overall mass collection efficiency is obtained by summing the mass of particles collected in each size band of the histogram.

Corrections to Theoretical Predictions

In the preceding section, a basis for calculating theoretical collection efficiencies has been described. This section will discuss some of the non-idealities which exist in full-scale electrostatic precipitators and describe calculation procedures for estimating the effects of such conditions on predicted collection efficiencies. The factors which have been considered are: (1) gas velocity distribution, (2) gas sneakage, (3) rapping re-entrainment.

Effect of Gas Velocity Distribution

Although it is widely known that a poor velocity distribution gives a lower than anticipated efficiency, it is difficult to apply a numerical description for gas flow quality. This discussion will describe an approach to the calculation of degradation of performance based upon the velocity distribution, the theoretical or ideal efficiency, and the Deutsch equation.

The Deutsch equation can be rearranged to allow calculation of the corrected penetration of a given size particle as a function of the efficiency expected with a uniform velocity and the actual velocity distribution. This can be accomplished as follows:

1. Calculate a constant k from the efficiency predicted under ideal conditions:

$$k = u_a \ln \frac{1}{1 - \eta/100} \qquad (8)$$

2. Calculate the mean penetration:

$$p = \frac{1}{Nu_a} \sum_{i=1}^{N} u_i \,(1 - \eta_i/100) \qquad (9)$$

$$p = \frac{1}{Nu_a} \sum_{i=1}^{N} u_i e^{-\frac{k}{u_i}} \qquad (10)$$

where

u_a = average velocity, m/sec
p = corrected penetration fraction of a given size particle
N = number of points or channels with a given velocity
u_i = point values of velocity
η_i = point values of efficiency

For any practical velocity distribution and efficiency, the mean penetration obtained by summation over the velocity traverse will be higher than the calculated penetration based on an average velocity. If an apparent migration velocity for a given particle size is computed based upon the mean penetration and the Deutsch equation, the result will be a value lower than the value used for calculation of the single point values of penetration. The ratio of the original migration velocity to the reduced migration velocity is a numerical measure of the performance degradation caused by a non-uniform velocity distribution. An expression for this ratio may be obtained by setting the penetration based on the average velocity equal to the corrected penetration obtained from a summation of the point values of penetration, and solving for the required correction factor, which will be a divisor for the migration velocity.

The correction factor "F" may be obtained from:

$$\exp\left(-\frac{k}{F}\cdot\frac{1}{u_a}\right) = \frac{1}{Nu_a} \sum_{i=1}^{N} u_i \exp\,(-k/u_i) = p \qquad (11)$$

Therefore

$$F = -\frac{k}{u_a(\ln p)} \qquad (12)$$

Whether the quantity F correlates reasonably well with statistical measures of velocity non-uniformity is yet to be established. A limited number of traverse calculations seem to indicate a correlation between the factor F and the normalized standard deviation of the velocity traverse. Figure 2 shows F as a function of the ideal efficiency for several values of gas velocity standard deviation. These curves were obtained by computer evaluation of equation 12, and the data on which the calculations are based were obtained from Preszler and Lajos.[7] The standard deviations have been normalized to represent a fraction of the mean. The overlapping of the curves for standard deviations of 1.01 and 0.98 indicates that the standard deviation above does not completely determine the relationship between F and collection efficiency.

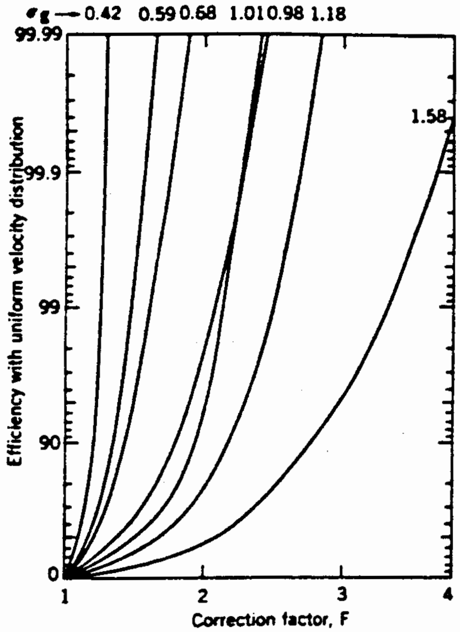

Figure 2. "F" as a function of ideal efficiency and gas flow standard deviation.

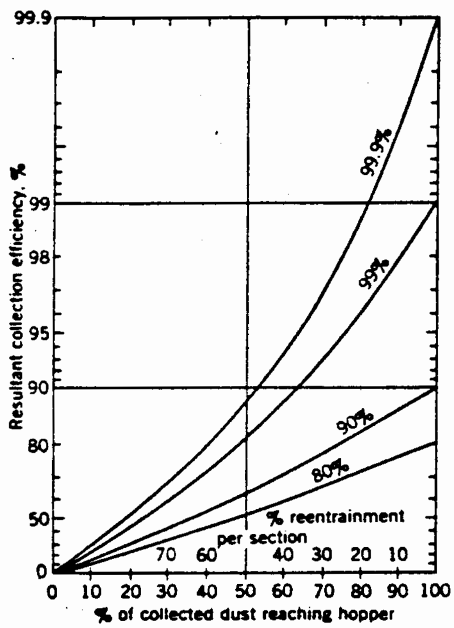

Figure 3. Effect of reentrainment on the efficiency of a four-section precipitator designed for a no reentrainment efficiency as indicated for a monodisperse particulate.

Gas Sneakage and Dust Re-entrainment

Gas sneakage occurs when gas bypasses the electrified areas of an electrostatic precipitator by flowing through the hoppers or through the high voltage insulation space. Sneakage can be reduced by frequent baffles which force the gas to return to the main gas passages between the collection plates. If there were no baffles, the percent sneakage would establish the minimum possible penetration because it would be the percent volume having zero collection efficiency. With baffles, the sneakage remixes with part of the main flow and then re-by-passes in the next unbaffled area. The limiting penetration due to sneakage will therefore depend on the amount of sneakage gas per section, the degree of re-mixing, and the number of sections.

If we make the simplifying assumption that perfect mixing occurs following each baffled section, a simple expression may be derived which relates penetration to the fractional amount of sneakage per section, the ideal efficiency, and the number of stages over which the by-passage is assumed to occur.

The expression is:

$$p_s = [S + (1 - S)\,(1 - \eta/100)^{1/N_s}]^{N_s} \qquad (13)$$

where

p_s = penetration corrected for sneakage
S = fractional amount of gas sneakage per section
N_s = number of baffled sections
η = collection efficiency of a given particle size obtained with no sneakage.

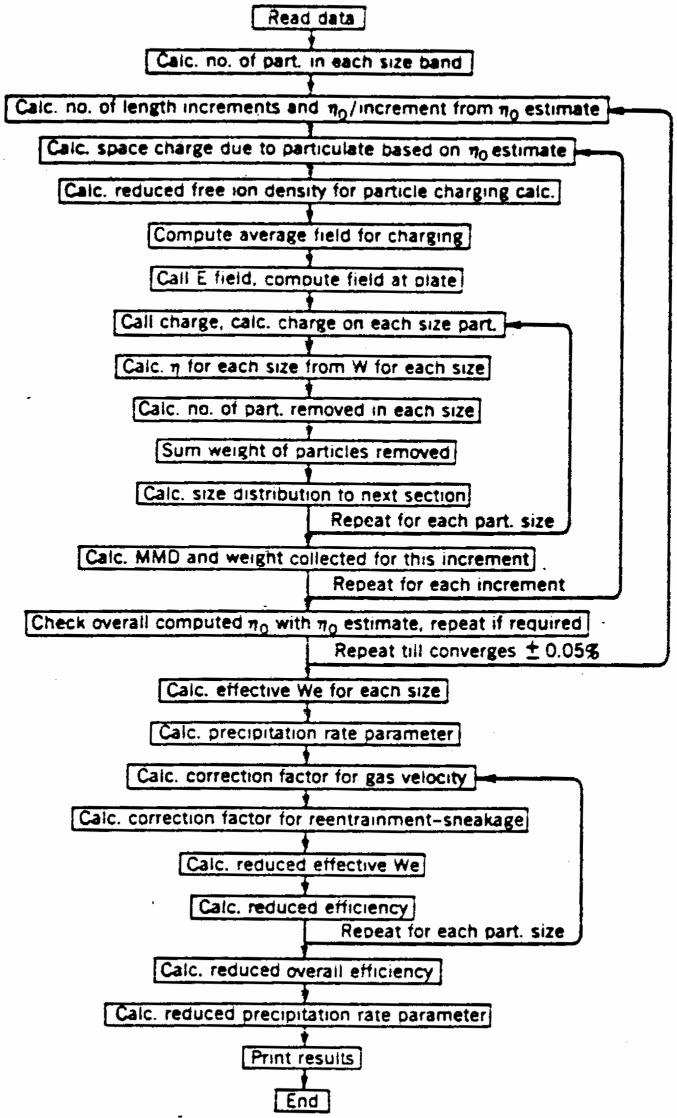

Figure 4. Simplified flow diagram of precipitator model computer program.

Rapping reentrainment is concerned with the amount of material that is recaptured by the gas stream after being dislodged from the collection plates by rapping. If we make the simplifying assumptions that a fixed fraction of the collected material of a given particle size is reentrained, and that the fraction does not vary with length through the precipitator, an expression can be derived identical in form to that obtained for gas sneakage:

$$p_R = [R + (1 - R)(1 - \eta/100)^{1/N_R}]^{N_R} \qquad (14)$$

where

p_R = penetration corrected for reentrainment
R = fraction of material reentrained per section
N_R = number of stages over which reentrainment is assumed to occur
η = collection efficiency of a given particle size obtained with no reentrainment.

Figure 3 shows the effect on resultant efficiency for a given particle size of various degrees of reentrainment for a four-section precipitator with the indicated values of no-reentrainment efficiency.

Since both the expressions for reentrainment and sneakage result in a reduction of the expected collection efficiency under ideal conditions, it is possible to define a correction factor for the Deutsch equation, which is a divisor for the migration velocity, analogous to that defined for the effect of a nonuniform gas velocity distribution. The expression for the correction factor is, in terms of reentrainment:

$$B = \frac{\ln(1 - \eta/100)}{N_R \ln[R + (1 - R)(1 - \eta/100)^{1/N_R}]} \qquad (15)$$

The foregoing expressions for reentrainment and gas sneakage are oversimplifications, but it is believed this analysis will be useful by providing a basis for estimating the order of magnitude of efficiency losses caused by these phenomena. If experimental data on these losses become available, it should be possible to develop more sophisticated models.

Description of Model

Figure 4 gives a simplified block diagram of the precipitator model computer program. The program is structured around three major loops, the outermost of which is a direct iteration that converges on the overall mass efficiency. An initial estimate of overall mass efficiency is required because the space charge on the particulate at any point in the precipitator is a function of the particle charge and the number of particles remaining in the gas. The program contains a calculation procedure which estimates the effect of particulate space charge on the average free ion density and the electric field near the collecting electrode. The second major loop includes the calculations which must be performed in each incremental length, and the innermost loop contains the calculations dependent on particle size. After the theoretical collection efficiencies for each particle size have been obtained, the program calculates an effective migration velocity for each particle size from the relationship:

$$w_e = \frac{Q}{A} \ln\left(\frac{100}{100 - \eta}\right) \qquad (16)$$

where η is again the collection efficiency of the particle size under consideration. At this point, a table of ideal or theoretical efficiencies and effective migration velocities is available for the representative particle sizes in the histogram of the particle size distribution. The program then evaluates the correction factors for gas velocity and reentrainment-sneakage, and calculates reduced effective migration velocities and collection efficiencies for each particle size. Overall mass efficiency is obtained by summing over the particle size distribution.

Results

For the purpose of developing typical performance relationships for electrostatic precipitators collecting coal fly ash, a representative particle size distribution and secondary voltage vs current relationship were chosen for input data to the computer model. Operating current densities from 5 to 40 na/cm² were selected. Theoretical values of effective migration velocity as a function of particle size are given in Figure 5. The values shown were computed with a specific collection area of 200 ft²/1000 cfm. Particle charging dynamics become significant at the lower current densities, and the indicated values for effective migration velocities would decrease somewhat if the program were executed with a lower value of specific collecting area.

If the previously discussed corrections are utilized in the computer model, the predicted values of overall mass efficiency are reduced to the range of values obtained from field measurements.

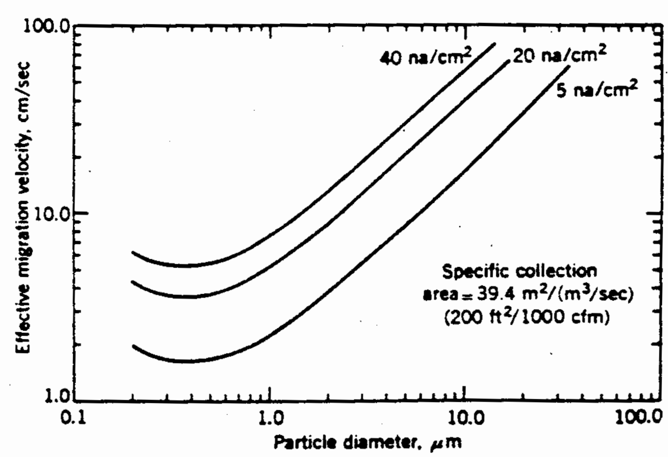

Figure 5. Theoretical effective migration velocity as a function of current density and particle size.

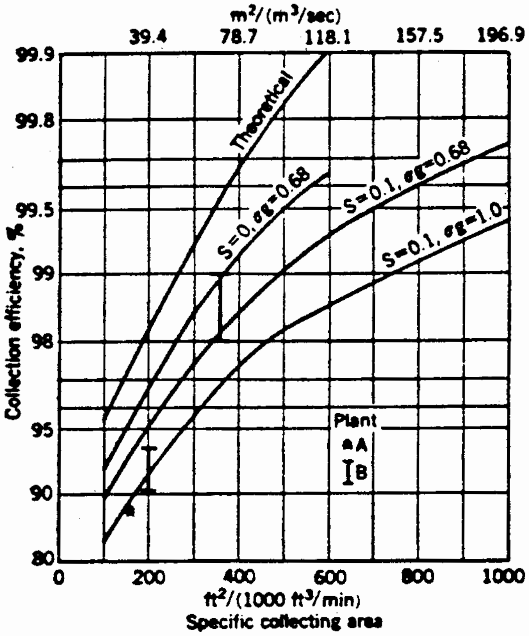

Figure 6. Computed performance curves at 5 na/cm².

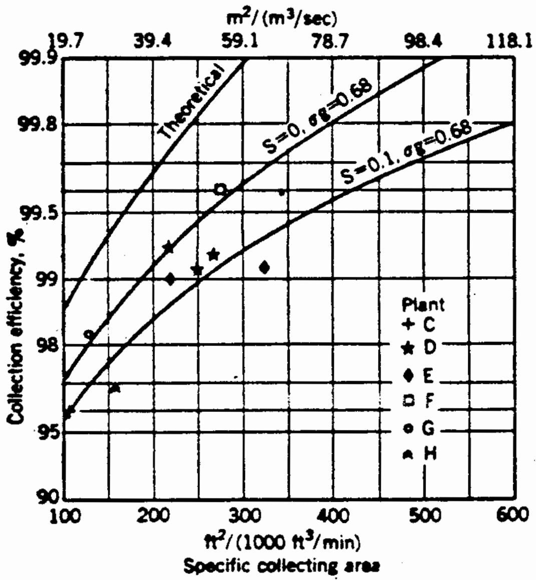

Figure 7. Computed performance curves at 20 na/cm².

Figures 6, 7, and 8 provide comparisons between test data from coal fired power boilers and computed values of efficiency as a function of specific collection area for the indicated current densities. It can be seen that the theoretical overall mass efficiency curves are consistently higher than the field data. The value of 0.68 for the normalized gas velocity standard deviation represents a poor gas velocity distribution, and it can be seen that this has a major effect on predicted performance. It should be noted that the correction factors for gas velocity were obtained from an empirical fit to the data in Figure 2, which is based on a pilot-plant gas flow study. Similar data from full scale units would probably produce significantly different relationships between efficiency, standard deviation, and the quantity F. Since reentrainment and sneakage effects are estimated with identical mathematical expressions, these effects were combined by assuming 10% loss per stage over four effective stages.

The relationships presented in Figures 6, 7, and 8 are based on 9 in. plate to plate spacing, and a secondary volt-amp curve and particle size distribution considered to be typical for a coal-fired power boiler.

As an illustration of the trends predicted by the program, the effect of changes in particle size distribution are presented in Figures 9 and 10. These calculations are based on the assumption that the change in space charge suppression of corona current with size distribution is not a significant factor. This assumption is valid only if the dust loading in the fine particle range is not unusually large.

Log normal particle size distributions with mass median diameters of 25, 10, 5, and 2 μm and a geometric standard deviation of 2.8 were used as input data to the computer model along with a current density of 20 na/cm², a gas velocity standard deviation of 0.68, and a reentrainment-sneakage factor of 0.1 over 4 stages. The results from these computer simulations are given in Figure 9. As would be expected, the computed performance is a strong function of the mass median diameter of the distribution.

It is also of interest to examine the variation in predicted performance caused by varying the standard deviation of a log normal size distribution with a given mass median diameter. Figure 10 presents results from computer simulations using a log normal particle size distribution with a mass median diameter of 10.0 μm and standard deviations ranging from 1.0 (a monodisperse distribution) to 5.0. Figure 10 indicates that predicted performance decreases with increasing values of particle size standard deviation. This decrease results from the influence of

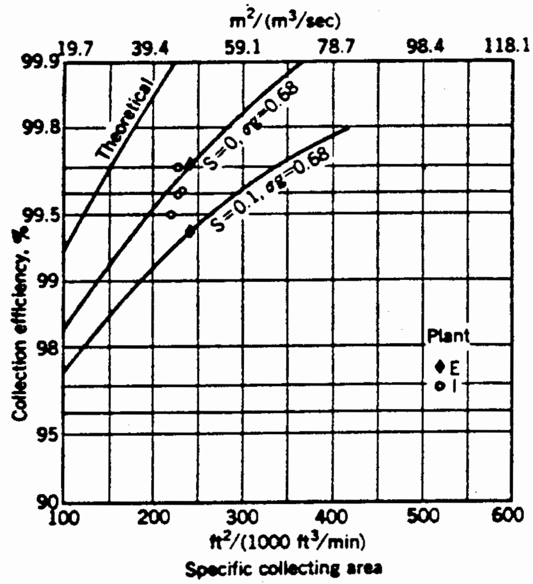

Figure 8. Computed performance curves at 40 na/cm².

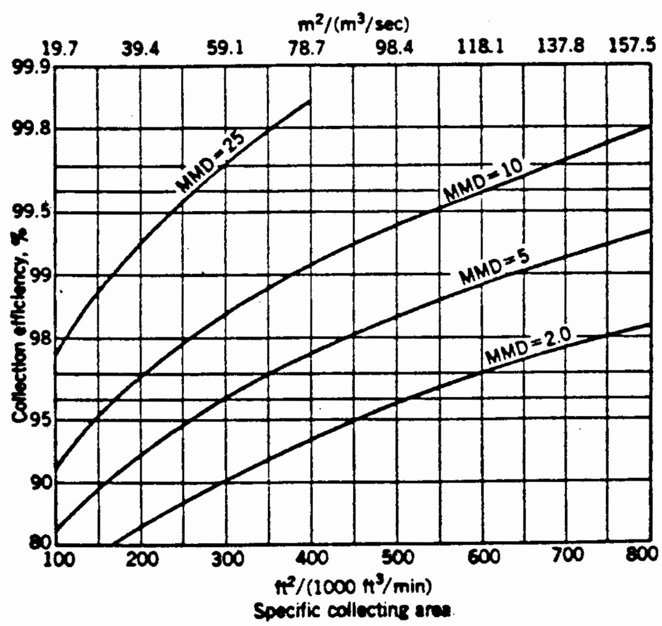

Figure 9. Effect of mass median diameter on computed performance ($\sigma_p = 2.8$).

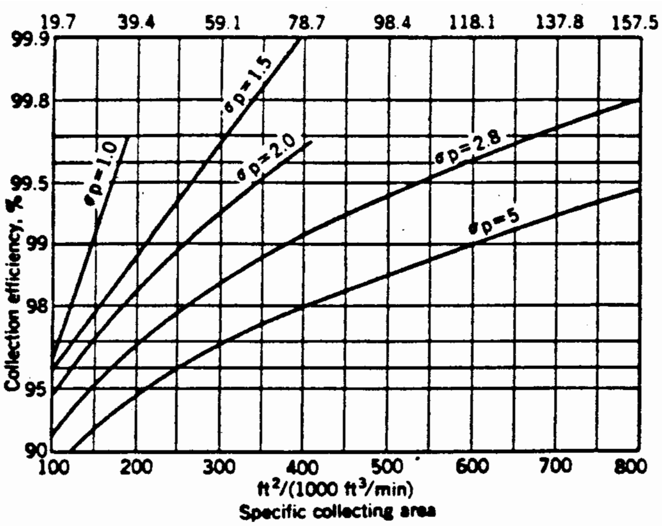

Figure 10. Effect of particle size distribution standard deviation on computed performance (MMD = 10 μm).

the increasing proportions of fine particulate which are present with the larger values of standard deviation. Note that the use of a monodisperse distribution with a diameter of 10.0 μm gives results vastly different from those obtained with realistic values of standard deviation.

Conclusions

Calculation of overall collection efficiency of polydisperse particulate in an electrostatic precipitator from theoretical relationships gives results considerably higher than those obtained from performance measurements on full-size units for coal-fired power boilers. Corrections to the idealized or theoretical collection efficiency to estimate the effects of non-uniform gas flow, rapping reentrainment, and gas by-passing the electrified range of values obtained from field measurements. These calculations suggest that the theoretical model may be used as a basis for quantifying performance under field conditions if sufficient data on the major non-idealities become available.

Acknowledgments

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Adhesive Behavior of Dust in Electrostatic Precipitation

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In 2-stage precipitators particles are driven to the collecting plates by electrostatic forces but then the electrostatic force reverses and tends to pull the particles off so that dust is held on the collecting electrodes only by adhesion. In Cottrell or single-stage precipitators the corona current can provide a significant force tending to hold the collected dust to the electrode provided that the resistivity of the dust is 10^{10} ohm-cm or more. Adhesion is still essential in the collection of lower resistivity dust and is of vital importance in the transfer of dust from the collecting electrodes to the hopper. As the dust falls from the plates to the hopper it must be held in agglomerations or chunks. There are many peculiarities in the adhesive behavior of electrostatically collected dust. A better understanding of this adhesive behavior is essential if we are to improve the transfer of dust from the collecting electrodes to the hopper.