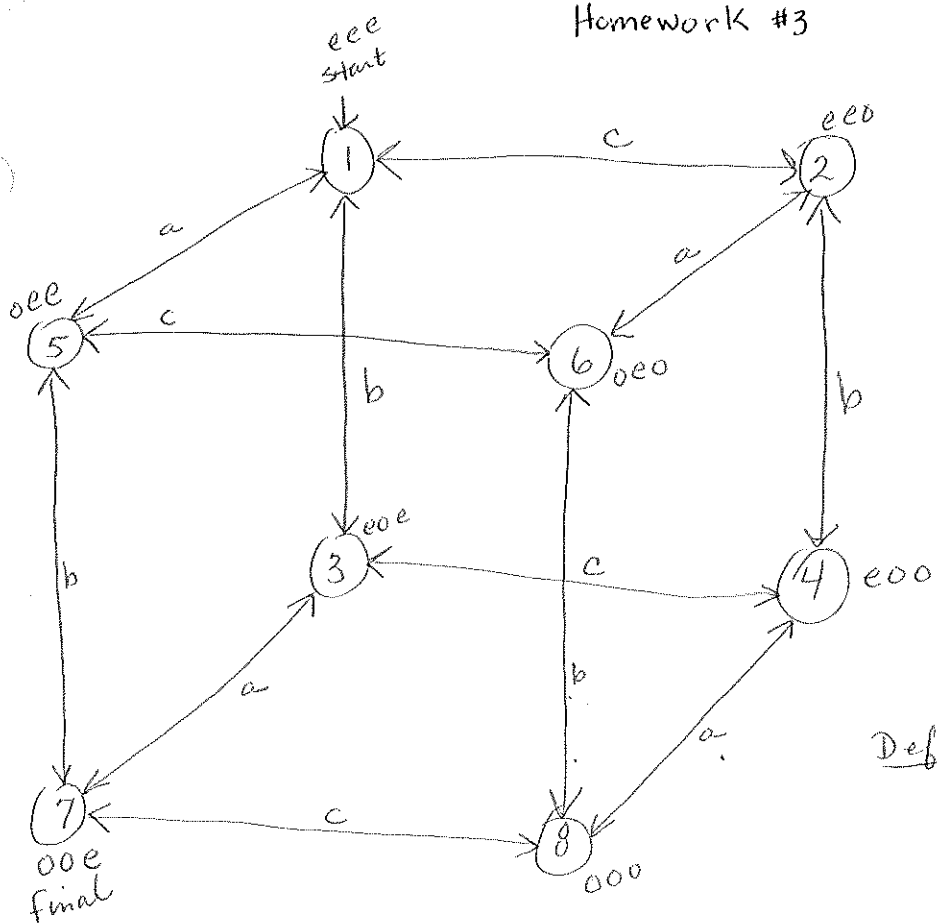


Homework #3

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CS 553 20/20



Final answer can be found on pages 10 thru 14

Notes: + is | (OR)

Def. R_{ij}^k is the set of strings which take M from q_i to q_j without passing through a node numbered ~~i, j~~ higher than k. So R_{ij}^n denotes the set of all strings which take M from q_i to q_j . (M is DFA, q_i, q_j are states)

$$R_{ij}^0 = \begin{cases} \{a; \delta(q_i, a) \rightarrow q_j\} & i \neq j \\ \{a; \delta(q_i, a) \rightarrow q_j\} \cup \{\epsilon\} & i = j \end{cases}$$

$$R_{ij}^k = R_{ik}^{k-1} (R_{kk}^{k-1})^* R_{kj}^{k-1} + R_{ij}^{k-1}$$

$$R_{17}^8 = R_{18}^7 (R_{88}^7)^* R_{87}^7 + R_{17}^7$$

$$R_{18}^7 = R_{17}^6 (R_{77}^6)^* R_{78}^6 + R_{18}^6$$

$$R_{88}^7 = R_{87}^6 (R_{77}^6)^* R_{78}^6 + R_{88}^6$$

$$R_{87}^7 = R_{87}^6 (R_{77}^6)^* R_{77}^6 + R_{87}^6$$

$$R_{17}^7 = R_{17}^6 (R_{77}^6)^* R_{77}^6 + R_{17}^6$$

$$R_{17}^6 = R_{16}^5 (R_{66}^5)^* R_{67}^5 + R_{17}^5$$

$$R_{77}^6 = R_{76}^5 (R_{66}^5)^* R_{67}^5 + R_{77}^5$$

$$R_{78}^6 = R_{76}^5 (R_{66}^5)^* R_{68}^5 + R_{78}^5$$

$$R_{18}^6 = R_{16}^5 (R_{66}^5)^* R_{68}^5 + R_{18}^5$$

$$R_{87}^6 = R_{86}^5 (R_{66}^5)^* R_{67}^5 + R_{87}^5$$

$$R_{88}^6 = R_{86}^5 (R_{66}^5)^* R_{68}^5 + R_{88}^5$$

$$R_{16}^5 = R_{15}^4 (R_{55}^4)^* R_{56}^4 + R_{16}^4 -$$

$$R_{66}^5 = R_{65}^4 \quad R_{56}^4 + R_{66}^4 -$$

$$R_{67}^5 = R_{65}^4 \quad R_{57}^4 + R_{67}^4 -$$

$$R_{17}^5 = R_{15}^4 \quad R_{57}^4 + R_{17}^4 -$$

$$R_{76}^5 = R_{75}^4 \quad R_{56}^4 + R_{76}^4 -$$

$$R_{77}^5 = R_{75}^4 \quad R_{57}^4 + R_{77}^4 -$$

$$R_{68}^5 = R_{65}^4 \quad R_{58}^4 + R_{68}^4 -$$

$$R_{78}^5 = R_{75}^4 \quad R_{58}^4 + R_{78}^4 -$$

$$R_{18}^5 = R_{15}^4 \quad R_{58}^4 + R_{18}^4 -$$

$$R_{86}^5 = R_{85}^4 \quad R_{56}^4 + R_{86}^4 -$$

$$R_{87}^5 = R_{85}^4 \quad R_{57}^4 + R_{87}^4 -$$

$$R_{88}^5 = R_{85}^4 \quad R_{58}^4 + R_{88}^4 -$$

$$R_{15}^4 = R_{14}^3 (R_{44}^3)^* R_{45}^3 + R_{15}^3 -$$

$$R_{55}^4 = R_{54}^3 \quad R_{45}^3 + R_{55}^3 -$$

$$R_{56}^4 = R_{54}^3 \quad R_{46}^3 + R_{56}^3 -$$

$$R_{16}^4 = R_{14}^3 \quad R_{46}^3 + R_{16}^3 -$$

$$R_{65}^4 = R_{64}^3 \quad R_{45}^3 + R_{65}^3 -$$


$$R_{66}^4 = R_{64}^3 \quad R_{46}^3 + R_{66}^3 -$$


$$R_{57}^4 = R_{54}^3 \quad R_{47}^3 + R_{57}^3 -$$

$$R_{67}^4 = R_{64}^3 \quad R_{47}^3 + R_{67}^3 -$$

$R_{17}^4 = R_{14}^3$	$(R_{44}^3)^*$	$R_{47}^3 + R_{17}^3 -$
$R_{75}^4 = R_{74}^3$		$R_{45}^3 + R_{75}^3 -$
$R_{76}^4 = R_{74}^3$		$R_{46}^3 + R_{76}^3 -$
$R_{77}^4 = R_{74}^3$		$R_{47}^3 + R_{77}^3 -$
$R_{58}^4 = R_{54}^3$		$R_{48}^3 + R_{58}^3 -$
$R_{68}^4 = R_{64}^3$		$R_{48}^3 + R_{68}^3 -$
$R_{78}^4 = R_{74}^3$		$R_{48}^3 + R_{78}^3 -$
$R_{18}^4 = R_{14}^3$		$R_{48}^3 + R_{18}^3 -$
$R_{85}^4 = R_{84}^3$		$R_{45}^3 + R_{85}^3 -$
$R_{86}^4 = R_{84}^3$		$R_{46}^3 + R_{86}^3 -$
$R_{87}^4 = R_{84}^3$		$R_{47}^3 + R_{87}^3 -$
$R_{88}^4 = R_{84}^3$		$R_{48}^3 + R_{88}^3 -$

$R_{14}^3 = R_{13}^2$	$(R_{33}^2)^*$	$R_{34}^2 + R_{14}^2 \checkmark$
$R_{44}^3 = R_{43}^2$		$R_{34}^2 + R_{44}^2 \checkmark$
$R_{45}^3 = R_{43}^2$		$R_{35}^2 + R_{45}^2 -$
$R_{15}^3 = R_{13}^2$		$R_{35}^2 + R_{15}^2 -$
$R_{54}^3 = R_{53}^2$		$R_{34}^2 + R_{54}^2 \checkmark$
$R_{55}^3 = R_{53}^2$		$R_{35}^2 + R_{55}^2 \checkmark$
$R_{46}^3 = R_{43}^2$		$R_{36}^2 + R_{46}^2 -$
$R_{56}^3 = R_{53}^2$		$R_{36}^2 + R_{56}^2 -$
$R_{16}^3 = R_{13}^2$		$R_{36}^2 + R_{16}^2 \checkmark$
$R_{64}^3 = R_{63}^2$		$R_{34}^2 + R_{64}^2 \checkmark$
$R_{65}^3 = R_{63}^2$		$R_{35}^2 + R_{65}^2 -$
$R_{66}^3 = R_{63}^2$		$R_{36}^2 + R_{66}^2 -$
$R_{47}^3 = R_{43}^2$		$R_{37}^2 + R_{47}^2 -$
$R_{57}^3 = R_{53}^2$		$R_{37}^2 + R_{57}^2 \checkmark$
$R_{67}^3 = R_{63}^2$		$R_{37}^2 + R_{67}^2 -$
$R_{17}^3 = R_{13}^2$		$R_{37}^2 + R_{17}^2 -$
$R_{74}^3 = R_{73}^2$		$R_{34}^2 + R_{74}^2 \checkmark$
$R_{75}^3 = R_{73}^2$		$R_{35}^2 + R_{75}^2 -$
$R_{76}^3 = R_{73}^2$		$R_{36}^2 + R_{76}^2 -$

$R_{77}^3 = R_{73}^2$	$(R_{33}^2)^*$ 	$R_{37}^2 + R_{77}^2 -$
$R_{48}^3 = R_{43}^2$		$R_{38}^2 + R_{48}^2 -$
$R_{58}^3 = R_{53}^2$		$R_{38}^2 + R_{58}^2 -$
$R_{68}^3 = R_{63}^2$		$R_{38}^2 + R_{68}^2 -$
$R_{78}^3 = R_{73}^2$		$R_{38}^2 + R_{78}^2 -$
$R_{18}^3 = R_{13}^2$		$R_{38}^2 + R_{18}^2 -$
$R_{84}^3 = R_{83}^2$		$R_{34}^2 + R_{84}^2 -$
$R_{85}^3 = R_{83}^2$		$R_{35}^2 + R_{85}^2 -$
$R_{86}^3 = R_{83}^2$		$R_{36}^2 + R_{86}^2 -$
$R_{87}^3 = R_{83}^2$		$R_{37}^2 + R_{87}^2 -$
$R_{88}^3 = R_{83}^2$	$R_{38}^2 + R_{88}^2 -$	

$2-13 = 1-12$	$(1-22)^*$ 	$1-23 + 1-13 ✓$
$2-33 = 1-32$		$1-23 + 1-33 ✓$
$2-34 = 1-32$		$1-24 + 1-34 ✓$
$2-14 = 1-12$		$1-24 + 1-14 ✓$
$2-43 = 1-42$		$1-23 + 1-43 ✓$
$2-44 = 1-42$		$1-24 + 1-44 ✓$
$2-35 = 1-32$		$1-25 + 1-35 ✓$
$2-45 = 1-42$		$1-25 + 1-45 ✓$
$2-15 = 1-12$		$1-25 + 1-15 ✓$
$2-53 = 1-52$		$1-23 + 1-53 ✓$
$2-54 = 1-52$		$1-24 + 1-54 ✓$
$2-55 = 1-52$		$1-25 + 1-55 ✓$
$2-36 = 1-32$		$1-26 + 1-36 ✓$
$2-46 = 1-42$		$1-26 + 1-46 ✓$
$2-56 = 1-52$		$1-26 + 1-56 ✓$
$2-16 = 1-12$		$1-26 + 1-16 ✓$
$2-63 = 1-62$		$1-23 + 1-63 ✓$
$2-64 = 1-62$		$1-24 + 1-64 ✓$
$2-65 = 1-62$		$1-25 + 1-65 ✓$
$2-66 = 1-62$		$1-26 + 1-66 ✓$
$2-37 = 1-32$	$1-27 + 1-37 ✓$	
$2-47 = 1-42$	$1-27 + 1-47 ✓$	
$2-57 = 1-52$	$1-27 + 1-57 ✓$	

2-67 = 1-62	(1-22)*	1-27 + 1-67 ✓
2-17 = 1-12		1-27 + 1-17 ✓
2-73 = 1-72		1-23 + 1-73 ✓
2-74 = 1-72		1-24 + 1-74 ✓
2-75 = 1-72		1-25 + 1-75 ✓
2-76 = 1-72		1-26 + 1-76 ✓
2-77 = 1-72		1-27 + 1-77 ✓
2-38 = 1-32		1-28 + 1-38 ✓
2-48 = 1-42		1-28 + 1-48 ✓
2-58 = 1-52		1-28 + 1-58 ✓
2-68 = 1-62		1-28 + 1-68 ✓
2-78 = 1-72		1-28 + 1-78 ✓
2-18 = 1-12		1-28 + 1-18 ✓
2-83 = 1-82		1-23 + 1-83 ✓
2-84 = 1-82		1-24 + 1-84 ✓
2-85 = 1-82		1-25 + 1-85 ✓
2-86 = 1-82		1-26 + 1-86 ✓
2-87 = 1-82		1-27 + 1-87 ✓
2-88 = 1-82		1-28 + 1-88 ✓

1-12 = 0-11	(0-11)*	0-12 + 0-12 = $\epsilon \epsilon^* c + c = c$
1-22 = 0-21		0-12 + 0-22 = $c \epsilon^* c + \epsilon = (cc + \epsilon)$
1-23 = 0-21		0-13 + 0-23 = $c \epsilon^* b + \phi = cb$
1-13 = 0-11		0-13 + 0-13 = $\epsilon \epsilon^* b + b = b$
1-32 = 0-31		0-12 + 0-32 = $b \epsilon^* c + \phi = bc$
1-33 = 0-31		0-13 + 0-33 = $b \epsilon^* b + \epsilon = (bb + \epsilon)$
1-24 = 0-21		0-14 + 0-24 = $c \epsilon^* \phi + b = b$
1-34 = 0-31		0-14 + 0-34 = $b \epsilon^* \phi + c = c$
1-14 = 0-11		0-14 + 0-14 = $\epsilon \epsilon^* \phi + \phi = \phi$
1-42 = 0-41		0-12 + 0-42 = $\phi \epsilon^* c + b = b$
1-43 = 0-41		0-13 + 0-43 = $\phi \epsilon^* b + c = c$
1-44 = 0-41		0-14 + 0-44 = $\phi \epsilon^* \phi + \epsilon = \epsilon$
1-25 = 0-21		0-15 + 0-25 = $c \epsilon^* a + \phi = ca$
1-35 = 0-31		0-15 + 0-35 = $b \epsilon^* a + \phi = ba$
1-45 = 0-41		0-15 + 0-45 = $\phi \epsilon^* a + \phi = \phi$

- 1-15 = 0-11
- 1-52 = 0-51
- 1-53 = 0-51
- 1-54 = 0-51
- 1-55 = 0-51
- 1-26 = 0-21
- 1-36 = 0-31
- 1-46 = 0-41
- 1-56 = 0-51
- 1-16 = 0-11
- 1-62 = 0-61
- 1-63 = 0-61
- 1-64 = 0-61
- 1-65 = 0-61
- 1-66 = 0-61
- 1-27 = 0-21
- 1-37 = 0-31
- 1-47 = 0-41
- 1-57 = 0-51
- 1-67 = 0-61
- 1-17 = 0-11
- 1-72 = 0-71
- 1-73 = 0-71
- 1-74 = 0-71
- 1-75 = 0-71
- 1-76 = 0-71
- 1-77 = 0-71
- 1-28 = 0-21
- 1-38 = 0-31
- 1-48 = 0-41
- 1-58 = 0-51
- 1-68 = 0-61

(0-11)*

- 0-15 + 0-15 = $\epsilon \epsilon^* a + a = a$
- 0-12 + 0-52 = $a \epsilon^* c + \phi = ac$
- 0-13 + 0-53 = $a \epsilon^* b + \phi = ab$
- 0-14 + 0-54 = $a \epsilon^* \phi + \phi = \phi$
- 0-15 + 0-55 = $a \epsilon^* a + \epsilon = (aa + \epsilon)$
- 0-16 + 0-26 = $c \epsilon^* \phi + a = a$
- 0-16 + 0-36 = $b \epsilon^* \phi + \phi = \phi$
- 0-16 + 0-46 = $\phi - + \phi = \phi$
- 0-16 + 0-56 = $a \epsilon^* \phi + c = c$
- 0-16 + 0-16 = $\epsilon \epsilon^* \phi + \phi = \phi$
- 0-12 + 0-62 = $\phi - + a = a$
- 0-13 + 0-63 = $\phi - + \phi = \phi$
- 0-14 + 0-64 = $\phi - + \phi = \phi$
- 0-15 + 0-65 = $\phi - + c = c$
- 0-16 + 0-66 = $\phi - + \epsilon = \epsilon$
- 0-17 + 0-27 = $c \epsilon^* \phi + \phi = \phi$
- 0-17 + 0-37 = $b \epsilon^* \phi + a = a$
- 0-17 + 0-47 = $\phi - + \phi = \phi$
- 0-17 + 0-57 = $a \epsilon^* \phi + b = b$
- 0-17 + 0-67 = $\phi - + \phi = \phi$
- 0-17 + 0-17 = $-\phi + \phi = \phi$
- 0-12 + 0-72 = $\phi - + \phi = \phi$
- 0-13 + 0-73 = $\phi - + a = a$
- 0-14 + 0-74 = $\phi - + \phi = \phi$
- 0-15 + 0-75 = $\phi - + b = b$
- 0-16 + 0-76 = $\phi - + \phi = \phi$
- 0-17 + 0-77 = $\phi - + \epsilon = \epsilon$
- 0-18 + 0-28 = $c \epsilon^* \phi + \phi = \phi$
- 0-18 + 0-38 = $b \phi + \phi = \phi$
- 0-18 + 0-48 = $\phi - + a = a$
- 0-18 + 0-58 = $\phi - + \phi = \phi$
- 0-18 + 0-68 = $\phi - + c = c$



$1-78 = 0-71$	$(0-11)^*$	$0-18$	$+ 0-78$	$= \phi - + c$	$= c$
$1-18 = 0-11$		$0-18$	$+ 0-18$	$= \epsilon\epsilon^*\phi + \phi$	$= \phi$
$1-82 = 0-81$		$0-12$	$+ 0-82$	$= \phi - + \phi$	$= \phi$
$1-83 = 0-81$		$0-13$	$+ 0-83$	$= \phi - + \phi$	$= \phi$
$1-84 = 0-81$		$0-14$	$+ 0-84$	$= \phi - + a$	$= a$
$1-85 = 0-81$		$0-15$	$+ 0-85$	$= \phi - + \phi$	$= \phi$
$1-86 = 0-81$		$0-16$	$+ 0-86$	$= \phi - + c$	$= c$
$1-87 = 0-81$		$0-17$	$+ 0-87$	$= \phi - + c$	$= c$
$1-88 = 0-81$		$0-18$	$+ 0-88$	$= \phi - + \epsilon$	$= \epsilon$

For R_{ij}^1 , where ij is:

- | | | | | | |
|------|-----------|-----------|------|-----------|------|
| 12 c | 22 (cc+ε) | 32 bc | 42 b | 52 ac | 62 a |
| 13 b | 23 cb | 33 (bb+ε) | 43 c | 53 ab | 63 φ |
| 14 φ | 24 b | 34 c | 44 ε | 54 φ | 64 φ |
| 15 a | 25 ca | 35 ba | 45 φ | 55 (aa+ε) | 65 c |
| 16 φ | 26 a | 36 φ | 46 φ | 56 c | 66 ε |
| 17 φ | 27 φ | 37 a | 47 φ | 57 b | 67 φ |
| 18 φ | 28 φ | 38 φ | 48 a | 58 φ | 68 c |
-
- | | |
|------|------|
| 72 φ | 82 φ |
| 73 a | 83 φ |
| 74 φ | 84 a |
| 75 b | 85 φ |
| 76 φ | 86 c |
| 77 ε | 87 c |
| 78 c | 88 ε |

For R_{ij}^2 where ij is:

- 13 $c(cc)^*cb + b$
- 14 $c(cc)^*b + \epsilon$
- 15 $c(cc)^*ca + a$
- 16 $c(cc)^*a$
- 17 ~~$c(cc)^*$~~ ϕ
- 18 ϕ

- 33 $bc(cc)^*cb + bb + \epsilon$
- 34 $bc(cc)^*b + c$
- 35 $bc(cc)^*ca + ba$
- 36 $bc(cc)^*a$
- 37 ~~$bc(cc)^*$~~ a
- 38 ~~$bc(cc)^*$~~ ϕ

- 43 $b(cc)^*cb + c$
- 44 $b(cc)^*b + \epsilon$
- 45 ~~$b(cc)^*ca + ba$~~ $b(cc)^*ca$
- 46 $b(cc)^*a$
- 47 ϕ
- 48 ~~$b(cc)^*$~~ a

- 53 $ac(cc)^*cb + ab$
- 54 $ac(cc)^*b$
- 55 $ac(cc)^*ca + aa + \epsilon$
- 56 $ac(cc)^*a + c$
- 57 ~~$ac(cc)^*$~~ b
- 58 ϕ

- 63 $a(cc)^*cb$
- 64 $a(cc)^*b$
- 65 $a(cc)^*ca + c$
- 66 $a(cc)^*a + \epsilon$
- 67 ~~$a(cc)^*$~~ ϕ
- 68 c

For R_{ij}^3 where ij is:

- 73 a
- 74 ϕ
- 75 b
- 76 ϕ
- 77 ϵ
- 78 c

14 $(c(cc)^*cb + b)(bc(cc)^*cb + bb)^*(bc(cc)^*b + c) + c(cc)^*b$

15 $(c(cc)^*cb + b)(bc(cc)^*cb + bb)^*(bc(cc)^*ca + ba) + c(cc)^*ca + a$

16 (") $\textcircled{3}$ $(bc(cc)^*a) + c(cc)^*a$

17 (") $\textcircled{3}$ a

18 ϕ

- 83 ϕ
- 84 a
- 85 ϕ
- 86 c
- 87 c
- 88 ϵ

44 $(b(cc)^*cb + c)(bc(cc)^*cb + bb)^*(bc(cc)^*b + c) + b(cc)^*b + \epsilon$

45 $(b(cc)^*cb + c)(bc(cc)^*cb + bb)^*(bc(cc)^*ca + ba) + b(cc)^*ca$

46 (") $\textcircled{3}$ $(bc(cc)^*a) + b(cc)^*a$

47 (") $\textcircled{3}$ a

48 ~~(")~~ $\textcircled{3}$ a

54 $(ac(cc)^*cb + ab)\textcircled{3}(bc(cc)^*b + c) + ac(cc)^*b$

55 (") $\textcircled{3}$ $(bc(cc)^*ca + ba) + ac(cc)^*ca + aa + \epsilon$

56 (") $\textcircled{3}$ $(bc(cc)^*a) + ac(cc)^*a + c$

57 (") $\textcircled{3}$ $a + b$

58 ϕ

- 64 $(a(cc)^*cb)\textcircled{3}(bc(cc)^*b + c) + a(cc)^*b$
- 65 (") $\textcircled{3}$ $(bc(cc)^*ca + ba) + a(cc)^*a + \epsilon$
- 66 (") $\textcircled{3}$ $(bc(cc)^*a) + a(cc)^*a + \epsilon$
- 67 (") $\textcircled{3}$ a
- 68 c
- 74 $a\textcircled{3}(bc(cc)^*b + c)$
- 75 $a\textcircled{3}(bc(cc)^*ca + ba) + b$
- 76 $a\textcircled{3}(bc(cc)^*a)$
- 77 $a\textcircled{3}a + \epsilon$
- 78 c

- 84 a
- 85 ϕ
- 86 c
- 87 c
- 88 ϵ

Let $\textcircled{3} = (bc(cc)^*cb + bb)^*$

For R^4 where ij is $\det(4) = ((b(cc)^*cb + c) \textcircled{3} (bc(cc)^*b + c) + b(cc)^*b) \textcircled{9}^*$

$$15 \quad (3-14) \textcircled{4} (3-45) + (3-15)$$

$$16 \quad (3-14) \textcircled{4} (3-46) + (3-16)$$

$$17 \quad (3-14) \textcircled{4} (3-47) + (3-17)$$

$$18 \quad (3-14) \textcircled{4} a$$

$$55 \quad (3-54) \textcircled{4} (3-45) + (3-55)$$

$$56 \quad (3-54) \textcircled{4} (3-46) + (3-56)$$

$$57 \quad (3-54) \textcircled{4} (3-47) + (3-57)$$

$$58 \quad (3-54) \textcircled{4} a$$

$$65 \quad (3-64) \textcircled{4} (3-45) + (3-65)$$

$$66 \quad (3-64) \textcircled{4} (3-46) + (3-66)$$

$$67 \quad (3-64) \textcircled{4} (3-47) + (3-67)$$

$$68 \quad (3-64) \textcircled{4} a + c$$

$$75 \quad (3-74) \textcircled{4} (3-45) + (3-75)$$

$$76 \quad (3-74) \textcircled{4} (3-46) + (3-76)$$

$$77 \quad (3-74) \textcircled{4} (3-47) + (3-77)$$

$$78 \quad (3-74) \textcircled{4} a + c$$

$$85 \quad a \textcircled{4} (3-45)$$

$$86 \quad a \textcircled{4} (3-46) + c$$

$$87 \quad a \textcircled{4} (3-47) + c$$

$$88 \quad a \textcircled{4} a + \varepsilon$$

$$\det(5) = ((3-54) \textcircled{4} (3-45) + (3-55))^*$$

For R^5 where ij is

$$16 \quad ((3-14) \textcircled{4} (3-45) + (3-15)) \textcircled{5} ((3-54) \textcircled{4} (3-46) + (3-56)) + ((3-14) \textcircled{4} (3-46) + (3-16))$$

$$17 \quad ((3-14) \textcircled{4} (3-45) + (3-15)) \textcircled{5} ((3-54) \textcircled{4} (3-47) + (3-57)) + ((3-14) \textcircled{4} (3-47) + (3-17))$$

$$18 \quad ((3-14) \textcircled{4} (3-45) + (3-15)) \textcircled{5} ((3-54) \textcircled{4} a) + ((3-14) \textcircled{4} a)$$

$$66 \quad ((3-64) \textcircled{4} (3-45) + (3-65)) \textcircled{5} ((3-54) \textcircled{4} (3-46) + (3-56)) + ((3-64) \textcircled{4} (3-46) + (3-66))$$

$$67 \quad ((3-64) \textcircled{4} (3-45) + (3-65)) \textcircled{5} ((3-54) \textcircled{4} (3-47) + (3-57)) + ((3-64) \textcircled{4} (3-47) + (3-67))$$

$$68 \quad ((3-64) \textcircled{4} (3-45) + (3-65)) \textcircled{5} ((3-54) \textcircled{4} a) + ((3-64) \textcircled{4} a + c)$$

$$76 \quad ((3-74) \textcircled{4} (3-45) + (3-75)) \textcircled{5} ((3-54) \textcircled{4} (3-46) + (3-56)) + ((3-74) \textcircled{4} (3-46) + (3-76))$$

$$77 \quad ((3-74) \textcircled{4} (3-45) + (3-75)) \textcircled{5} ((3-54) \textcircled{4} (3-47) + (3-57)) + ((3-74) \textcircled{4} (3-47) + (3-77))$$

$$78 \quad ((3-74) \textcircled{4} (3-45) + (3-75)) \textcircled{5} ((3-54) \textcircled{4} a) + ((3-74) \textcircled{4} a + c)$$

$$86 \quad (a \textcircled{4} (3-45)) \textcircled{5} ((3-54) \textcircled{4} (3-46) + (3-56)) + (a \textcircled{4} (3-46) + c)$$

$$87 \quad (a \textcircled{4} (3-45)) \textcircled{5} ((3-54) \textcircled{4} (3-47) + (3-57)) + (a \textcircled{4} (3-47) + c)$$

$$88 \quad (a \textcircled{4} (3-45)) \textcircled{5} ((3-54) \textcircled{4} a) + (a \textcircled{4} a + \varepsilon)$$

R₁₇⁸ =

$$8-17 = (7-18)(7-88)^* (7-87) + (7-17)$$

$$= \left((6-17)(6-77)^* (6-78) + (6-18) \right) \left((6-87)(6-77)^* (6-78) + (6-88) \right)^* \\ \left((6-87)(6-77)^* \overset{(6-87)}{\cancel{(6-87)}} + \overset{(6-87)}{\cancel{(6-87)}} \right) + \left((6-17)(6-77)^* (6-77) + (6-17) \right)$$

$$= \left(\left((5-16)(5-66)^* (5-67) + (5-17) \right) \left((5-76)(5-66)^* (5-67) + (5-77) \right)^* \right. \\ \left. \left((5-76)(5-66)^* (5-68) + (5-78) \right) + \left((5-16)(5-66)^* (5-68) + (5-18) \right) \right) \\ \left(\left((5-86)(5-66)^* (5-67) + (5-87) \right) \left((5-76)(5-66)^* (5-67) + (5-77) \right)^* \right. \\ \left. \left((5-76)(5-66)^* (5-68) + (5-78) \right) + \left((5-86)(5-66)^* (5-68) + (5-88) \right) \right)^* \\ \left(\left((5-86)(5-66)^* (5-67) + (5-87) \right) \left((5-76)(5-66)^* (5-67) + (5-77) \right)^* \right. \\ \left. \left((5-76)(5-66)^* (5-67) + (5-77) \right) + \left((5-86)(5-66)^* (5-67) + (5-87) \right) \right) \\ + \left(\left((5-16)(5-66)^* (5-67) + (5-17) \right) \left((5-76)(5-66)^* (5-67) + (5-77) \right)^* \right. \\ \left. \left((5-76)(5-66)^* (5-67) + (5-77) \right) + \left((5-16)(5-66)^* (5-67) + (5-17) \right) \right)$$

~~let $(6) = (5-66)^*$~~

This is the final answer with these each defined on the following pages.

Final answer details

(4) + (5) are defined on page 14

(11)

$$\begin{aligned}
 5-16 = R_{16}^5 &= ((((((c(cc)^*eb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + e(cc)^*b) \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + b(cc)^*b)^*) \\
 &(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba)) + \\
 &(c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + e(cc)^*ca+a)) \textcircled{5} \\
 &(((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)(bc(cc)^*a) + b(cc)^*a) + \\
 &((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + ac(cc)^*a+c)) + \\
 &(((c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4} \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + b(cc)^*a) + \\
 &((c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + e(cc)^*a))))))
 \end{aligned}$$

$$\begin{aligned}
 5-17 = R_{17}^5 &= ((((((c(cc)^*eb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + e(cc)^*b) \textcircled{4} \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + ~~e(cc)^*ca+a~~) \textcircled{5} \\
 &(c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + e(cc)^*ca+a)) \textcircled{5} \\
 &((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*a) + \\
 &((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*a + b)) + \\
 &(((c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + e(cc)^*b) \textcircled{4} \\
 &((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*a) + \\
 &((c(cc)^*cb+b)(bc(cc)^*cb+bb)^*a))))))
 \end{aligned}$$

$$\begin{aligned}
 5-18 = R_{18}^5 &= ((((((c(cc)^*eb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + e(cc)^*b) \textcircled{4} \\
 &(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + \\
 &(c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + e(cc)^*ca+a)) \textcircled{5} \\
 &((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} a) \\
 &+ (((c(cc)^*cb+b)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + e(cc)^*b) \\
 &\textcircled{4} a))))))
 \end{aligned}$$

$$\begin{aligned}
5-66 = R_{66}^5 &= (((((a(cc)^*ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} \\
&((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) + \\
&((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + a(cc)^*a+\epsilon) \textcircled{5} \\
&((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\
&(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + b(cc)^*a) + \\
&(ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + ac(cc)^*a+c)) + \\
&((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} \\
&(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + b(cc)^*a) + \\
&(a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + a(cc)^*a+\epsilon))))) \textcircled{12}
\end{aligned}$$

$$\begin{aligned}
5-67 = R_{67}^5 &= (((((a(cc)^*ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} \\
&((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) + \\
&((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + a(cc)^*a+\epsilon) \textcircled{5} \\
&((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\
&(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*a) + \\
&(ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*a+b)) + \\
&((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} \\
&(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*a) + (a(cc)^*b(b(cc)^*cb+bb)^*a))))) \textcircled{4}
\end{aligned}$$

$$\begin{aligned}
5-68 = R_{68}^5 &= (((((a(cc)^*ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} \\
&((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) + \\
&((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + a(cc)^*a+\epsilon) \textcircled{5} \\
&((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\
&+ ((a(cc)^*cb)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + a(cc)^*b) \textcircled{4} a+c))))) \textcircled{4}
\end{aligned}$$

(13)

$$5-76 = R_{76}^5 = (((((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4})$$

$$((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) +$$

$$(a(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b)) \textcircled{5})$$

$$((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4})$$

$$(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*a + b(cc)^*a) +$$

$$(ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + ac(cc)^*a+c)) +$$

$$(((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4})$$

$$(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*a) + b(cc)^*a) +$$

$$(a(bc(cc)^*cb+bb)^*(bc(cc)^*a))))))$$

$$5-77 = R_{77}^5 = (((((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4})$$

$$((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) +$$

$$(a(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b)) \textcircled{5})$$

$$((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4})$$

$$(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(a) +$$

$$(ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(a+b)) +$$

$$(((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4})$$

$$(b(cc)^*cb+c)(bc(cc)^*cb+bb)^*a) +$$

$$(a(bc(cc)^*cb+bb)^*a + \varepsilon))))))$$

$$5-78 = R_{78}^5 = (((((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4})$$

$$((b(cc)^*cb+c)(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b(cc)^*ca) +$$

$$(a(bc(cc)^*cb+bb)^*(bc(cc)^*ca+ba) + b)) \textcircled{5})$$

$$((ac(cc)^*cb+ab)(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4})$$

$$+ (((a(bc(cc)^*cb+bb)^*(bc(cc)^*b+c) \textcircled{4}) a + c))))))$$

$$5-86 = R_{86}^5 = (((((a \textcircled{4}) (b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*ca+ba) + b(cc)^*ca) \textcircled{5}) \\ ((ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4}) \\ (b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*a) + b(cc)^*a) + \\ (ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*a) + ac(cc)^*a+c)) + \\ (a \textcircled{4} ((b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*a) + b(cc)^*a) + c))))$$

$$5-87 = R_{87}^5 = (((((a \textcircled{4}) (b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (b(cc)^*ca+ba) + b(cc)^*ca) \textcircled{5}) \\ ((ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4}) \\ (b(cc)^*cb+c) (bc(cc)^*cb+bb)^* a + \\ (ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* a+b)) + \\ (a \textcircled{4} ((b(cc)^*cb+c) (bc(cc)^*cb+bb)^* a) + c))))$$

$$5-88 = R_{88}^5 = (((((a \textcircled{4}) (b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*ca+ba) + b(cc)^*ca) \textcircled{5}) \\ ((ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4}) \\ a) + (a \textcircled{4} a + \varepsilon))))$$

And used in all the above statements:

$$\textcircled{4} = ((b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*b+c) + b(cc)^*b)^*$$

$$\textcircled{5} = ((ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*b+c) + ac(cc)^*b) \textcircled{4} \\ ((b(cc)^*cb+c) (bc(cc)^*cb+bb)^* (bc(cc)^*ca+ba) + b(cc)^*ca) \\ + ((ac(cc)^*cb+ab) (bc(cc)^*cb+bb)^* (bc(cc)^*ca+ba) + ac(cc)^*ca + aa + \varepsilon)$$