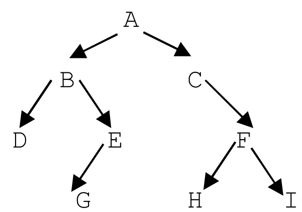
CSS 343 Exam The actual exam has space for your work. Be as clear as possible. You don't have to comment code, but use meaningful names and proper indentation. Show all work!!!!! Don't erase.

1. Assume a Binary Tree class has a member function call play and a tree has been built as shown. What's the output of main? Show the execution tree. (10 pts)

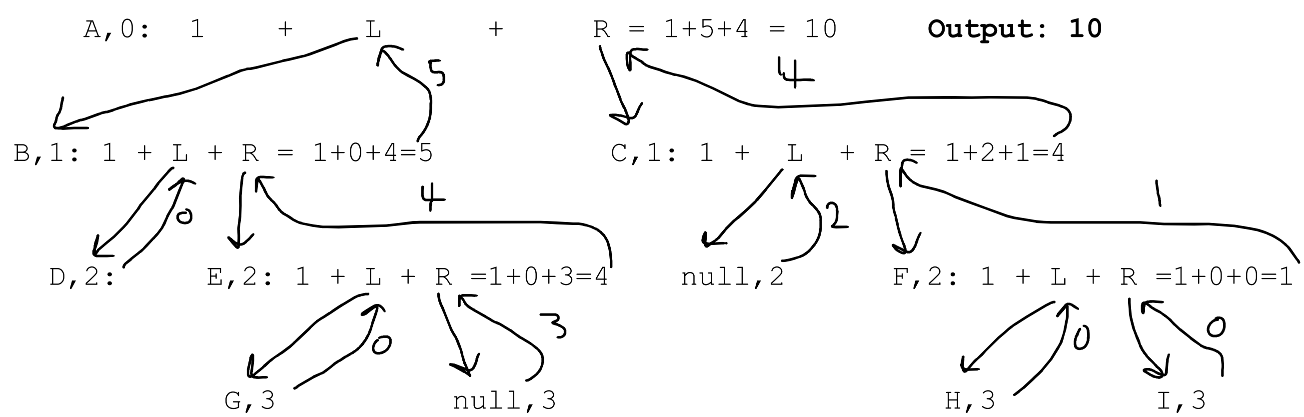
```
int BinTree::play() const {
   int n = 0;
   return helper(root, n);
}
```

main:
```
BinaryTree T;
...
cout << T.play() << endl;
```

```
int BinTree::helper(Node* current, int n)const {
   if (current == NULL)
      return n;
   if (current->right != NULL || current->left != NULL)
      return 1 + helper(current->left, n+1) + helper(current->right, n+1);
   return 0;
}
```



Execution tree (the = designates return values at internal nodes):



2. Give the complexity (tight big-oh) of the following. You need not show work. (15 pts)

(a). Find the smallest value in an AVL tree of n items. O(log n)

(b). Find the smallest value in a binary heap of n items. O(1)

(c). Insert one item into a binary heap of n items. O(log n)

(d). Destructor for a binary search tree of n items. O(n)

(e). Remove one edge in a graph of n nodes and E edges stored in an adjacency matrix. O(1)

(f). The maximum number of unique edges in an undirected graph of n nodes. O(n^2)

(g). Breadth-first ordering on a graph with n nodes and E edges stored in an adjacency matrix. O(n^2)

3. Given the following characters and frequency of occurrence in a message, use the Huffman encoding algorithm to find a unique encoding for the characters. Show all work. (10 pts)

```
Letter   Frequency
 a          10 ←
 b          15 ← start with these two, a and b, then delete from list, add "ab 25"
 c          20 ← these would be next, c and d, delete, add "cd 42", and so on
 d          22 ←
 e          30
```

Final answer:

```
                     cdabe(97)
           0 /               \ 1                       a 100
            /                 abe(55)                  b 101
           /              0 /    \                     c 00
       cd(42)           ab(25)    \ 1                  d 01
        /   \            /   \      \                  e 11
    0 /      \ 1     0 /      \ 1    \
    c(20)   d(22)   a(10)    b(15)    e(30)
```

4. Demonstrate the heap sort (done efficiently) on the values: 10 5 7 11 6 3 2 13 15 1 4. Show only the first three passes of the algorithm (three values are sorted). There is no code writing. Redraw your heaps when needed for clarity. (15 pts)

Build a heap using the O(n) algorithm (work is not shown):

```
Final heap:          1
                 /       \
             4               2
           /   \           /   \
        11       5       3       7
       / \      / \
     13   15   6   10
```

Three deleteMins (work is not shown) (highlighted values are at the end of the array, but not a part of the heap):

```
                     2
                 /       \
             4               3
           /   \           /   \
        11       5       10      7
       / \      / \
     13   15   6   1

                     3
                 /       \
             4               6
           /   \           /   \
        11       5       10      7
       / \      / \
     13   15   2   1

                     4
                 /       \
             5               6
           /   \           /   \
        11      15       10      7
       / \      / \
     13   3    2   1
```

5. Given the following AVL tree structure (letters represent appropriate values).
Suppose a value was added as shown (in the P position).
Show the balanced tree after the insertion. Show each step. (5 pts)

```
           A
       /       \
     B            C
   /   \         /   \
  D     E       F     G
   \   / \    /         \
   H  I   J  K           L
     /   / \
    M   N   O
           /
          P
```

Which node is **unbalanced**? **B**

Do a right-right rotation.

Final answer:

```
             A
         /       \
      E              C
    /   \          /   \
   B      J       F     G
 /   \   / \     /       \
D     I N   O   K         L
 \   /      /
  H M      P
```

7. Consider your Polynomial class only the array content contains double : (10 pts)

```
class Poly {
   friend ...
public:
   Poly(int coeff=0, int maxExp=0);         // constructor, sets size=maxExp+1
   ~Poly();                                 // destructor
   Poly(const Poly &);                      // copy constructor
   void integrate();                        // integrate poly
   ...
private:
   double* ptr;                              // pointer to first array element
   int size;                                 // size of the array
};
```

Write the member function integrate which replaces the current polynomial with its integral (as in calculus). E.g.,

```
Poly A(0,4);
cin >> A;                  // user enters values so A = +10x^4 -7x^2 +3
A.integrate();
cout << A << endl;         // outputs: +2.00x^5 -2.33x^3 +3.00x
```

```
void Poly::integrate() {
   double* temp = new double[size+1];
   temp[0] = 0.0;
   for (int i = 0; i < size; i++) {
      temp[i+1] = ptr[i]/(i+1);
   }
   delete [] ptr;
   ptr = temp;
   temp = NULL;
   size++;
}
```

8. Implement function(s) for a BSTree class that determine whether or not the tree is fully complete (all levels filled completely). Assume usual Node, with member data: `NodeData* data, Node* left, Node* right` Any function you use, you must write. For example: (20 pts)

```
T1:      A
       /    \
     B        C
   /  \      /  \
  D    E    F    G
 / \  / \  / \   / \
H  I J  K L  M N   P
```

Sample main:

```
BSTree T1;
BSTree T2;
bool complete;
...
complete = T1.isComplete();      // returns true
complete = T2.isComplete();      // returns false
```

```
T2:      A
       /    \
     B        C
   /   \    /   \
  D     E  F     G
 / \   /
H   I J
```

```
bool BSTree::isComplete() const {
   return depthBalanceHelper(root) != -1;
}

int BSTree::depthBalanceHelper(Node* current) const {
   if (current == NULL)
      return 0;

   int leftDepth = depthBalanceHelper(current->left);
   if (leftDepth == -1)
      return -1;

   int rightDepth = depthBalanceHelper(current->right);
   if (rightDepth == -1)
      return -1;

   if (leftDepth != rightDepth)
      return -1;

   return leftDepth+1;
}
```