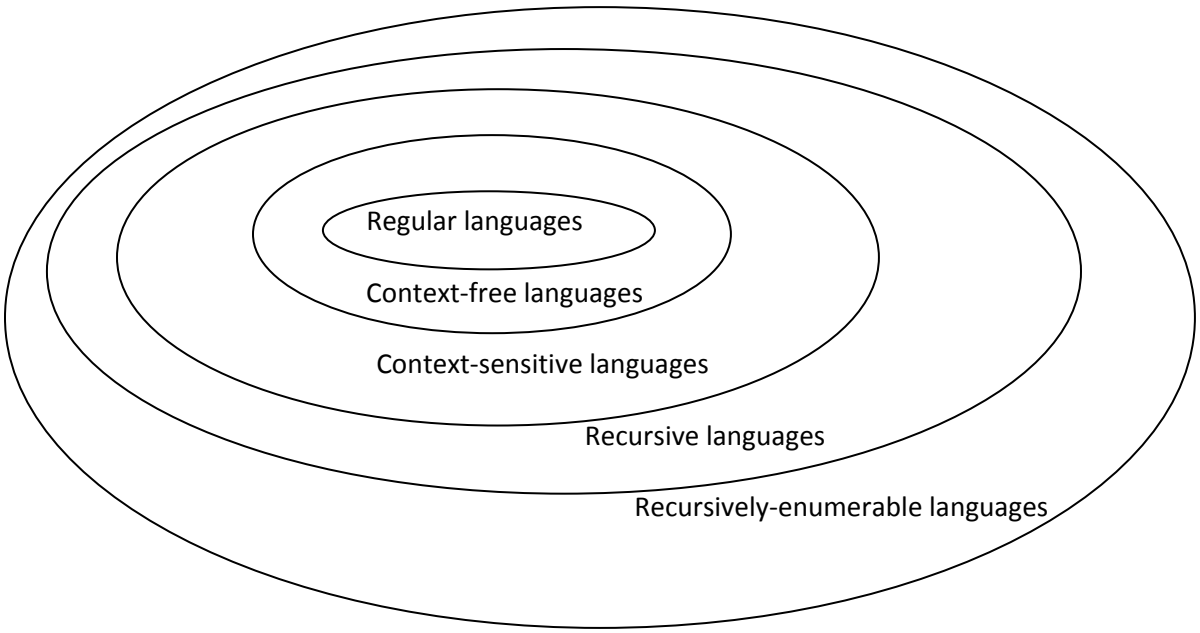
Languages (Introduction, Regular Expressions) Carol Zander

This topic is about modeling computation. Models are important for several reasons. They let us think about whether problems can even be solved using a computer (some can't). They also let us think abstractly about the method to solve a problem. First, we'll model languages. These are very important from a programming language definition and compilation standpoint. All programming languages and their components are defined by different types of languages.

First, what is a language? **A language is a set of strings.** For example, one program you write is one long string. Or one sentence you write is another long string. Different types of languages give us different powers of expression. Powers of expression refers to what can be represented using that form of expression, i.e., what you can specify using the given representation. If you think of representation of a language as sort of an algebra, some representations allow us to generate more complex strings than others. We can express more complex thoughts using English than we can in C++.

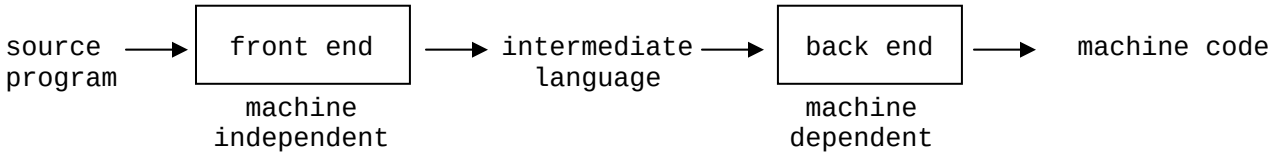
Noam Chomsky (the well-known political activist) is by profession a well-known linguist. He was interested in formal languages and whether or not they could represent properties of human language. While he was not interested in computers, he came up with a language hierarchy that turned out to be useful in computer science (for the compilation process). This is called the Chomsky hierarchy. It captures the power of expression of languages. A Venn diagram represents the power of these languages, the weakest at the center. Regular languages are subsets of every other category. For example, every language that is regular is context-free, but not conversely.



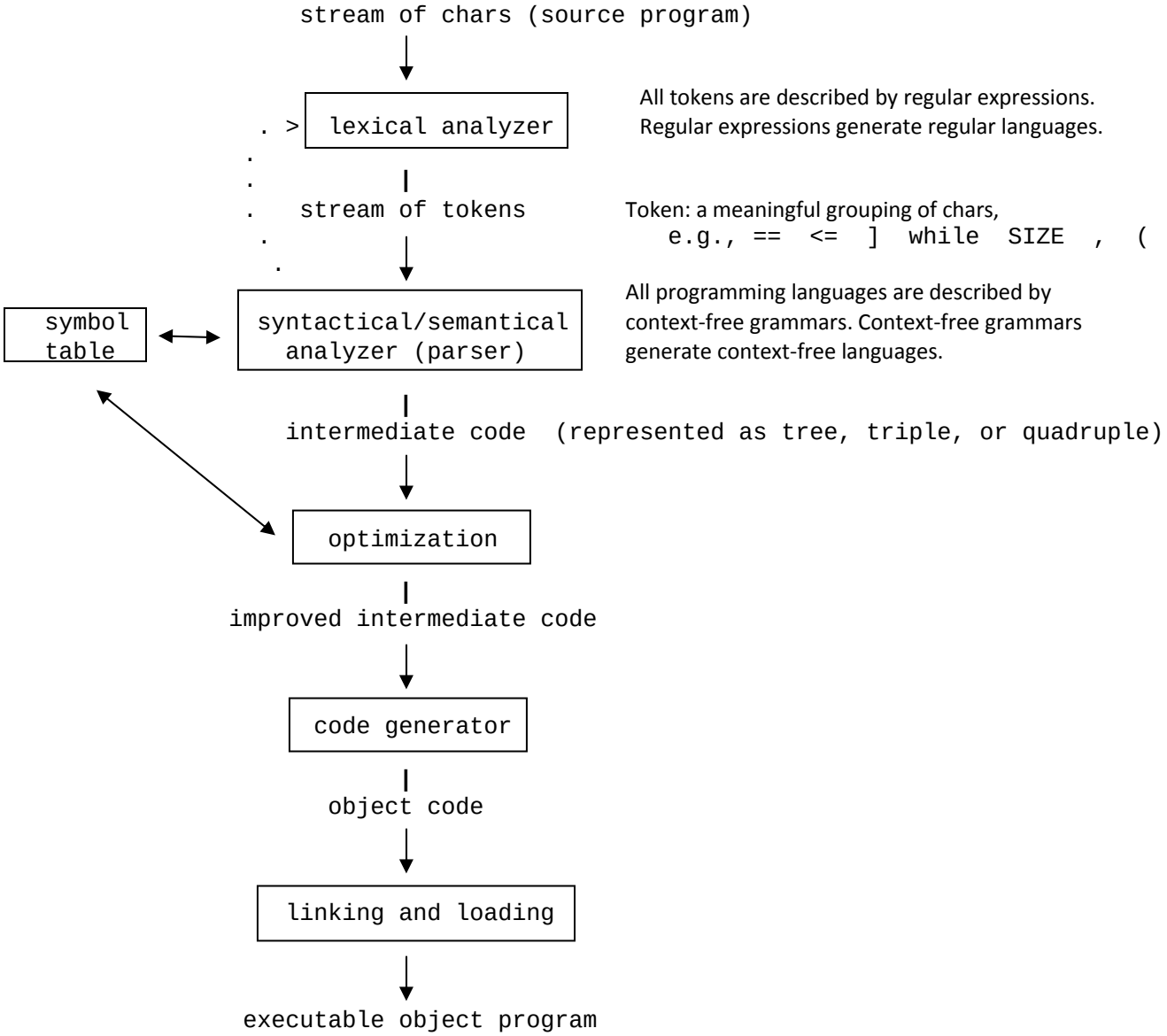
Chomsky defined four language categories:

- Regular (Type 3)
- Context-free (Type 2)
- Context-sensitive (Type 1)
- Recursively-enumerable (Type 0)

We will study languages from the standpoint of the compiling process. **Big Picture:**



Phases of a Compiler:



All tokens are described by regular expressions. Regular expressions generate regular languages.

Token: a meaningful grouping of chars,
e.g., == <=] while SIZE , (

All programming languages are described by context-free grammars. Context-free grammars generate context-free languages.

What makes the language representation so important is that *every* token can be represented by a regular expression. *Every* computing programming language can be represented by a context-free grammar.

While the phases looks linear, in a compiler program, the lexical analyzer and parser phases are really more of a loop. The lexical analyzer keeps feeding the parser tokens, one at a time, while the parser puts the tokens together to try to recognize a statement in the language.

Regular expressions

The simplest type of language is the regular language. Regular languages are generated by regular expressions. These are commonly used for pattern matching. Regular expressions are usually defined recursively. All languages are defined over some alphabet, say A (often you will see sigma, Σ, used for the alphabet). For an alphabet A, the regular expressions over this alphabet (or set) consist of the following:

- λ (the empty string; sometimes epsilon, ε, is used)
- Any character in A
- If X and Y are regular expressions over A, then so are the following, listed from highest operator precedence to lowest:

highest precedence	(X)	Parentheses are used to group subexpressions
↓	X*	Kleene star operator: zero or more repetitions of X
	XY	Concatenation
lowest precedence	X|Y	Alternation, OR (sometimes + is used)

Let's start with a simple alphabet A = {a, b, c}. The regular expression (a|b|c)* would generate all possible strings of a, b, and c. Inside the parens to build a string, you can choose an 'a' or 'b' or 'c' . The Kleene start says you can repeat zero or more times. So, you get any combination of the chars. The regular expression c (a|b|c)*c would generate all possible strings of a, b, and c that start and end in a 'c' . The shortest string is in the language is "cc" because you must choose a 'c' at the beginning and end. Between them, you can choose any of the characters 'a' or 'b' or 'c' .

Let's generate a regular expression for **unsigned binary numbers**. For this case, our alphabet A = {0, 1}.
Our expression is: (0|1) (0|1)*
The first term forces our string to start with a digit (note that the empty string is not in the language). The second term allows us to have as many digits, in any order, as we want.

Now generate a regular expression for **signed binary numbers** (the sign is optional for positive numbers.)
For this case, our alphabet is A = {+, -, 0, 1}. Our expression is: (+|-|λ) (0 |1) (0|1)*
We need the empty string (λ) in the first set of options to allow for positive numbers with an implied sign. Choosing λ in the first term essentially says to choose nothing. Then we start with a digit and more if desired. This could be written without λ as follows: (+|-) (0 1) (0|1)* | (0|1) (0|1)*

Suppose you wanted unsigned binary numbers, but wanted to disallow more than one leading zero. Now string such as 0000 or 000101 would not be in the language. Start with what you know that strings that start with one are no different than we've seen: 1(0|1)* Then think about starting with zero. In that case, force a one after the zero: 01(0|1)* Now the only string missing is one zero. The final answer is
1(0|1)* | 01(0|1)* | 0 which is equivalent to: (1|01)(0|1)* | 0
Or, another correct answer is: (11|01|10)(0|1)* | 0 | 1

All the languages we've seen so far are infinite languages (an infinite number of strings can be generated). All the keywords in a computing language are tokens in that language. Suppose the alphabet is all the characters that can be used to write a program. Then a keyword such as "while" is generated by the regular expression:
while
This is a finite language with exactly one string in it.

Let's do some more interesting regular expressions for computer languages. Let's write a regular expression for identifiers with a simplified alphabet of {a, b, c, 1, 2, 3, _}. First, write a regular expression for C-like identifiers. These follow the rules:

1. Must start with a letter
2. Then there is either a letter, number, or underscore

(a|b|c) (a|b|c|1|2|3|_)*

Now try Ada-like identifiers (language named after Ada Augusta, often called the first programmer):

1. Must start with a letter
2. Then there is either a letter, number, or underscore, but there cannot be two consecutive underscores

We will go over the solution in lecture.

There are housekeeping things to check when you generate a regular expression:

- Can I generate all of the strings in the language?
- Can I generate anything that isn't in the language?
- Boundary conditions warrant special consideration.

Often I ask: what is the shortest string in the language that my expression generates? Is the empty string in the language? Example boundary conditions to consider are: what can it start with? What can it end with?

For boundary conditions, often something is added to handle a special condition. For example, suppose you can generate all strings with what you have written, except you don't allow them to end is some character, say 'x' . You can fix up what you have by adding the term (x|λ) at the end of it. It is common to write a regular expression and then fix the boundaries..

Let's play a bit more with regular expressions. You start to get a feel for the expressiveness of regular expressions. You can't keep track of many things, for example, you can't count, but you can keep track of state. For example, whether a string is even or odd is state information.

Write a regular expression for the alphabet {a,b} for all strings that have an even number of a's. The logic is that there can be a 'b' anywhere, no restrictions, but every 'a' must have a *buddy* 'a' to keep it even
(b | aa)*. Close, but what about the string "aba" ? You must allow characters in between the paired a's:
(b | ab*a)*

Write a regular expression for strings that start with an 'a', end in 'b', and have an even number of a's in total.
ab*a(b | ab*a)*b

Now write the regular expression for strings that have an even number of a's and an even number of b's.
The a's and b's can be in any order, e.g., aabb, abaaab, ababbb, babaaaaababbab, aaaaaaaa, abbbbbbbba .

(aa | bb | (ab|ba)(aa|bb)*(ab|ba))*

The regular expression has three terms to allow for pairs of a's, pairs of b's, or mixed up a's and b's. To build a string, allow "aa" or "bb" anywhere. To make sure the a's and b's that are mixed up have a buddy 'a' or 'b' force each "ab" or "ba" to have a matching "ab" or "ba" so the count always remains even.

To be sure all possible strings are generated, allow "aa"s and "bb"s to come between mixed up 'a' and 'b' terms. While this regular expression is hard to come up with, once you see it, it has nice symmetry. I was given a similar problem – alphabet {a,b,c}, come up with a regular expression for even a's, even b's, and even c's. It is not small and elegant like the one above, but is excessively long and not at all interesting.

Now write a regular expression for all strings that are of the form $a^n b^n$, in set notation $\{ a^n b^n \mid \text{integer } n \geq 0\}$, for example: λ ab aabb aaabbb aaaabbbb aaaaabbbbb

It doesn't take you long to decide this is difficult. In fact, this is a trick question as this language is not regular. There is no regular expression for $a^n b^n$. The language $a^n b^m$, where the number a's and b's don't have to necessarily be the same is generated by the regular expression a*b*, but there is no way to count characters.