

HW # 2

1. General equation

$$F_{in} - F_{out} + \text{Generation} = \text{accumulation}$$

$$\text{Generation} = (-r_A) \Delta V (1-\epsilon)$$

$$F_{A,V} - F_{A,V+\Delta V} - r_A \Delta V (1-\epsilon) = 0$$

$$\Rightarrow \boxed{\frac{dF_A}{dV} = -r_A (1-\epsilon)}$$

2. $F_i - F_{out} + G = \frac{dN_A}{dt}$

$$F_i = 0$$

$$G_j = 2.5 \times 10^{-7} \text{ mole/m}^2 \text{ s}$$

$$C_{0, \text{stink}} = 5 \times 10^{-3} \text{ mole/m}^3$$

$$v_0 = 0.1 \text{ m}^3/\text{s}$$

$$F_{out} = v_0 C_{s, out}$$

$$V = 5 \times 8 \times 2.5 = 100 \text{ m}^3$$

$$A = 5 \times 8 \text{ m}^2$$

(a) steady-state

$$\cancel{F_i} - F_{out} + G = \cancel{\frac{dN_A}{dt}}$$

$$-v_0 C_{s, out} + G_j A = 0$$

$$-0.1 \times C_{s, out} + 2.5 \times 10^{-7} \times 40 = 0$$

$$\boxed{C_{s, out} = 10^{-4} \text{ mole/m}^3}$$

(b) not steady state

$$\cancel{F_i} - F_{out} + G = \frac{dN_A}{dt}$$

$$-F_{out} + G = \frac{dC_s V}{dt}$$

$$\frac{-v_0 C_s + G_j A}{V} = \frac{dC_s}{dt}$$

$$-\frac{v_0}{V} dt = \frac{dC_s}{\left(C_s - \frac{G_j A}{v_0}\right)}$$

$$\int_0^{3600} (-10^{-4}) dt = \int_{C_{s,0}}^{C_s} \frac{dC_s}{(C_s - 10^{-7})}$$

$$\boxed{C_s = 2.34 \times 10^{-4} \text{ mole/m}^3}$$

3. $A \rightarrow B$

$$X = 0.99$$

$$F_{A0} = 5 \text{ mol / h}$$

$$\nu_0 = 10 \text{ dm}^3 / \text{h}$$

$$C_{A0} = 0.5 \text{ mole / dm}^3$$

$$\text{For CSTR } V = \frac{F_{A0}X}{-r_A}$$

$$\text{For PFR } \frac{dF_A}{dV} = r_A$$

$$F_A = F_{A0}(1-X) \rightarrow dF_A = -F_{A0}dX$$

$$F_{A0} \frac{dX}{dV} = -r_A$$

$$(a) -r_A = k \quad k = 0.05$$

0

$$\text{For CSTR } V = \frac{5 \times 0.99}{0.05} = \boxed{99 \text{ dm}^3}$$

$$\text{For PFR } 5 \int_0^{0.99} \frac{dX}{0.05} = \int dV$$

$$\boxed{V = 99 \text{ dm}^3}$$

$$(b) -r_A = kC_A \quad k = 0.0001 \text{ s}^{-1}$$

$$\text{For CSTR } V = \frac{F_{A0}X}{-r_A} = \frac{F_{A0}X}{kC_A} = \frac{\nu_0 C_{A0}X}{kC_{A0}(1-X)} = \frac{\nu_0 X}{k(1-X)} = \boxed{2750 \text{ dm}^3}$$

$$\frac{dF_A}{dV} = -F_{A0} \frac{dX}{dV} = r_A = -kC_A$$

$$\text{For PFR } \nu_0 C_{A0} \frac{dX}{dV} = kC_{A0}(1-X)$$

$$\frac{\nu_0}{k} \int_0^{0.99} \frac{dX}{(1-X)} = \int dV$$

$$V = \boxed{127.92 \text{ dm}^3}$$

$$(c) -r_A = kC_A^2 \quad k = 3 \text{ dm}^3/\text{mol h}$$

$$\text{For CSTR } V = \frac{F_{A0}X}{-r_A} = \frac{\nu_0 C_{A0}X}{kC_{A0}^2(1-X)^2} = \frac{\nu_0 X}{kC_{A0}(1-X)^2} = \boxed{66000 \text{ dm}^3}$$

For PFR

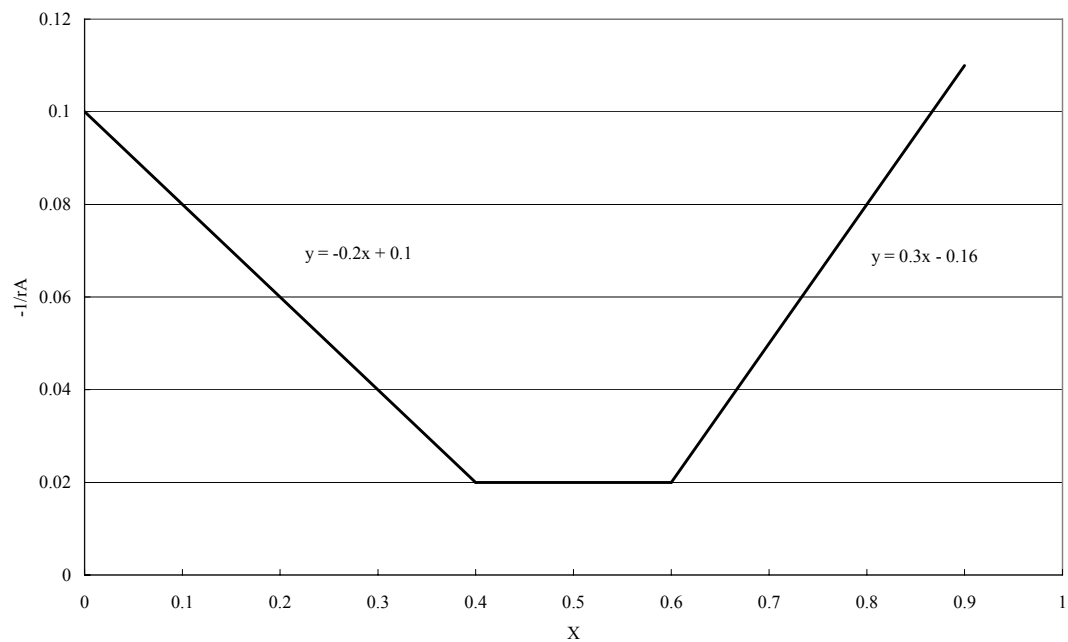
$$\frac{dF_A}{dV} = -F_{A0} \frac{dX}{dV} = r_A = -kC_A^2 = -kC_{A0}^2(1-X)^2$$

$$\frac{v_0}{kC_{A0}} \frac{dX}{(1-X)^2} = dV$$

$$\boxed{V = 660 \text{ dm}^3}$$

4. (for 3rd edition)

| | | | | | | |
|------------------|-----|-------|------|------|------|------|
| X | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 0.9 |
| r _A | 10 | 16.67 | 50 | 50 | 12.5 | 90.9 |
| 1/r _A | 0.1 | 0.06 | 0.02 | 0.02 | 0.8 | 0.11 |



(a)

$$V_{\text{CSTR}} = \frac{F_{A0}X}{-r_A} = \frac{300 \times 0.4}{50} = \boxed{2.4 \text{ dm}^3}$$

$$V_{\text{PFR}} = F_{A0} \int_0^X \frac{dX}{-r_A} = 300 \times \frac{(0.1 + 0.02)}{2} \times 0.4 = \boxed{7.2 \text{ dm}^3}$$

(b) Between 0.4 ~ 0.6, because the rate is constant over this conversion range.

(c) $V_{\text{CSTR}} = 10.5 \text{ dm}^3$

$$= \frac{F_{A0}X}{-r_A} = 300 \times \frac{X}{-r_A}$$

$$\Rightarrow \frac{X}{-r_A} = 0.035$$

You can trial and error until $\frac{X}{-r_A} = 0.035$, or just calculate area from the plot.

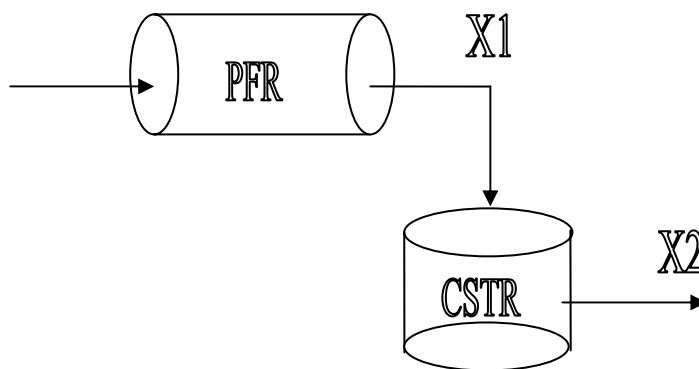
From $X=0.4$ $V_1 = 2.4$

$X=0.6$ $V_2 = 3.6$

$$10.5 = 300 X (0.3X - 0.16)$$

Maximum conversion $X = 0.7$

(d)

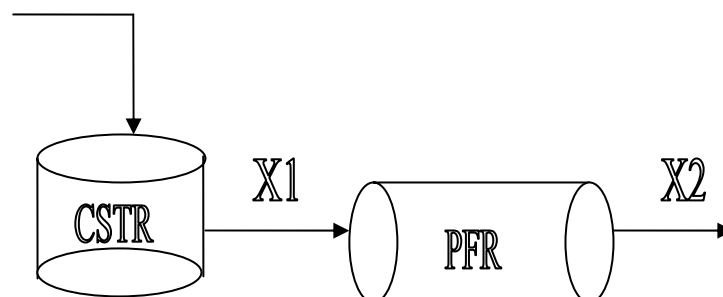


From part (a) we know $X_1 = 0.4$

$$V_{CSTR} = 2.4 = 300(X_2 - 0.4)(0.3X_2 - 0.16)$$

$X = 0.64$

(e)

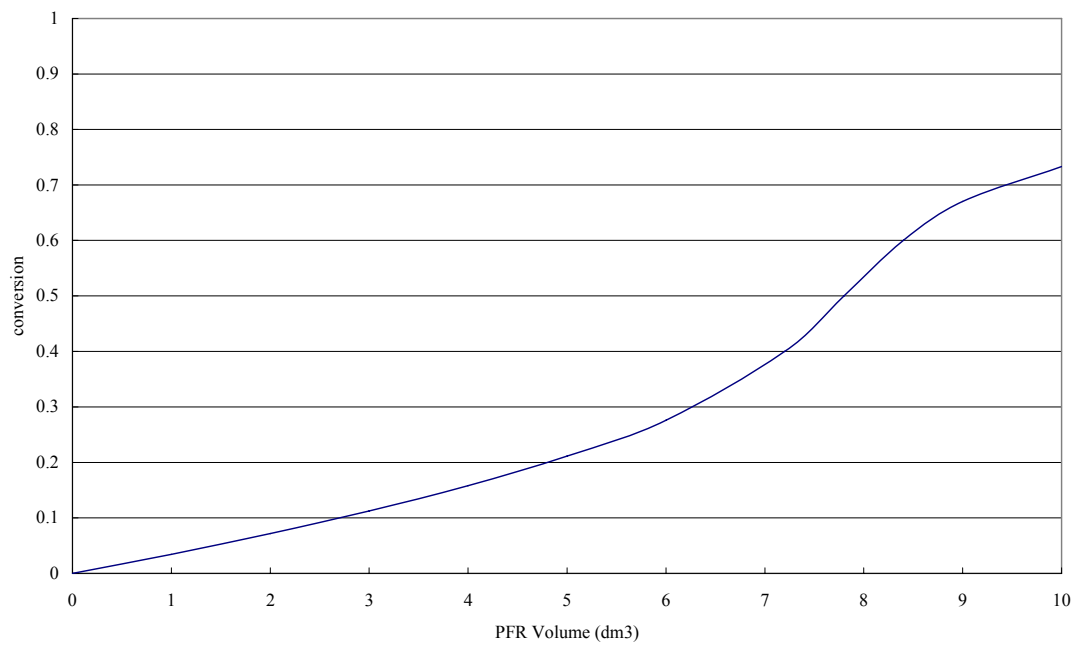
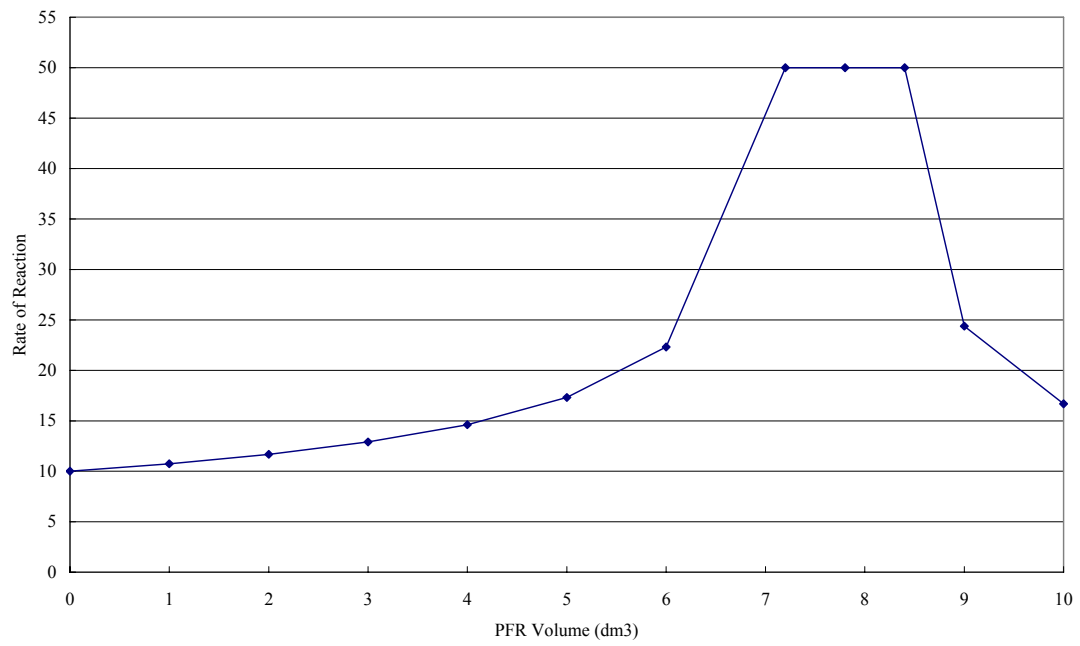


From part (a) we know $X_1 = 0.4$

$$V_{\text{PFR}} = 7.2 = F_{A0} \int_0^X \frac{dX}{-r_A} = 300 \int_{0.4}^{X_2} \frac{dX}{-r_A} = 1.2 + 300 \int_{0.6}^{X_2} (0.3X - 0.16) dX$$

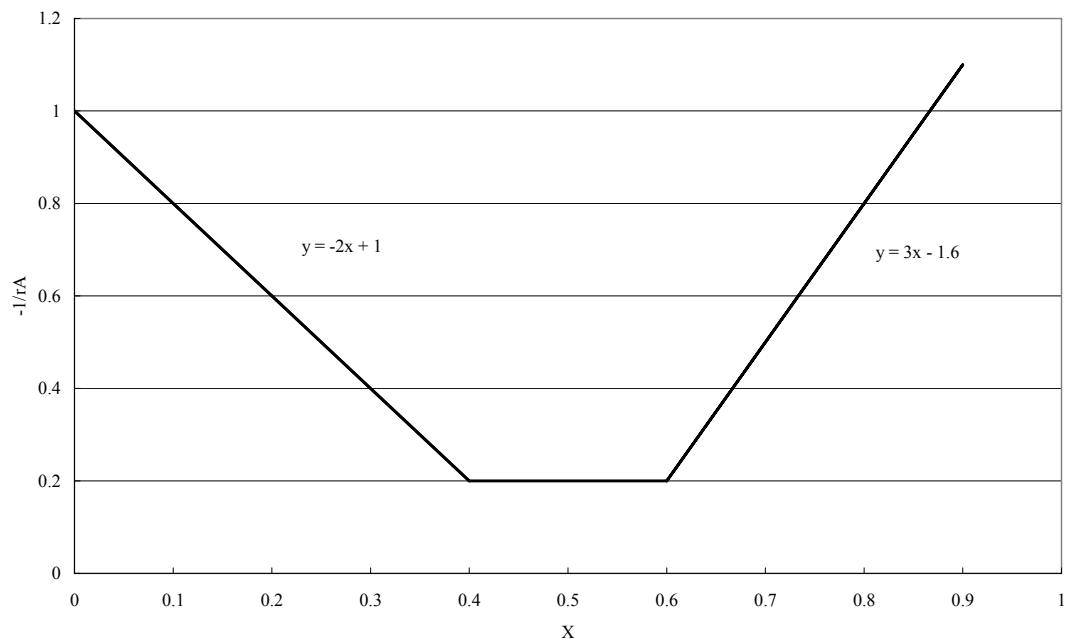
$$X_2 = 0.905$$

(f)



4. for 4th edition

| | | | | | | |
|---------|---|------|-----|-----|------|------|
| X | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 0.9 |
| r_A | 1 | 1.67 | 5.0 | 5.0 | 1.25 | 0.91 |
| $1/r_A$ | 1 | 0.6 | 0.2 | 0.2 | 0.8 | 1.1 |



(a)

$$V_{\text{CSTR}} = \frac{F_{A0} X}{-r_A} = \frac{300 \times 0.4}{5} = \boxed{24 \text{ dm}^3}$$

$$V_{\text{PFR}} = F_{A0} \int_0^X \frac{dX}{-r_A} = 300 \times \frac{(1 + 0.2)}{2} \times 0.4 = \boxed{72 \text{ dm}^3}$$

(b) Between 0.4 ~ 0.6, because the rate is constant over this conversion range.

(c) $V_{\text{CSTR}} = 10.5 \text{ dm}^3$

$$= \frac{F_{A0} X}{-r_A} = 300 \times \frac{X}{-r_A}$$

$$\Rightarrow \frac{X}{-r_A} = 0.035$$

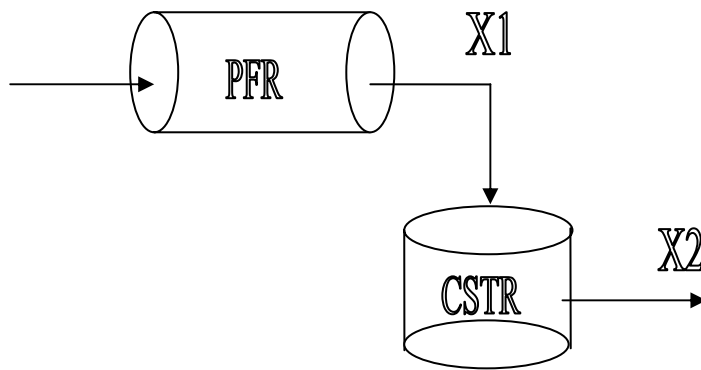
You can trial and error until $\frac{X}{-r_A} = 0.035$, or just calculate area from the plot.

From $X=0.4$ $V_1 = 24$

$$10.5 = 300 X (-2X+1)$$

Maximum conversion $\boxed{X = 0.0378}$

(d)

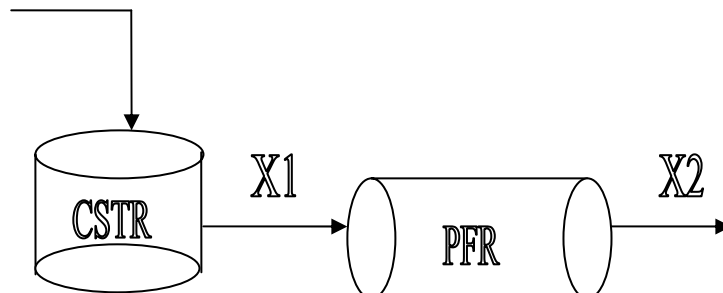


From part (a) we know $X_1 = 0.4$

$$V_{\text{CSTR}} = 24 = 300(X_2 - 0.4)(3X_2 - 1.6)$$

$$\boxed{X_2 = 0.64}$$

(e)

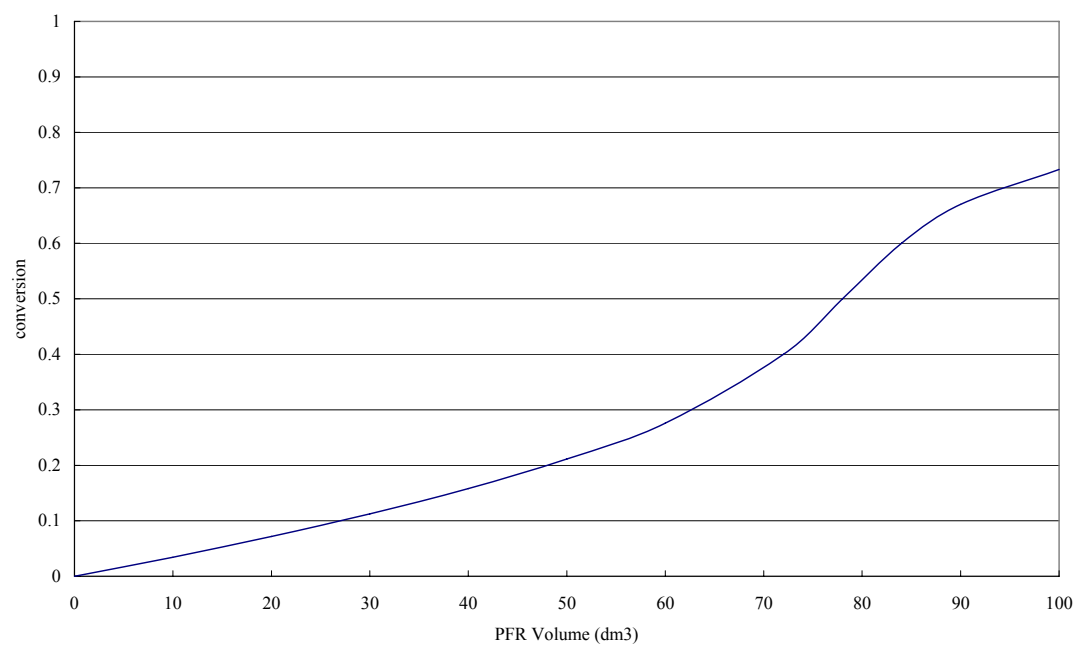
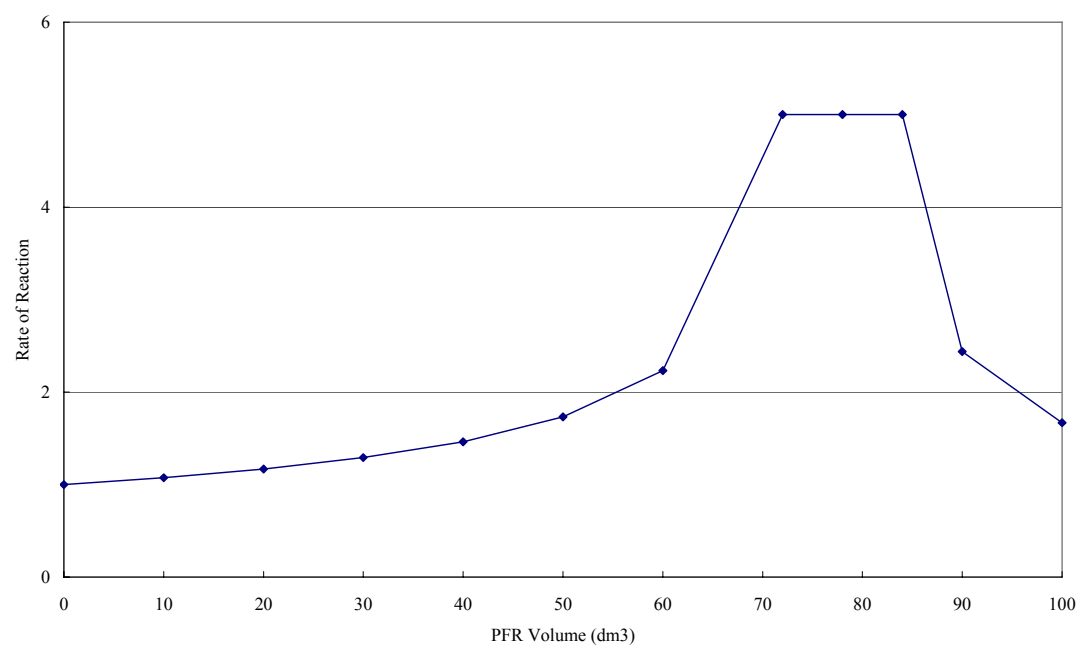


From part (a) we know $X_1 = 0.4$

$$V_{\text{PFR}} = 72 = F_{A0} \int_0^{X_2} \frac{dX}{-r_A} = 300 \int_{0.4}^{X_2} \frac{dX}{-r_A} = 12 + 300 \int_{0.6}^{X_2} (3X - 1.6) dX$$

$$\boxed{X_2 = 0.905}$$

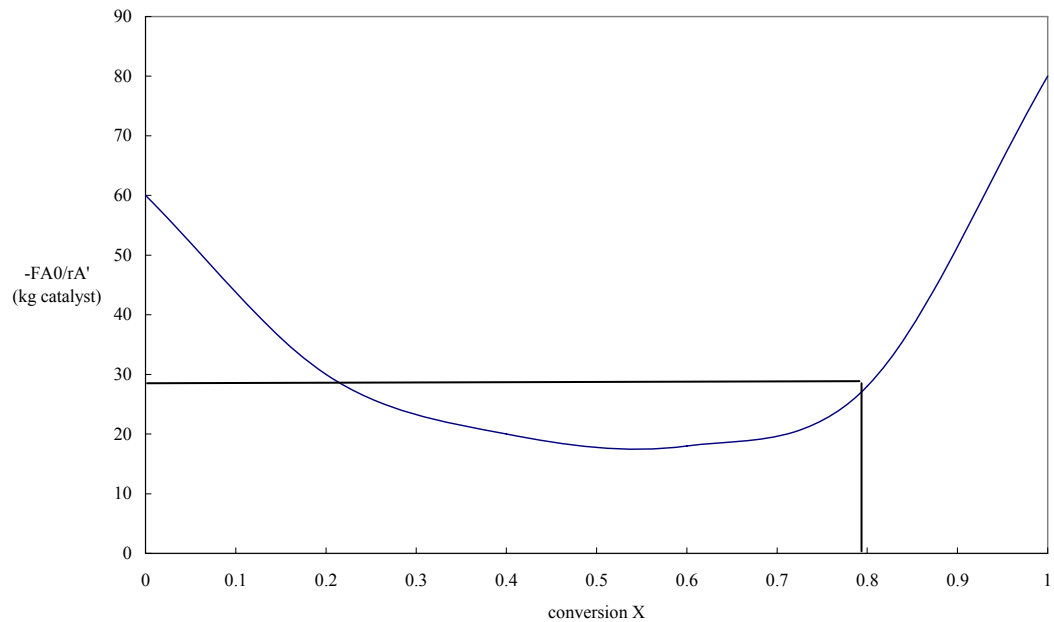
(f)



5.

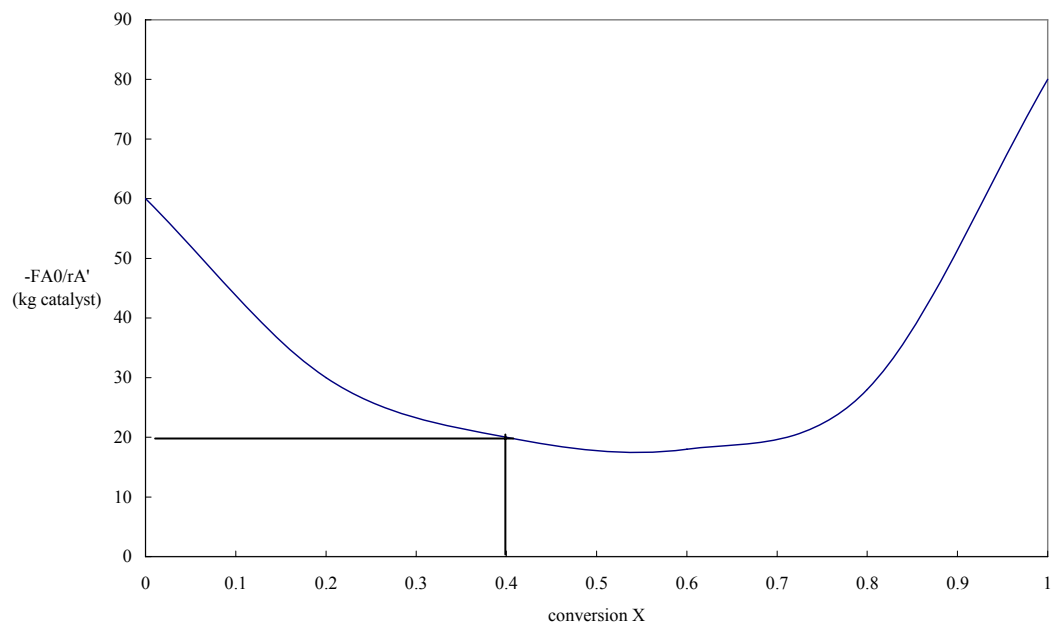
(a) CSTR is used before PFR.

(b) One can calculate the amount of catalyst needed to carry out the same reaction to 80 % conversion using a single CSTR by determining the area of the rectangle.



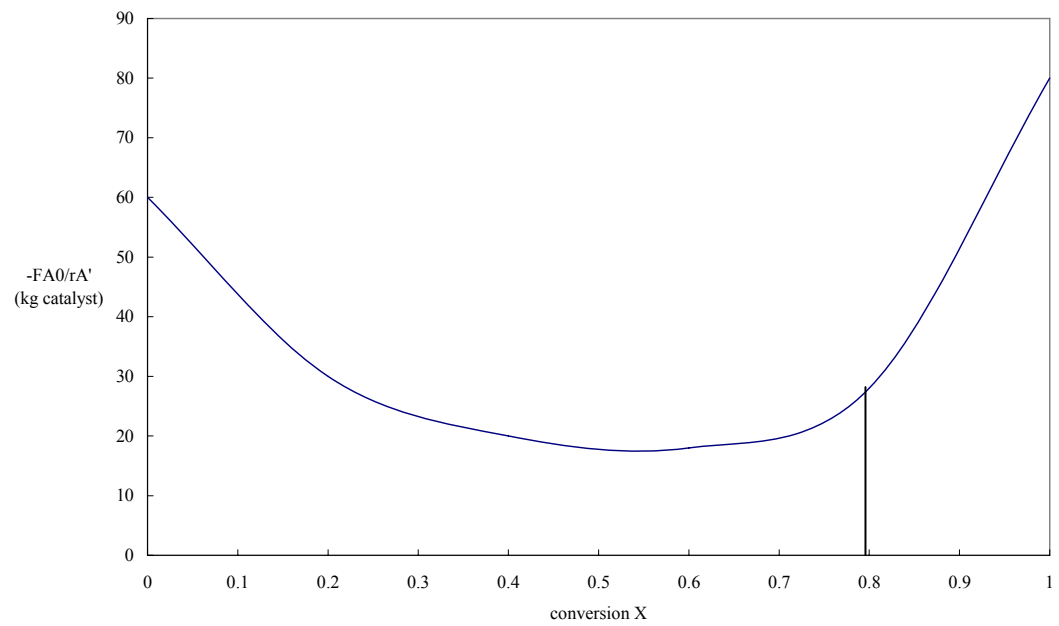
approximately 22.4 kg catalyst.

(c) for 40 % conversion



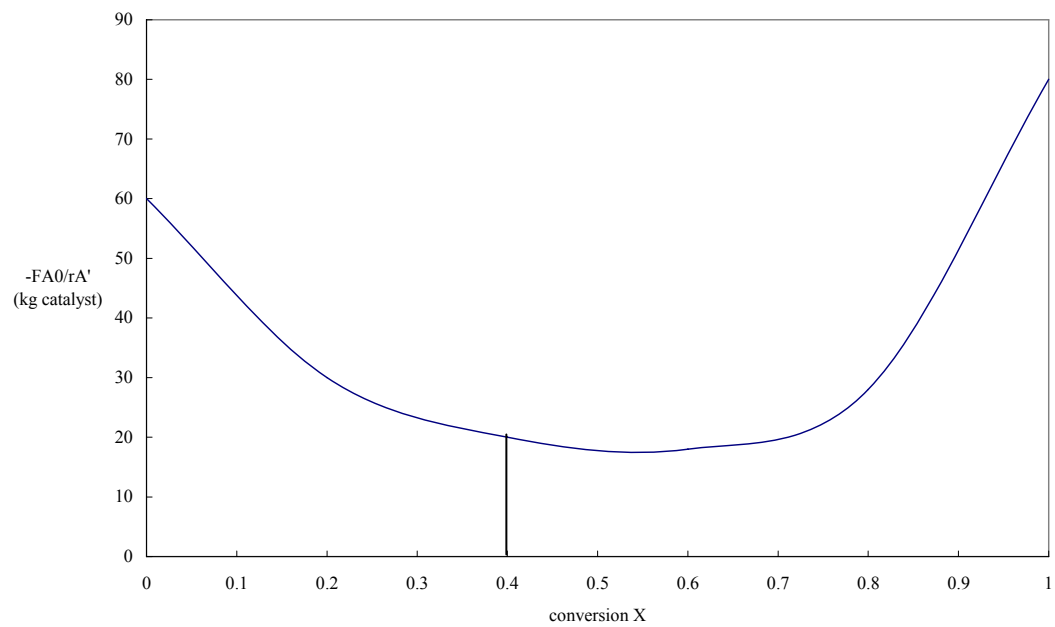
→ 8 kg catalyst

(d)



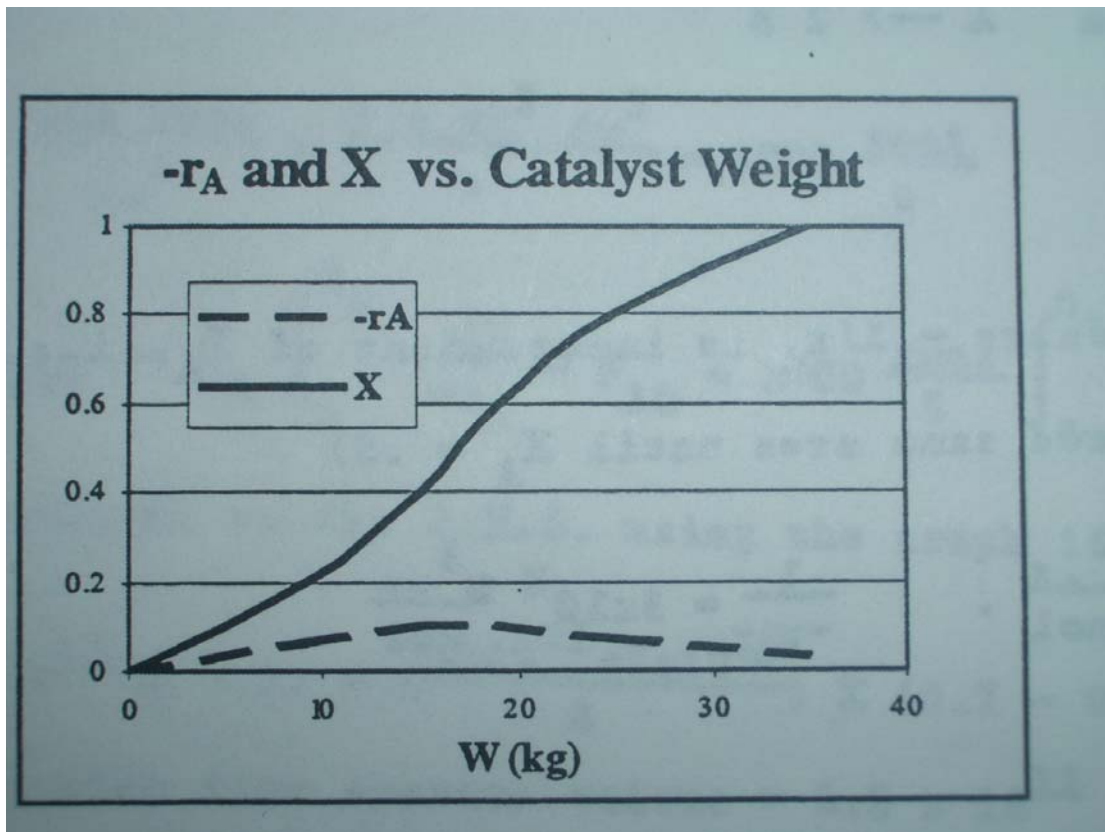
Calculate the area under the curve between 0 and 0.8. Approximately 30 kg

(e)



Calculate the area under the curve between 0 and 0.4. Approximately 15 kg

(f)



(g) For different $(-r_A)$ vs. (X) curves, reactors should be arranged so that the smallest amount of catalyst is needed to give the maximum conversion. This can be done by minimizing the area that is occupied by a given reactor. One useful heuristic is that for curves with a negative slope, it is generally better to use a CSTR. Similarly, when the curve has a positive slope, it is generally better to use a PFR.