# Chem E 465 <br> Reactor Design 

## EXAM 1

Open book, open notes. Calculator allowed, but not wireless or IR transmitting device.
Please do your work directly on the exam. An extra page is included at the end.

## Problem 1 ( 30 Pts )

Both parts of this problem refer to the inverse rate versus conversion data plotted below.
(a) What is the minimum reactor volume required to achieve $80 \%$ conversion for a feed stream with 50 moles/sec of species A flowing. (15 pts)

The minimum volume for the reaction is given by the $\mathrm{F}_{\mathrm{A}, 0} *$ [area under the curve]
$\mathrm{V}=50 \mathrm{~mole} / \mathrm{s} *[0.4 * 2$ liter $-\mathrm{s} / \mathrm{mole}+(0.8-0.4) * 3$ liter $-\mathrm{s} / \mathrm{mole}]=50 *[0.8+1.2]$ liters $=100$ liters
(b) There are a few different CSTR and/or PFR reactor options (either alone or with two reactors in series) that can achieve $80 \%$ conversion at this minimum total reactor volume. Please list all the different reactor options that can produce $80 \%$ conversion in the minimum volume (note the type of reactor/reactors and the range of conversion it/they should operate over). (15 pts)
(1) A single PFR that runs up to $X=0.8$
(2) A CSTR that runs to $X=0.4$ followed by a PFR that runs from $0.4<X \leq 0.8$
(3) Some students may also say two PFRs, where the first one runs to $\mathrm{X}=0.4$ followed by a second PFR that runs from $0.4<\mathrm{X} \leq 0.8$


## Problem 2 ( 35 Pts)

Fish produce bodily wastes that builds-up in their tank, making their environment toxic. Studies show that fish eat less (and produce less bodily waste) as the waste becomes more concentrated in the tank. The rate that one fish produces waste $\left(\mathrm{r}_{\mathrm{w}}\right)$ is given by the expression

$$
\mathrm{r}_{\mathrm{w}}(\mu \mathrm{~mole} / \text { day } / \mathrm{fish})=100\left[1-\mathrm{k} \mathrm{C}_{\mathrm{w}}\right],
$$

where $\mathrm{C}_{\mathrm{w}}$ ( $\mu$ mole/liter) is the concentration of accumulated body waste in the tank and k (liter $/ \mu$ mole) is a constant that tells how sensitive the particular fish species is to living in its own waste.
(a) Write the appropriate mole balance equation that describes how waste accumulates in the fish tank. State any assumptions. ( 15 pts )

The fish tank is taken to be a constant volume batch reactor that is well mixed by the swimming fish. Since the rate of generation is given per fish, not per liter, we need to multiply the rate by the number of fish per liter, so
$\qquad$
where N is the number of fish in the tank, and V is the tank volume.
(b) For goldfish, the lethal concentration of body wastes is $\mathrm{C}_{\mathrm{W}}=200 \mu \mathrm{~mole} /$ liter, at which point they die and cease to produce any more waste. What is k for goldfish? ( 5 pts )

Since they produce no more waste at the lethal concentration, we set $r_{w}=0$ and solve $r_{w}=0=100\left[1-k^{*} 200 \mu\right.$ mole/liter], or $k=0.005$ liter $/ \mu$ mole for goldfish.
(c) A good fish owner will change the water in the fish tank when the concentration of waste is $50 \%$ of the lethal concentration. If you have 2 goldfish in a 10 liter tank, how often should you change the water? ( 15 pts )
$\mathrm{dC}_{\mathrm{w}} / \mathrm{dt}=100\left[1-0.005 * \mathrm{C}_{\mathrm{w}}\right] * 2 / 10=(20 \mu \mathrm{~mol} /$ liter-day $)-\left(0.1 \mathrm{day}^{-1}\right)^{*} \mathrm{C}_{\mathrm{w}}$
SOLVE the 1 st order ode using an integrating factor $\quad \mathrm{d} / \mathrm{dt}\left\{\mathrm{C}_{\mathrm{w}} * \mathrm{e}^{0.1 \mathrm{t}}\right\}=20 * \mathrm{e}^{0.1 \mathrm{t}}$
$\mathrm{C}_{\mathrm{w}}=200+\mathrm{K} * \mathrm{e}^{-0.1 \mathrm{t}} \quad$ where K is an integration constant
Use the initial condition $\mathrm{C}_{\mathrm{w}}=0$ when $\mathrm{t}=0$, to find $\mathrm{K}=-200 \mu \mathrm{~mol} / \mathrm{liter}$, and thus
$\mathrm{C}_{\mathrm{w}}=200 *\left(1-\mathrm{e}^{-0.1 \mathrm{t}}\right) \quad \mu \mathrm{mole} / \mathrm{liter}$

The time to $50 \%$ lethal dose $\left(\mathrm{C}_{\mathrm{w}}=100 \mu\right.$ mole/liter $)$ is $100=200 *\left(1-\mathrm{e}^{-0.1 \mathrm{t}}\right)$
$t=-(1 / 0.1) \ln (0.5)=6.9$ days (about once per week)

## Problem 3 (35 Pts)

We are interested in carrying out the elementary gas phase reaction

$$
\mathrm{A}(\mathrm{~g})+\mathrm{B}(\mathrm{~g}) \rightarrow \mathrm{C}(\mathrm{~g})
$$

in a continuously stirred tank reactor. Unfortunately, species A is too dangerous to pump and handle as a gas, but is much less dangerous as a liquid. To get around this problem, we pump species A as a liquid into the reactor, and allow it to evaporate to form gaseous A that can react with B, see Figure below.

How much liquid $\mathrm{A}\left(\mathrm{F}_{\mathrm{A}, 0}\right)$ must be pumped into the reactor at steady state for any given exit conversion $\mathrm{X}_{\mathrm{B}}$ ?

## You may need some of this data:

$\mathrm{F}_{\mathrm{B}, 0}=1 \mathrm{~mole} / \mathrm{s}, \mathrm{P}_{\mathrm{T}}=1 \mathrm{~atm}, \mathrm{P}_{\mathrm{vap}, \mathrm{A}}=0.2 \mathrm{~atm},$.

Thermodynamics dictates the vapor-phase mole fractions
$\mathrm{y}_{\mathrm{A}}=\mathrm{P}_{\text {vap }, \mathrm{A}} / \mathrm{P}_{\mathrm{T}}=0.2=\mathrm{F}_{\mathrm{A}} / \mathrm{F}_{\mathrm{T}}$


So now we can find the total moles flowing in the gas phase using this data, definitions for $\mathrm{F}_{\mathrm{T}}$, and the stoichiometry of the reaction:
$\mathrm{F}_{\mathrm{T}}=\mathrm{F}_{\mathrm{A}}+\mathrm{F}_{\mathrm{B}}+\mathrm{F}_{\mathrm{C}}=0.2 \mathrm{~F}_{\mathrm{T}}+\mathrm{F}_{\mathrm{B}}+\mathrm{F}_{\mathrm{C}}$
Thus, $0.8 \mathrm{~F}_{\mathrm{T}}=\mathrm{F}_{\mathrm{B}}+\mathrm{F}_{\mathrm{C}}=\mathrm{F}_{\mathrm{B}, 0}\left(1-\mathrm{X}_{\mathrm{B}}\right)+\mathrm{F}_{\mathrm{B}, 0} \mathrm{X}_{\mathrm{B}}$
resulting in the total molar flow being $\mathrm{F}_{\mathrm{T}}=1.25 \mathrm{~F}_{\mathrm{B}, 0}$
The total molar flow of A leaving the reactor is thus,
$\mathrm{F}_{\mathrm{A}}=0.2 \mathrm{~F}_{\mathrm{T}}=0.25 \mathrm{~F}_{\mathrm{B}, 0}$
So, to find the inlet amount of A we can use the reaction stoichiometry and definition of conversion
$\mathrm{F}_{\mathrm{A}}=\mathrm{F}_{\mathrm{A}, 0}-\mathrm{F}_{\mathrm{B}, 0} \mathrm{X}_{\mathrm{B}}$
Insert $\mathrm{F}_{\mathrm{A}}$ into this expression to get,
$0.25 \mathrm{~F}_{\mathrm{B}, 0}=\mathrm{F}_{\mathrm{A}, 0}-\mathrm{F}_{\mathrm{B}, 0} \mathrm{X}_{\mathrm{B}}$
Rearrange to find the inlet A flow needed to balance A that leaves in the gas and A that reacts
$\mathrm{F}_{\underline{\mathrm{A}}, 0}=\mathrm{F}_{\underline{B}, 0}\left(0.25+\mathrm{X}_{\underline{B}}\right)=(1 \mathrm{~mol} / \mathrm{s})^{*}\left(0.25+\mathrm{X}_{\underline{B}}\right)$

Thus, if we want to add liquid to the tank, and keep it operating at steady state, we need the flow to exactly match what we've calculated, other wise it will slowly fill up with liquid (if we flow more in) or completely dry-out (if we flow less in).

