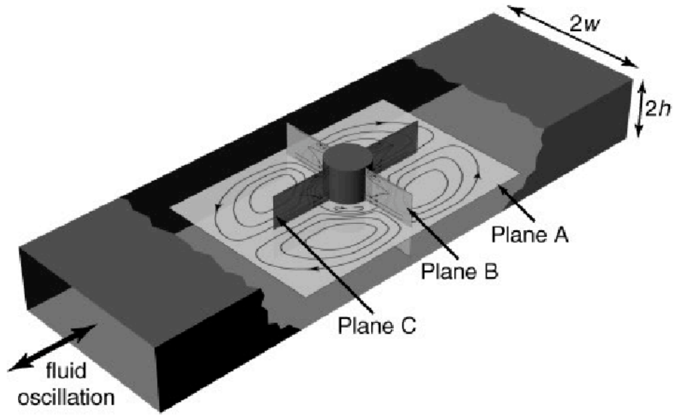
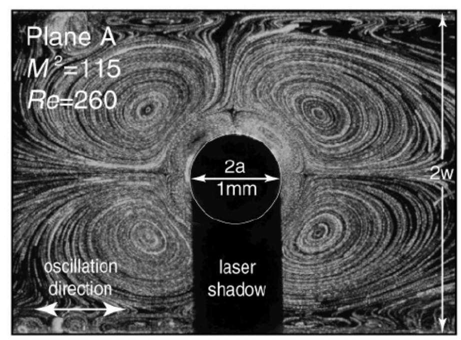
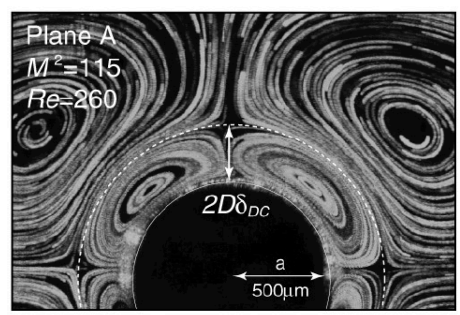
Project Description Chem E 530

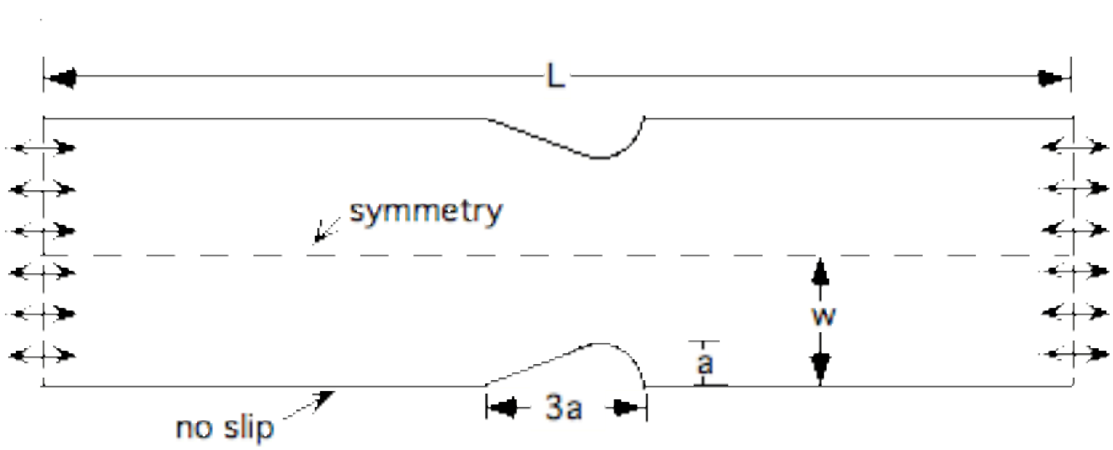


On the first day of class, you saw a microfluidic device like that shown schematically, right. Far upstream, the fluid was driven as a pure sinusoidal oscillation in a rectangular duct. As you've seen from homework, at high enough frequencies, the oscillating duct flow looks like a planar acoustic wave. In the vicinity of an obstruction (a cylinder, here), the fluid oscillations drive a steady recirculating flow in the vicinity of the vertical cylinder. The flow is not unidirectional anymore. Most of our interest is focused on the flow that occurs in Plane A (see schematic). The experimentally observed steady recirculating flow in Plane A is shown immediately to the right. The flow is seen to be symmetric, with four recirculating eddies near the cylinder, and four larger eddies further from the cylinder. The flow we see in Plane A is, for all practical purposes, identical to the flow we predict if the cylinder is infinitely long. Thus, we can model this flow as a 2-D problem in the plane marked A.





The geometry I would like you to solve for is shown below. Instead of placing a cylinder in the middle of the flow stream, we putting half a "tear drop" on each of the walls. The dimensions of this device are going to be: width w=2.5 mm, the length L=10 mm, and "tear drop" height a=0.5 mm. We take x to be in the direction of L and y to be in the direction of w.



Your job is to model the oscillating and steady flows generated by a pure, planar cosine oscillation at the entrance and exit of the channel. This two dimensional oscillating flow will have a velocity of the form:

(1) $$\mathbf{V} = U(x,y,t)\,\mathbf{e_x} + V(x,y,t)\,\mathbf{e}_y$$

where bold characters denote vectors. The entrance and exit conditions are pure oscillations of the form:

(2) $$\mathbf{V} = s\omega * \cos(\omega t)\,\mathbf{e_x}\,,$$

where s is the oscillation displacement amplitude (mm) and ω is the oscillation frequency (radians/s).

The Navier-Stokes and continuity equations should be nondimensionalized using the lengths ξ = x/a and η = y/a, time $\tau = \omega t$, velocities $u = U/\omega a$, $v = V/\omega a$, and pressure $P = p/\rho\omega^2 a^2$.

TASK 1 (Monday, Dec. 1): Non-dimensionalize the x- and y-components of the Navier-Stokes Equations and continuity equation, and check results. You should get a dimensionless parameter we call $M^2 = a^2\omega/\nu$, where ν is the kinematic viscosity.

TASK 2 (Monday, Dec. 1) Determine the appropriate boundary conditions, and their non-dimensional form.

We are going to use small amplitude fluid oscillations, such that the parameter ε=s/a is much less than unity ($\varepsilon \ll 1$). In this case, we can explicitly separate the oscillating time-dependent terms from the steady state terms using a regular perturbation expansions for the dimensionless velocity and pressure:

(3a,b,c)
$$\begin{aligned}
u &= \varepsilon(uc\cdot\cos\tau + us\cdot\sin\tau) + \varepsilon^2 ust + O(\varepsilon^3)\\
v &= \varepsilon(vc\cdot\cos\tau + vs\cdot\sin\tau) + \varepsilon^2 vst + O(\varepsilon^3)\\
P &= \varepsilon(Pc\cdot\cos\tau + Ps\cdot\sin\tau) + \varepsilon^2 Pst + O(\varepsilon^3)
\end{aligned}$$

Here, all the oscillating and steady velocities and pressure coefficients are assumed to be functions of (x,y). It is interesting to note that the steady flow is a *secondary* flow that is much smaller than the oscillating flow terms (steady flow effects show up as ε^2 terms).

TASK 3 (by Tuesday, Dec. 2) Insert Eqs. (3a-b) into the Navier-Stokes equations and determine the *O(ε)* equations for the in-phase and out-of-phase oscillating components of flow, and the *O(ε^2)* equations for the steady components of flow. You will need to have a handbook of trigonometric identities handy, since you will come across things like $\cos^2(\omega t)$ which is equal to $0.5 + 0.5*\cos(2\omega t)$. Ignore the *O($\varepsilon^2$)* oscillating flow. Do the same for the boundary conditions. Double check your results!

You should now have six coupled $O(\square)$ equations that are time-independent, plus boundary conditions, and 3 coupled $O(\square^2)$ steady flow equations.

TASK 4: Come to class on Wednesday, Dec. 3, ready to show the equations and boundary conditions you have derived. Any team with very close-to-correct equations and boundary conditions will get a tutorial from Haixia on how to code these equations into FEMLAB.

TASK 5: Come to class on Friday, Dec. 5 with a base-case solutions for $O(\square)$ and $O(\square^2)$ equations using the geometry given above and the parameter M^2=115. Your base-case solution should also have verification of the solution quality using grid-refinement techniques that show convergence. Those groups who have a suitable base-case solution by Friday or Monday (Dec. 8) will be given a range of geometric or frequency (M^2) variables to explore in order to have a more interesting and complete study.

TASK 6: Written reports will be due on the date of our final exam, Friday, Dec. 12 by 4 pm in the 3rd floor Chem E office. Do not underestimate the time necessary to write. For certain, at least one group member needs to be writing material up by Wednesday, Dec. 10.