

Monopolistic Competition

Econ 502, Fall 2004

The model

- 1) A continuum of households on the unit interval $[0, 1]$ each produces one type of differentiated good, i.e. goods that are imperfect substitutes for each other (e.g. think Dell laptop vs. say Apple.)
- 2) Each household i enjoys a consumption good basket C_i which consists of all goods ("love for variety")
- 3) Money is needed for transactions, so real money holding provides utility. (This is a convenient way to introduce money in the system. You will later see alternative approaches such as through the introduction of a cash-in-advance constraint.)
- 4) As goods are imperfect substitutes, each household producer acts like a monopolist given the demand function for the good (which in turn is affected by other producers' behavior. This fact makes the setup different from a typical monopoly situation, and is called "Monopolistic Competition"). This is an important feature of the model, as it gives the price-setting decision to individual firms.
- 5) To simplify things, we assume away a separate labor market but let each household produces its good using its own labor. Labor (work) causes disutility.

Formally:

Let the utility function of household i be:

$$U(C_i, \frac{M_i}{P}, L_i) = (\frac{C_i}{\alpha})^\alpha (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{L_i^\beta}{\beta}$$

where consumption index C_i and price index P are:

$$C_i = [\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)}$$

$$P = [\int_0^1 P_j^{1-\sigma} dj]^{1/(1-\sigma)}$$

This type of consumption function is called the Constant Elasticity of Substitution (CES) utility. The Elasticity of Substitution (b/w good variety j vs k as their relative price changes) is σ . If σ is large, then goods are close substitutes. Note also that σ must be greater than 1 to

guarantee a meaningful equilibrium. (Convince yourself that if $\sigma < 1$, each producer would produce nothing and charge a price of infinity.)

The consumer's budget constraint is:

$$\int_0^1 P_j C_{ij} dj + M_i = P_i Y_i + \bar{M}_i$$

where \bar{M}_i is initial money holding of household i. We can further assume a simple production function: $Y_i = L_i$

There are quite a few advantages to this setup:

Solution Method:

A convenient way to solve the model is to divide the solution procedure into three separate steps:

- 1) Let household i's consumption spending be X_i , and solve for its consumption demand for each good.
- 2) Household decides how to allocate between consuming and holding money
- 3) Consider household producer's production and pricing decisions, given the demand for their good.

1) Household i chooses all C_{ij} to maximize aggregate consumption basket

$$C_i = \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)} \text{ s.t. } \int_0^1 P_j C_{ij} dj = X_i$$

Note the following fact (it is easy to prove):

If $g(\cdot)$ is a strictly increasing positive transformation of another function $f(x)$, then if x^* solves $\max_x f(x)$, then x^* also solves $\max_x g(f(x))$

so, the maximization problem can be simplified to: $\max \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj \right] \text{ s.t. } \int_0^1 P_j C_{ij} dj = X_i$

(this simplification is certainly not necessary; it is not that much more complicated to keep the exponents around.)

FOC with respect to C_{ij} from the Lagrangian: $L = \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj \right] + \lambda \left[X_i - \int_0^1 P_j C_{ij} dj \right] :$

$$\frac{\sigma-1}{\sigma} C_{ij}^{-1/\sigma} = \lambda P_j \text{ for all } j$$

so: $\frac{C_{ij}}{C_{ik}} = \left(\frac{P_j}{P_k} \right)^{-\sigma}$ (this result shows that the elasticity of substitution between any

two goods j and k is indeed constant at σ .)

Note the above can be re-written as: $(\frac{C_{ij}}{C_{ik}})^{(\sigma-1)/\sigma} = (\frac{P_j}{P_k})^{1-\sigma}$

Integrate over all j on both sides and use the definition for C_i and price index P:

$$(\frac{C_i}{C_{ik}})^{(\sigma-1)/\sigma} = (\frac{P}{P_k})^{1-\sigma} \quad \text{so } C_{ik} = (\frac{P_k}{P})^{-\sigma} C_i \text{ for all variety } k$$

It is also easy to verify that $C_i P = X_i$ (i.e. multiply the above equation by P_k and integrate over all k). Note also that X_i is the minimum expenditure needed such that aggregate consumption level is C_i , given all prices. (X is the result from solving the "dual" problem of utility maximization.)

So you can think of the household as making a two-step decision: 1) it decides how much to consume (how big C_i should be) given P; 2) Given C_i , household i allocates demand for each good j in proportion to its relative price.

2) Allocation between aggregate C_i and M_i

Given $C_i P = X_i$, the original utility optimization problem can be re-written as:

$$\max (\frac{C_i}{\alpha})^\alpha (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{L_i^\beta}{\beta} \text{ s.t. } C_i P + M_i = P_i Y_i + \bar{M}_i$$

Given Y_i and \bar{M}_i , it is straight forward to derive the following (standard Cobb-Douglas results):

$$P C_i = \alpha (P_i Y_i + \bar{M}_i) \text{ and } M_i = (1 - \alpha) (P_i Y_i + \bar{M}_i)$$

(Note: C_{ij} can then be expressed as: $\frac{\alpha}{1-\alpha} \frac{M_i}{P} (\frac{P_j}{P})^{-\sigma}$)

3) Production and Pricing decisions:

We can plug C_i and M_i above back into the utility function, and use the production function to get:

$$U_i = \frac{P_i}{P} Y_i - \frac{Y_i^\beta}{\beta} + \bar{M}_i/P$$

Each good i has total demand from all household k, so total demand for good i is: (note subscript on C is ki ; that is, household k's demand for good i)

$$Y_i = \int_0^1 C_{ki} dk = \frac{\alpha}{1-\alpha} \frac{\int_0^1 M_k dk}{P} (\frac{P_i}{P})^{-\sigma} = \frac{\alpha}{1-\alpha} \frac{M}{P} (\frac{P_i}{P})^{-\sigma}$$

This gives the downward sloping demand function we used in class, " $Q_i = D_i(\frac{P_i}{P}, \frac{M}{P})$ "

(Money market-wise: let $\bar{M} = \int_0^1 \bar{M}_i di$ and in equilibrium $\bar{M} = M$)

The maximization problem, after plugging in Y_i above, gives:

$$\begin{aligned} \frac{P_i}{P} &= \frac{\sigma}{\sigma-1} Y_i^{(\beta-1)} \quad (\text{price} = \text{markup times the marginal cost}) \\ &= \left[\frac{\sigma}{\sigma-1} \left(\frac{\alpha}{1-\alpha} \right)^{(\beta-1)} \left(\frac{M}{P} \right)^{(\beta-1)} \right]^{1/(1+\sigma(\beta-1))} \end{aligned}$$

So, when there is an increase in M , how output and the relative price adjust depends on β and σ :

If $\beta = 1$, then relative price doesn't change, and any shift in demand is accommodated by output change only. As β gets larger (>1), an outward shift in demand would lead to a greater increase in the relative price (in addition to quantity adjustment).

General Equilibrium:

Given complete symmetry across all households, the relative price must be equal to 1. And aggregate output would satisfy the following:

$$1 = \frac{\sigma}{\sigma-1} Y^{(\beta-1)} \quad \text{and} \quad P = \frac{\alpha}{1-\alpha} \frac{M}{Y} \propto M$$

Note, output is a constant markup and MONEY IS NEUTRAL under monopolistic competition. This setup also results in output that is inefficiently low.

Adding Menu Cost to give monetary non-neutrality:

The intuition here is the same as the cases we discussed in class. With menu costs to price adjustment, if the change in M is small (implying a small change in the optimal price), there would be no change in prices but in output, giving us non-neutrality. As in the partial equilibrium model, the effect of a small price deviation from the optimal level has only a 2nd order effect on firms' profit, but a 1st order effect on output and social welfare.