Monopolistic Competition

Econ 502, Fall 2009

The model

1) A continuum of households on the unit interval [0, 1] each produces one type of differentiated good, i.e. goods that are imperfect substitutes for each other (e.g. think Dell laptop vs. say Apple.)

2) Each household i enjoys a consumption good basket C_i which consists of all goods ("love for variety")

3) Money is needed for transactions, so real money holding provides utility. (This is a convenient way to introduce money in the system. You will later see alternative approaches such as through the introduction of a cash-in-advance constraint.)

4) As goods are imperfect substitutes, each household producer acts like a monopolist given the demand function for the good (which in turn is affected by other producers' behavior. This fact makes the setup different from a typical monopoly situation, and is called "Monopolistic Competition"). This is an important feature of the model, as it gives the price-setting decision to individual firms.

5) To simplify things, we assume away a separate labor market but let each household produces its good using its own labor. Labor (work) causes disutility.

Formally:

Let the utility function of household i be:

$$U\left(C_i, \frac{M_i}{P}, L_i\right) = \left(\frac{C_i}{\alpha}\right)^{\alpha}\left(\frac{M_i/P}{1-\alpha}\right)^{1-\alpha} - \frac{L_i^{\beta}}{\beta} \tag{1}$$

where consumption index C_i and price index P are:

$$C_i = \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj\right]^{\sigma/(\sigma-1)} \tag{2}$$

$$P = \left[\int_0^1 P_j^{1-\sigma} dj\right]^{1/(1-\sigma)} \tag{3}$$

This type of consumption function is called the Constant Elasticity of Substitution (CES) utility. The Elasticity of Substitution (b/w good variety j vs k as their relative price changes) is σ. If σ is large, then goods are close substitutes. Note also that σ must be greater than 1 to guarantee a meaningful equilibrium. (Convince yourself that if $\sigma < 1$, each producer would produce nothing and charge a price of infinity.)

The consumer's budget constraint is:

$$\int_0^1 P_j C_{ij} dj + M_i = P_i Y_i + \overline{M}_i \tag{4}$$

where $\overline{M}_i$ is initial money holding of household i. We can further assume a simple production function: $Y_i = L_i$

There are quite a few advantages to this setup:

Solution Method:

A convenient way to solve the model is to divide the solution procedure into three separate steps:

1) Let household i's consumption spending be X_i, and solve for its consumption demand for each good.

2) Household decides how to allocate between consuming and holding money

3) Consider household producer's production and pricing decisions, given the demand for their good.

Household Consumption Demand:

Household i chooses all C_{ij} to maximize aggregate consumption basket $C_i = [\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj]^{\sigma/(\sigma-1)}$s.t. $\int_0^1 P_j C_{ij} dj = X_i$

Note the following fact (it is easy to prove):

If g(·) is a strictly increasing positive transformation of another function f(x), then if x* solves $\max_x f(x)$, then x* also solves $\max_x g(f(x))$

so, the maximization problem can be simplied to:

$$\max \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj\right] \quad \text{s.t.} \quad \int_0^1 P_j C_{ij} dj = X_i$$

(this simplification is certainly not necessary; it is not that much more complicated to keep the exponents around.)

FOC with respect to C_{ij} from the Lagrangian: $L = \left[\int_0^1 C_{ij}^{(\sigma-1)/\sigma} dj\right] + \lambda \left[X_i - \int_0^1 P_j C_{ij} dj\right]$:

$$\frac{\sigma-1}{\sigma} C_{ij}^{-1/\sigma} = \lambda P_j \quad \text{for all j} \tag{5}$$

so by choosing two arbitrary conditions at indices j and k:

$$\frac{C_{ij}}{C_{ik}} = \left(\frac{P_j}{P_k}\right)^{-\sigma} \tag{6}$$

(this result shows that the elasticity of substitution between any two goods j and k is indeed constant at σ.)

Equation 6 can be re-written as:

$$\left(\frac{C_{ij}}{C_{ik}}\right)^{(\sigma-1)/\sigma} = \left(\frac{P_j}{P_k}\right)^{1-\sigma} \tag{7}$$

Integrate equation 7 over all j on both sides and use the definition for C_i in equation 2 and price index P in equation 3:

$$\left(\frac{C_i}{C_{ik}}\right)^{(\sigma-1)/\sigma} = \left(\frac{P}{P_k}\right)^{1-\sigma} \qquad \text{so } C_{ik} = \left(\frac{P_k}{P}\right)^{-\sigma} C_i \text{ for all variety k}$$

It is also easy to verify that $C_i P = X_i$ (i.e.multiply the above equation by P_k and intergrate over all k). Note also that X_i is the minimum expenditure needed such that aggregate consumption level is C_i,given all prices. (X is the result from solving the "dual" problem of utility maximization.)

So you can think of the household as making a two-step decision: 1) it decides how much to consume (how big C_i should be) given P; 2) Given C_i, household i allocates demand for each good j in proportion to its relative price.

Allocation of Households between Consumption and Money

Given $C_i P = X_i$, the original utility optimization in equation 1 problem can be re-written as:

$$\max \quad \left(\frac{C_i}{\alpha}\right)^{\alpha}\left(\frac{M_i/P}{1-\alpha}\right)^{1-\alpha} - \frac{L_i^{\beta}}{\beta} \text{ s.t. } \quad C_i P + M_i = P_i Y_i + \overline{M}_i \tag{8}$$

Given Y_i and $\overline{M}_i$,it is straight forward to derive the following (standard Cobb-Douglas results):

$$PC_i = \alpha(P_i Y_i + \overline{M}_i) \tag{9}$$

$$M_i = (1-\alpha)(P_i Y_i + \overline{M}_i) \tag{10}$$

(Note: C_{ij} can then be expressed as: $\frac{\alpha}{1-\alpha}\frac{M_i}{P}\left(\frac{P_j}{P}\right)^{-\sigma}$)

Production and Pricing Decisions

We can plug C_i and M_i above back into the utility function, and use the production function to get:

$$U_i = \frac{P_i}{P}Y_i - \frac{Y_i^{\beta}}{\beta} + \overline{M}_i/P \tag{11}$$

Each good i has total demand from all household k, so total demand for good i is: (note subscript on C is ki; that is, household k's demand for good i)

$$Y_i = \int_0^1 C_{ki}dk = \frac{\alpha}{1-\alpha}\frac{\int_0^1 M_k dk}{P}\left(\frac{P_i}{P}\right)^{-\sigma} = \frac{\alpha}{1-\alpha}\frac{M}{P}\left(\frac{P_i}{P}\right)^{-\sigma} \tag{12}$$

This gives the downward sloping demand function we used in class, $Q_i = D_i\left(\frac{P_i}{P}, \frac{M}{P}\right)$

(Money market-wise: let $\overline{M} = \int_0^1 \overline{M}_i di$ and in equilibrium $\overline{M} = M$)

The maximization problem, after plugging in Y_i above, gives:

$$\frac{P_i}{P} = \frac{\sigma}{\sigma-1}Y_i^{(\beta-1)} \qquad \text{(price = markup times the marginal cost)} \tag{13}$$

$$= \left[\frac{\sigma}{\sigma-1}\left(\frac{\alpha}{1-\alpha}\right)^{(\beta-1)}\left(\frac{M}{P}\right)^{(\beta-1)}\right]^{1/(1+\sigma(\beta-1))} \tag{14}$$

So, when there is an increase in M, how output and the relative price adjust depends on β and σ :

If $\beta = 1$, then relative price doesn't change, and any shift in demand is accommodated by output change only. As β gets larger (>1), an outward shift in demand would lead to a greater increase in the relative price (in addition to quantity adjustment).

General Equilibrium:

Given complete symmetry across all households, the relative price must be equal to 1. And aggregate output would satisfy the following:

$$1 = \frac{\sigma}{\sigma - 1} Y^{(\beta - 1)} \qquad \text{and P} = \frac{\alpha}{1-\alpha}\frac{M}{Y} \propto M$$

Note, output is a constant markup and MONEY IS NEUTRAL under monopolistic competition. This setup also results in ouput that is inefficiently low.

Adding Menu Cost to give monetary non-neutrality:

The intuition here is the same as the cases we discussed in class. With menu costs to price adjustment, if the change in M is small (implying a small change in the optimal price), there would be no change in prices but in output, giving us non-neutrality. As in the partial equilibrium model, the effect of a small price deviation from the optimal level has only a 2nd order effect on firms' profit, but a 1st order effect on output and social welfare.