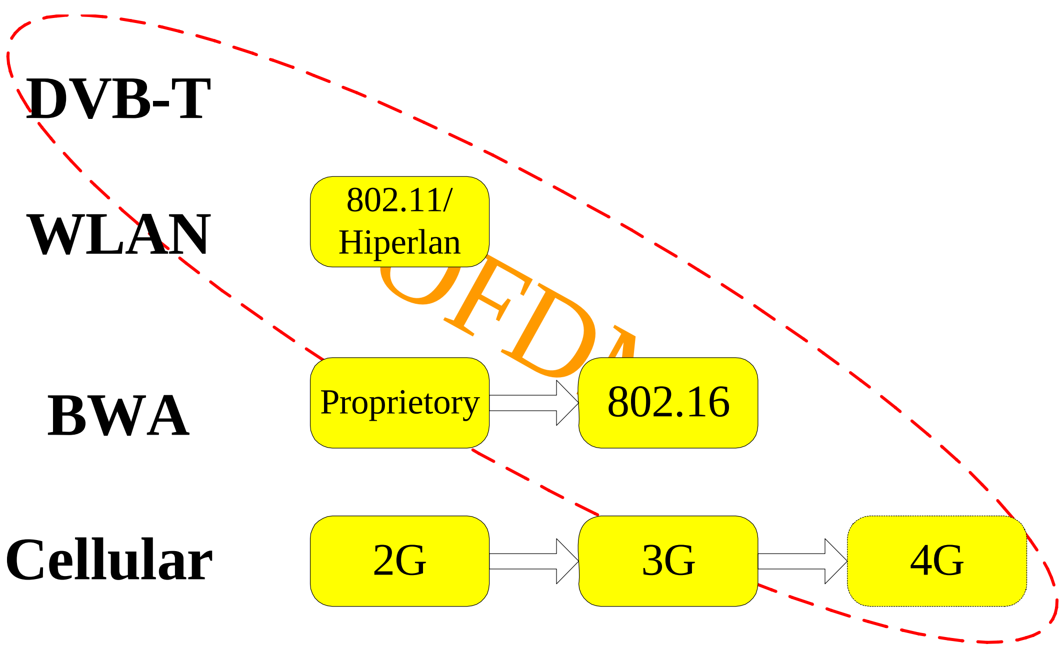
1 OFDM and its implementation

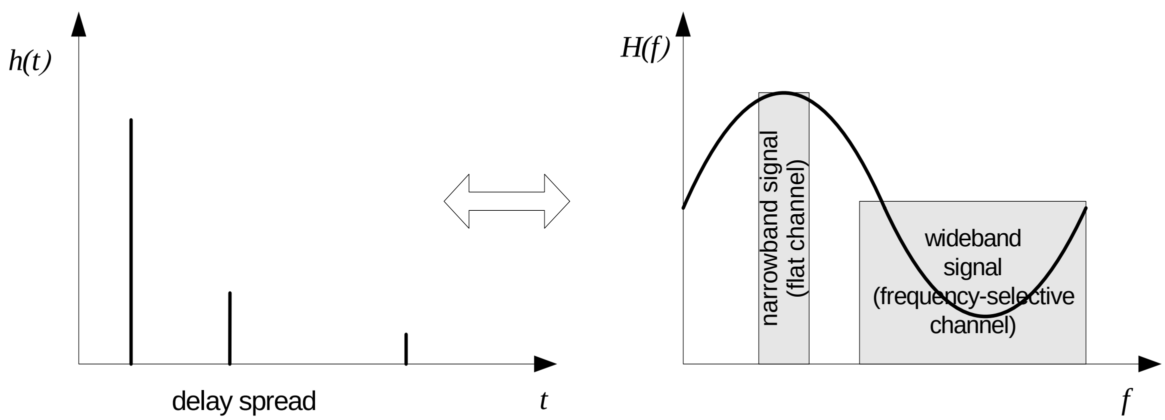
References H. Liu and G. Li, *OFDM-based broadband wireless networks*, Chapter 2

1.1 OFDM: canonical form of broadband communications

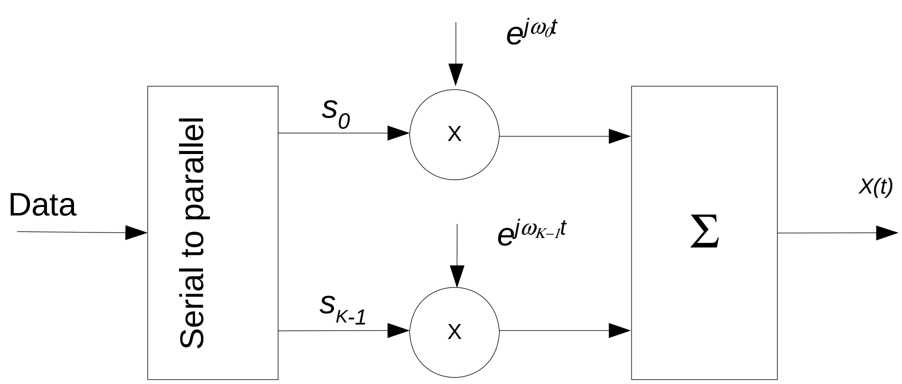


1.2 The good, old frequency-division multiplexing (FDM)

- multipath leads to frequency selectivity fading
- frequency selectivity leads to signal distortion (e.g., inter symbol interference)
- narrower signal bandwidth = lesser distortion
- sinusoid = no distortion



Achieving high data rate through *parallel, narrowband* streams!



1.3 The Math behind it

$$x(t) = \sum_{k=0}^{K-1} s_k e^{j2\pi kt/N}$$

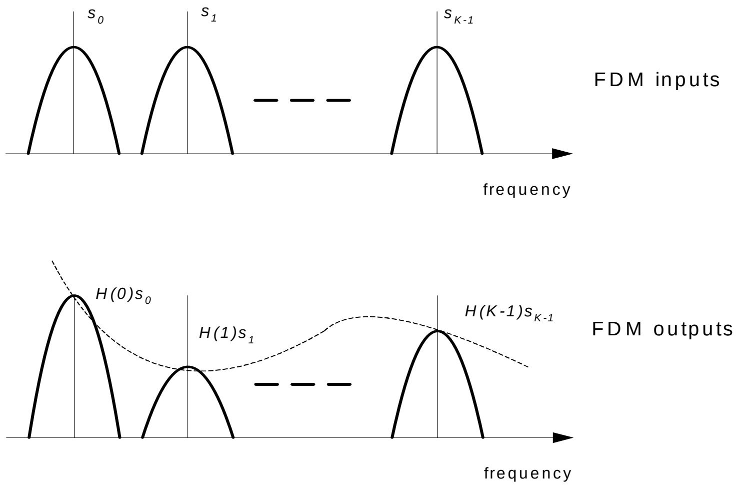
Input-output relation of linear system:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Eigen signals for linear systems: $x(t) = e^{st}$:

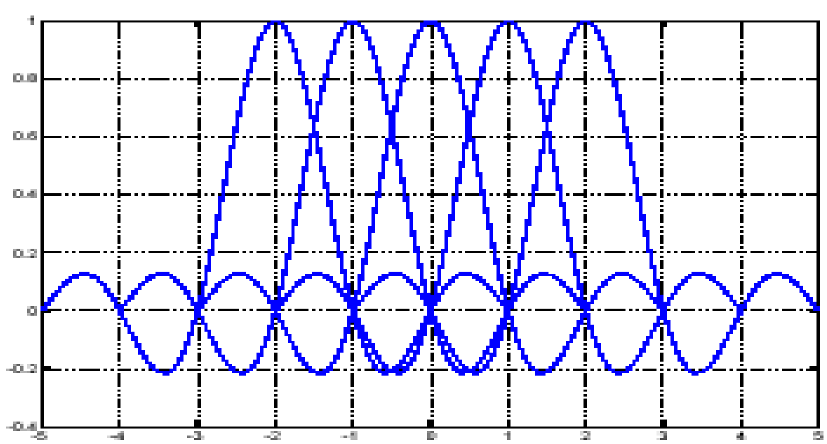
$$\begin{aligned} y(t) &= h(t) * x(t) \\ &= \int_{-\infty}^{\infty} h(\tau)e^{s(t-\tau)}d\tau \\ &= e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau = H(s)e^{st} \end{aligned}$$

1.4 The output



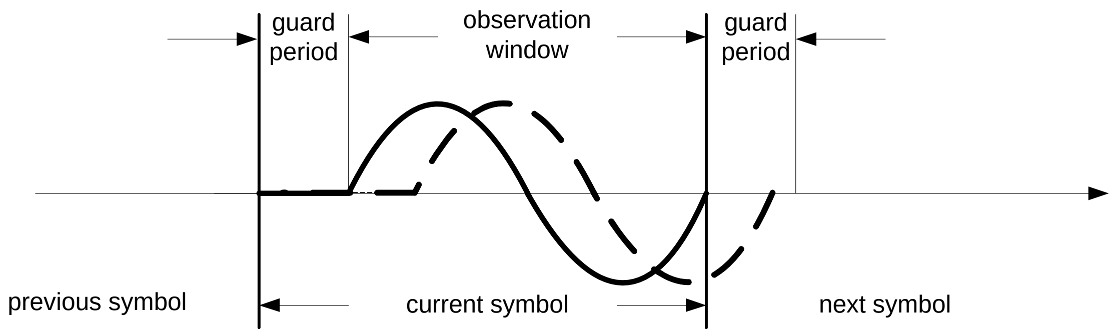
- effect of the channel is a mere “scaling” on each subchannel.
- the scalar ambiguity can be removed with channel estimation,

The FDM that satisfies the interference-free frequency spacing requirement is referred to as OFDM - *Orthogonal Frequency Division Multiplexing*.

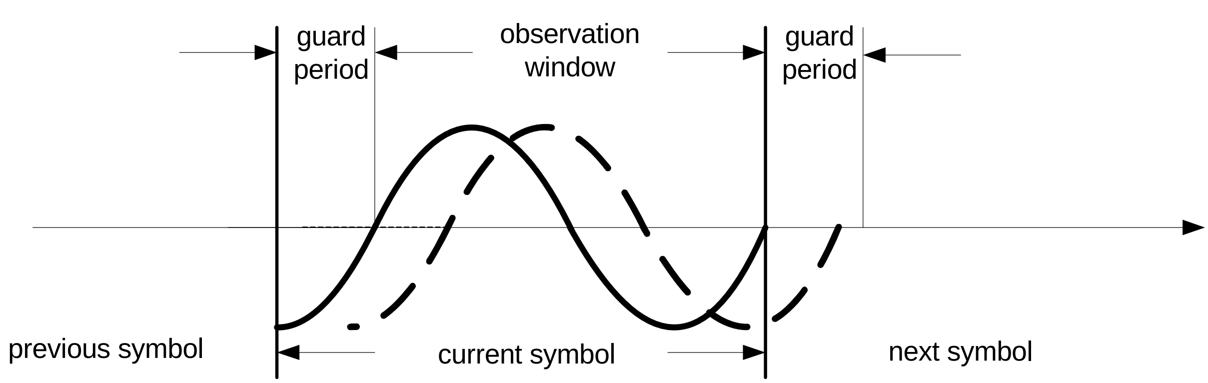


1.5 Finite-duration OFDM symbols

- complex exponential signals over multipath channels: no distortion except for the amplitude
- only valid for ever-lasting complex exponential.



- A cyclic-prefix *(CP)* of duration at least $\tau_{\max}$ at the transmitter.
- Multipath components with delays $< \tau_{\max}$ will maintain their waveforms within the observation window
- Immunity is achieved at the expense of an unused CP (at the receiver).



1.6 Discrete implementation

- A total number of K (usually power of 2) tones in the system
- the time-domain sampling rate is $N/T = 1/T_s$, $N = K$ hertz
- the channel delay spread is $L < N$ samples.

Within a time window, we would like to have

$$Y(k) = X(k)H(k), \qquad k = 0, ..., K-1$$

where $Y(k)$, $X(k)$,and $H(k)$ are the Discrete Fourier transforms (DFTs).

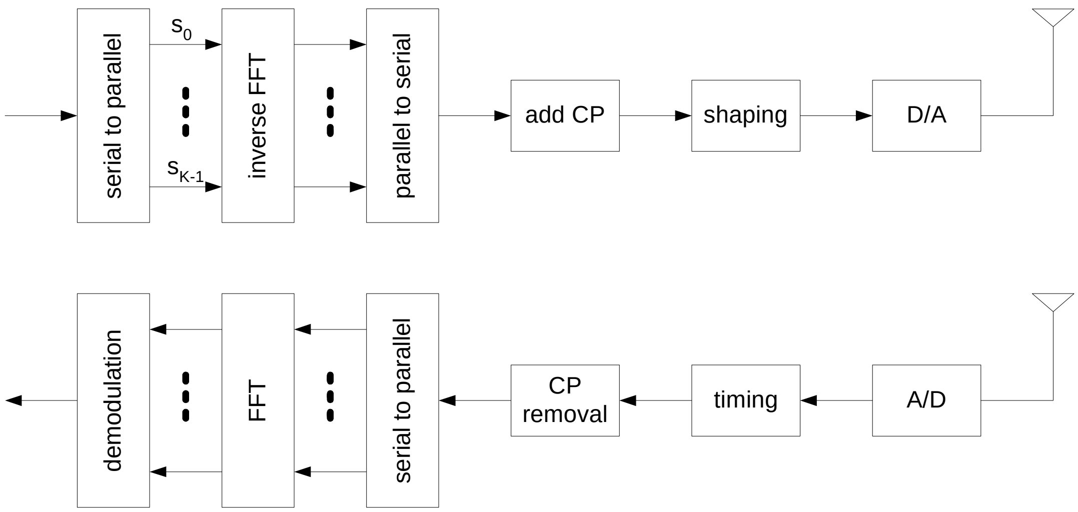
Note that

$$y_n = x_n \circledast h_n \Longleftrightarrow Y(k) = X(k)H(k)$$

However, circular convolution is NOT how real-world channel operates! In order to take advantage of DFT, the *effect* of circular convolution must be created artificially.

1.7 OFDM modulation

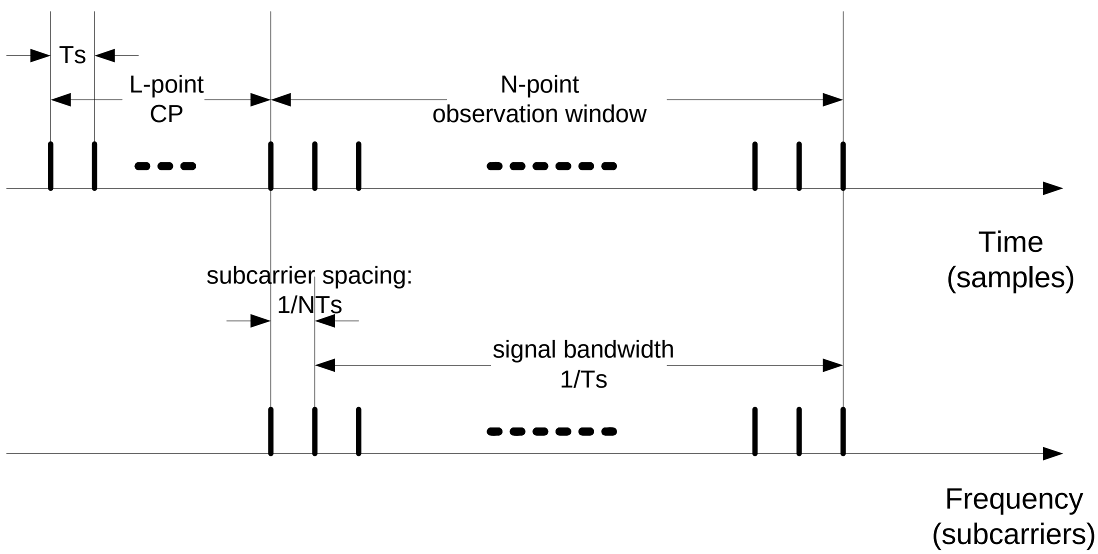
- A real-world channel only performs linear convolution, i.e., $y_n = x_n * h_n$,
- The circular convolutional effect is created within the N point time window by appending an L point "cyclic prefix" (CP), $x_{N-L}, x_{N-L+1}, \ldots, x_{N-1}$, to x_n.



- Unlike signal-carrier modulation, the OFDM modem is performed on a *block-by-block* basis.
- At TX, a block of symbols are serial-to-parallel converted onto K *subcarriers*.
- The orthogonal waveform modulation is carried out using an inverse FFT and a parallel-to-serial converter.
- The last L points are appended to the beginning of the sequence as the cyclic prefix.
- Each transmitted block is referred to as an *OFDM symbol*.

1.8 OFDM demodulation

- At RX, the sampled signals are processed to determine the proper demodulation window.
- By removing the CP (which now contains ISI), an N $(N = K)$ point sequence is serial-to-parallel converted and fed to the FFT.
- The output of the FFT are the symbols modulated on the K subcarriers, each multiplied by a complex channel gain.
- Regular demodulation (e.g., QPSK) is then carried out based on the channel information $H(k)$



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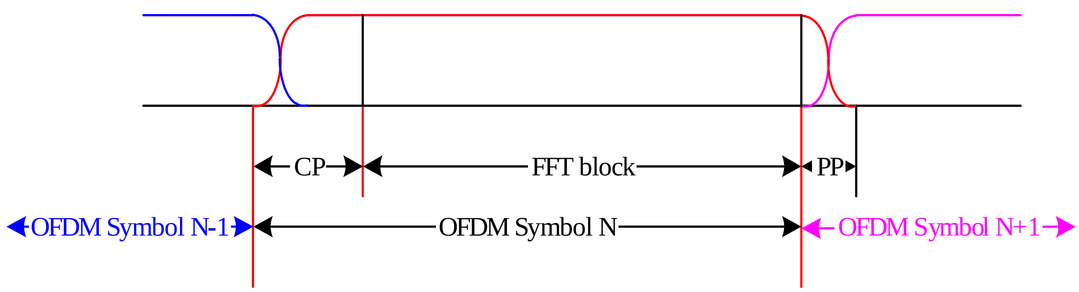
1.9 Practical Considerations

OFDM symbol stream



CP: Cyclic Pre-fix
PP: Cyclic Post-fix

OFDM pilots