Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 20

How Does Energy Detection Work?

- Energy detection uses the energy spectra of the received signal in order to identify the frequency locations of the transmitted signal
- Several steps are involved in producing the frequency representation of the intercepted signal
  - *Pre-filtering* of intercepted signal extracts frequency band of interest
  - *Analog-to-digital conversion* (ADC) converts filtered intercepted signal into discrete time samples
  - *Fast Fourier transform* (FFT) provides the frequency representation of the signal
  - *Square-law device* yields the square of the magnitude of the frequency response from the FFT output

Detector Implementation

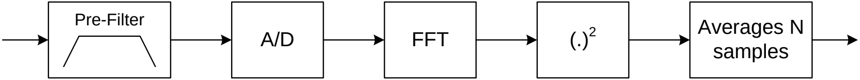

Figure : Schematic of an Energy Detector Implementation Employing Pre-Filtering and a Square-Law Device.

Detection Threshold

- Energy detection involves the *application* of a threshold in the frequency domain
  - Threshold is used to decide whether a transmission is present a specific frequency
- Any portion of the frequency band where the energy exceeds the threshold is considered to be occupied by a transmission
  - Binary decision making process
  - Two hypotheses: $\mathcal{H}_0$ (idle) or $\mathcal{H}_1$ (occupied)
- One of the major concerns of energy detection is the selection of an appropriate threshold
  - A threshold that may work for one transmission may not be sufficient for another
    - Transmitters employing different signal power levels
    - Transmission ranges may vary

An Example

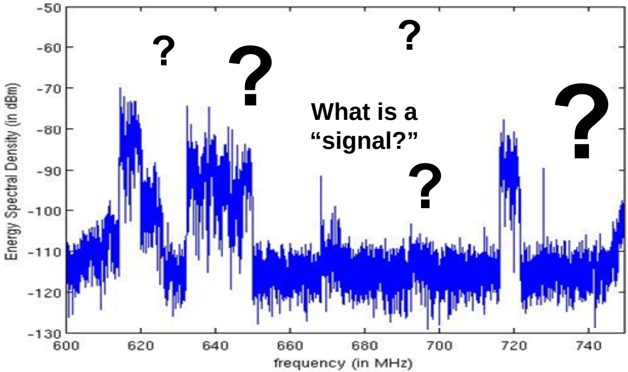

Figure : Spectrum Measurements from Springfield, MA during June 2009.

Applying the Detection Threshold

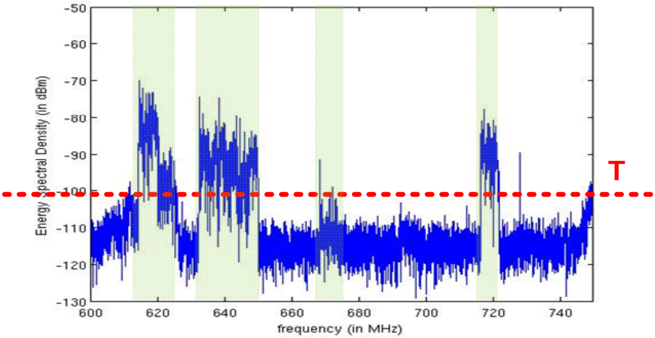

Figure : Energy Detection Threshold used to Identify Occupied Spectrum.

"False-Alarm" Scenario

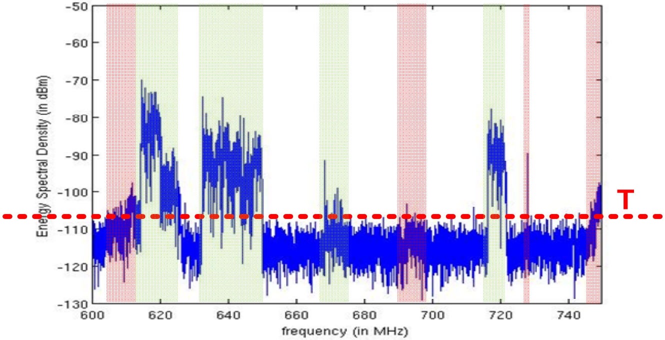

Figure : Energy Detection Threshold Level Yielding False Alarms.

"Missed Detection" Scenario

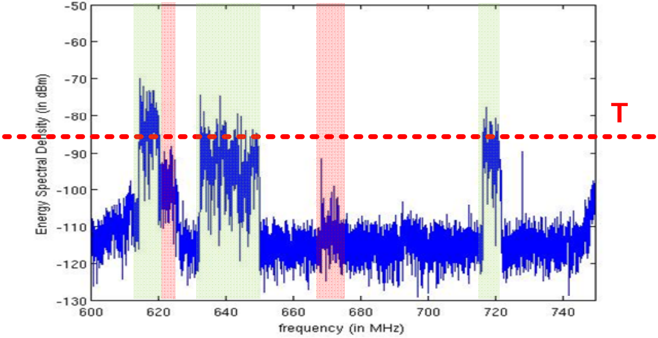

Figure : Energy Detection Threshold Level Yielding Missed Detection.

Hypothesis Testing Revisited

- Spectrum sensing involves distinguishing between two mutually independent and identically distributed Gaussian sequences:

$$\begin{aligned} \mathcal{H}_0: &\quad y(k) = w(k) \rightarrow \text{Idle} \\ \mathcal{H}_1: &\quad y(k) = s(k) + w(k) \rightarrow \text{Occupied} \end{aligned}$$

  where $w(k)$, $k = 1, \ldots, n$, is the noise signal sample, and $s(k)$, $k = 1, \ldots, n$, is a transmitted signal sample
- Both $w(k)$ and $s(k)$ are zero-mean complex Gaussian random variables with variances σ_w^2 and σ_s^2 per dimension
- Let us define the vector of the n observed samples:

$$\mathbf{y} = [y(1), \ldots, y(n)]'$$

Log-Likelihood Ratio

- Suppose we define the variances $\sigma_0^2 = \sigma_w^2$ and $\sigma_1^2 = \sigma_s^2 + \sigma_w^2$
- Neymann-Pearson detector is a threshold detector on either the likelihood ratio or the log-likelihood ratio (LLR):

$$\mathrm{LLR} = \log\left(\frac{p(\mathbf{y}|\mathcal{H}_1)}{p(\mathbf{y}|\mathcal{H}_0)}\right) > \tau' \qquad (1)$$

  where τ' is a suitably chosen threshold
- The detector is equivalent to deciding $\mathcal{H}_1$ given the independent and identical assumption if:

$$z = \frac{1}{2n\sigma_0^2}\sum_{k=1}^{n}|y(k)|^2 > \tau \qquad (2)$$

  where z is a statistic possessing a scaled version of a standard χ^2 distribution with $2n$ degrees of freedom

Computing Tail Probability

- Given that x_i are independent real Gaussian variables with zero means and unit variances, we get $x = \sum_{i=1}^{2n} x_i^2$ where x is χ^2 distributed with $2n$ degrees of freedom
- The χ^2 PDF with $2n$ degrees of freedom is:

$$p(x) = \frac{1}{2^n(n-1)!} x^{n-1} e^{-x/2} \tag{3}$$

- Using integration by parts, we compute $P(x > \tau)$:

$$\begin{aligned} P(x > \tau) &= \int_{\tau}^{\infty} \frac{1}{2^n(n-1)!} x^{n-1} e^{-x/2} dx \\ &= e^{-\tau/2} \sum_{k=0}^{n-1} \frac{1}{k!} \left(\frac{\tau}{2}\right)^k = \Gamma_u\left(\frac{\tau}{2}, n\right) \end{aligned}$$

Computing P_{FA} and P_{MD}

- What is $\Gamma_u()$?
  - The *upper incomplete gamma function* defined as:

$$\Gamma_u(a,n) = \frac{1}{\Gamma(n)} \int_a^{\infty} x^{n-1} e^{-x} dx$$

- Consequently, the test statistic $z \times 2n$ has the same PDF as a χ^2 variable with $2n$ degrees of freedom
- The probability of false alarm (P_{FA}) and probability of missed detection (P_{MD}) are equal to:

$$\epsilon = P_{\text{FA}} = \Gamma_u(n\tau, n)$$

$$\delta = P_{\text{MD}} = 1 - \Gamma_u\left(\frac{n\tau}{1+\sigma_s^2/\sigma_w^2}, n\right)$$