Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 09

Modulation Format

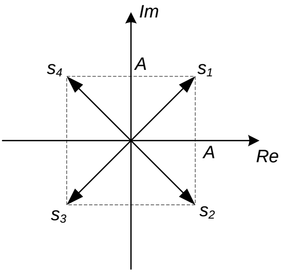

Figure : QPSK Signal Constellation.

- So far we have seen modulation schemes that consist of just one of two waveforms
  - We will now expand our signal constellation repertoire to four distinct waveforms per modulation scheme
- In QPSK modulation, a signal waveform possesses the following representation:

$$s_i(t) = \pm A \cdot \cos(\omega_c t + \theta) \pm A \cdot \sin(\omega_c t + \theta) \quad (1)$$

Computing $\varepsilon_{p,\text{QPSK}}$

- Solving for d^2_{min}, we obtain:

$$d^2_{\text{min}} = \int_0^T \Delta s^2(t)dt = 2A^2T \tag{2}$$

- To find $\bar{E}_b$, we need to average over all the signals, which is equal to:

$$\bar{E}_b = \frac{(E_{s_1} + E_{s_2} + E_{s_3} + E_{s_4})/4}{\log_2(M)} = \frac{A^2T}{2} \tag{3}$$

since $E_{s_1} = E_{s_2} = E_{s_3} = E_{s_4} = A^2T$

- Solving for the power efficiency, we get:

$$\varepsilon_{p,\text{QPSK}} = \frac{d^2_{\text{min}}}{\bar{E}_b} = 4 \tag{4}$$

Modulation Format

- Distance of signal constellation point to origin always a constant
- Consists of M equally spaced points on a circle
- General expression for an M-ary PSK waveform:

$$s_i(t) = A \cdot \cos\left(\omega_c t + \frac{2\pi i}{M}\right), \text{ for } i = 0, 1, 2, \ldots, M-1 \quad (5)$$

- There are advantages and disadvantages with this modulation scheme
  - As M increases, the spacing between signal constellation points decreases $\rightarrow$ error robustness decreases
  - Having information encoded in the phase results in constant envelope modulation, which is:
    - Good for non-linear power amplifiers
    - Robust to amplitude distortion channels

Computing $\varepsilon_{p,\mathrm{M-PSK}}$

- Given $s_1(t) = A \cdot \cos(\omega_c t)$ and $s_2(t) = A \cdot \cos(\omega_c t + 2\pi/M)$
  - Calculate $d^2_{\min} = E_{s_1} + E_{s_2} - 2\rho_{12}$ where:

$$E_{s_i} = \int_0^T s_i^2(t)dt = \frac{A^2 T}{2}, \text{ for } i = 1, 2 \tag{6}$$

and:

$$\rho_{12} = \int_0^T s_1(t)s_2(t)dt = \frac{A^2 T}{2} \cos\left(\frac{2\pi}{M}\right) \tag{7}$$

which yields $d^2_{\min} = A^2 T(1 - \cos(\frac{2\pi}{M}))$
  - The average bit energy $\bar{E}_b$ is equal to $\bar{E}_b = \frac{\bar{E}_s}{\log_2(M)} = \frac{\bar{E}_s}{b}$, where $\bar{E}_s = A^2 T/2$
  - Using the definition for the power efficiency, we see that $\varepsilon_{p,\mathrm{M-PSK}} = 2b(1 - \cos(\frac{2\pi}{M})) = 4b\sin^2(\frac{\pi}{2^b})$

Modulation Format

- We can expression the M-ary PAM waveform as:

$$s_i(t) = A_i \cdot p(t), \text{ for } i = 1, 2, \ldots, M/2 \tag{8}$$

where $A_i = A(2i - 1)$, $p(t) = u(t) - u(t - T)$, and $u(t)$ is the unit step function

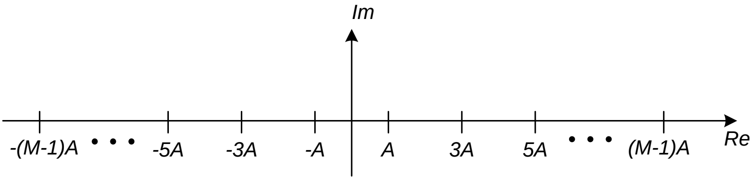

Figure : M-PAM Signal Constellation.

Computing $\varepsilon_{p,\mathrm{M-PAM}}$

- Selecting the d^2_{min} pair $s_1(t) = A \cdot p(t)$ and $s_2(t) = -A \cdot p(t)$
  - Solving $\Delta s(t) = 2A \cdot p(t) \rightarrow d^2_{\mathrm{min}} = 4A^2 T$
- Find $\bar{E}_s$ in general else only positive symbols due to symmetry of signal constellation, which yields:

$$
\begin{aligned}
\bar{E}_s &= \frac{2}{M} A^2 T \sum_{i=1}^{M/2} (2i-1)^2 \\
&= A^2 T \frac{(M^2-1)}{3} \quad \text{which is simplified via tables} \\
\rightarrow \bar{E}_b &= \frac{\bar{E}_s}{\log_2(M)} = \frac{A^2 T (2^{2b}-1)}{3b}
\end{aligned}
$$

- Solving for the power efficiency yields $\varepsilon_{p,\mathrm{M-PAM}} = \frac{12b}{2^{2b}-1}$

Simple Receiver Structure

- M-ary QAM is a popular modulation scheme due to its **simple** receiver structure
  - Each branch employs a $\sqrt{M}$-ary PAM detector

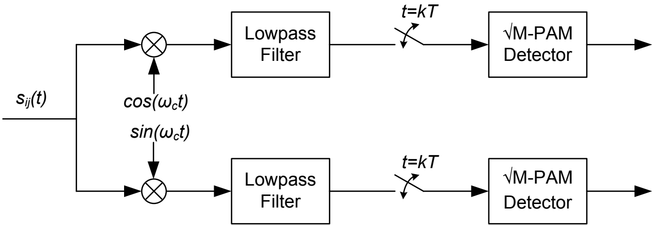

Figure : M-QAM Receiver Structure.

Modulation Format

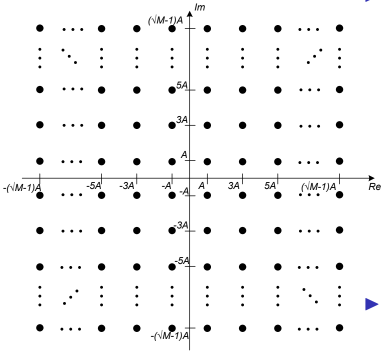

Figure : M-QAM Signal Constellation.

- M-ary QAM looks like two simultaneous $\sqrt{M}$-ary PAM signal constellations
  - One is acting in the real axis while the other is working on the imaginary axis
  - For instance, a 64-QAM signal constellation can be represented by two 8-PAM signal constellations
- We represent this linear modulation technique as:

$$s_{ij}(t) = A_i \cdot \cos(\omega_c t) + B_j \cdot \sin(\omega_c t) \quad (9)$$

Computing $\varepsilon_{p,\mathrm{M-QAM}}$

- To find the ε_p of M-ary QAM, we need to determine the following:
  - Calculate $d_{\min}^2 = \int_0^T \Delta s^2(t)dt = 2A^2T$ using:

$$s_1(t) = A \cdot \cos(\omega_c t) + A \cdot \sin(\omega_c t)$$
$$s_2(t) = 3A \cdot \cos(\omega_c t) + A \cdot \sin(\omega_c t)$$

  - For computing $\bar{E}_s$, use the expression from M-ary PAM and replace M with $\sqrt{M}$ such that $\bar{E}_s = A^2T\frac{M-1}{3}$
    - Solve for $\bar{E}_b = \frac{\bar{E}_s}{\log_2(M)} = A^2T\frac{2^b-1}{3b}$
  - The power efficiency is equal to $\varepsilon_{p,\mathrm{M-QAM}} = \frac{3!b}{2^b-1}$

Comparison of ε_p

- To determine how much power efficiency we are losing relative to $\varepsilon_{p,\text{QPSK}}$, we use $\delta\text{SNR} = 10 \cdot \log_{10}(\frac{\varepsilon_{p,\text{QPSK}}}{\varepsilon_{p,\text{other}}})$

Table : δSNR Values of Various Modulation Schemes.

M	b	M-ASK	M-PSK	M-QAM
2	1	0	0	0
4	2	4	0	0
8	3	8.45	3.5	(??)
16	4	13.27	8.17	4.0
32	5	18.34	13.41	(??)
64	6	24.4	18.4	8.45

Observations of ε_p

- Two dimensional modulation is better than one dimensional modulation
- All modulation schemes studied are linear modulation schemes$\rightarrow$similar receiver complexity