Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 06

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Frequency Domain Perspective

- It is sometimes more convenient to study a communication system in the frequency domain rather than the time domain
  - Mathematical analysis is more tractable
  - Operations such as convolution are transformed into simple multiplications
- Physically it makes sense to study wireless transmissions in the frequency domain
  - Digital communications is the transferral of data based on changes of electromagnetic wave characteristics such as frequency, amplitude, and phase

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Fourier Transform

- Mathematical relationship between a time domain waveform and weighted sum of sinusoidal components that constitute it, i.e., frequency domain representation
- Translating between time and frequency domains is achieved using the Fourier transform and inverse Fourier transform:

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \tag{1}$$

$$h(t) = \int_{-\infty}^{\infty} H(f) e^{j2\pi ft} df \tag{2}$$

Einstein-Wiener-Khinchin Theorem

- In particular, we are interested in the *power spectral density* (PSD) of a signal, which is related to the autocorrelation function via the *Einstein-Wiener-Khintchine* (EWK) Relations:

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau) e^{-j2\pi f \tau} d\tau \tag{3}$$

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f) e^{j2\pi f \tau} df \tag{4}$$

- Relating the PSD between input $x(t)$ and output $y(t)$ of a system $h(t)$, we have the following very important result:

$$S_Y(f) = |H(f)|^2 S_X(f) \tag{5}$$

Impulse Train Model

- Signal analysis and knowledge of the input PSD can help determine the output PSD of the transmitter
  - Determine the autocorrelation function of B_n from I_n and then apply EWK relations

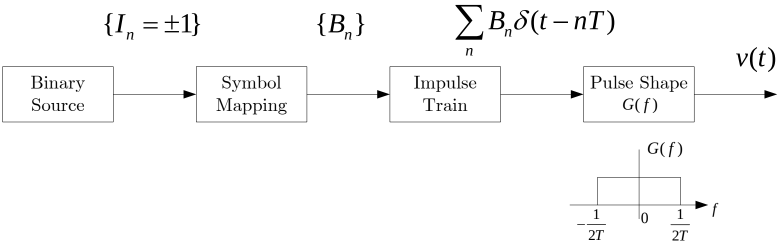

Figure : Impulse Train Model Used to Generate an Analog Waveform Based on an Input Binary Sequence.

PSD Properties

- Zero frequency scenario:

$$S_X(0) = \int_{-\infty}^{\infty} R_X(\tau)d\tau$$

- Mean-squared value:

$$E\{x^2(t)\} = \int_{-\infty}^{\infty} S_X(f)df$$

- Non-negative PSD: $S_X(f) \geq 0, \ \forall f$
- Real-valued process: $S_X(-f) = S_X(f)$
- Normalized PSD associated with a PDF:

$$p_X(f) = \frac{S_X(f)}{\int_{-\infty}^{\infty} S_X(f)df}$$

PSD Example

- Find $S_X(f)$ of the following random process:

$$x(t) = A\cos(2\pi f_c t + \Theta),$$

where Θ is uniformly distributed over the interval $[-\pi, \pi]$.

- First solve for $R_X(\tau)$ by using:

$$R_X(\tau) = E\{x(t+\tau)x(t)\} = \frac{1}{2}A^2\cos(2\pi f_c \tau)$$

- Then solve for the PSD using EWK relations:

$$\begin{aligned} S_X(f) &= \int_{-\infty}^{\infty} R_X(\tau)e^{-j2\pi f\tau}d\tau = \int_{-\infty}^{\infty} \frac{1}{2}A^2\cos(2\pi f_c\tau)e^{-j2\pi f\tau}d\tau \\ &= \int_{-\infty}^{\infty} \frac{1}{4}A^2\left(e^{j2\pi f_c\tau} + e^{-j2\pi f_c\tau}\right)e^{-j2\pi f\tau}d\tau \\ &= \frac{A^2}{4}(\delta(f-f_c) + \delta(f+f_c)) \end{aligned}$$

Identifying PSD Features

- PSD characteristics can reveal a substantial amount of info regarding a signal
  - Modulation format
  - Pulse shape filtering
  - Transmission bandwidth
- Time-varying behavior indicates network traffic levels
- PSD shape can uniquely identify specific wireless standards

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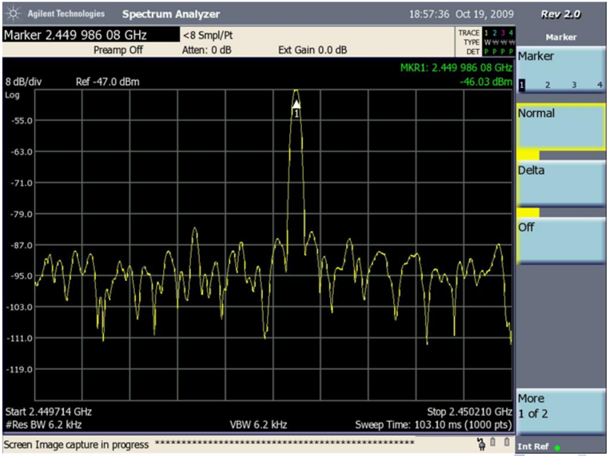
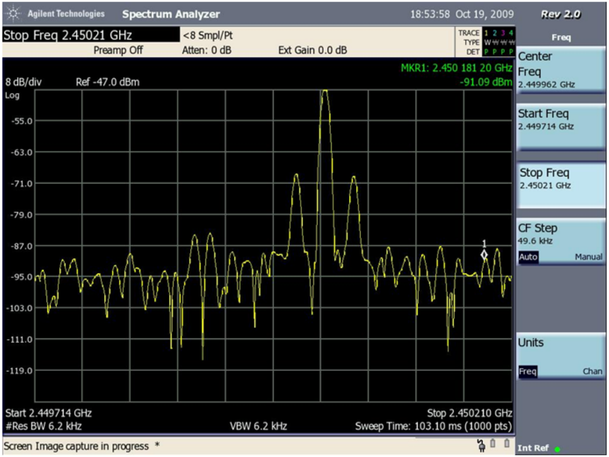
Constant Transmission

- Output of the baseband portion of the digital transmitter is: $s(t) = C$



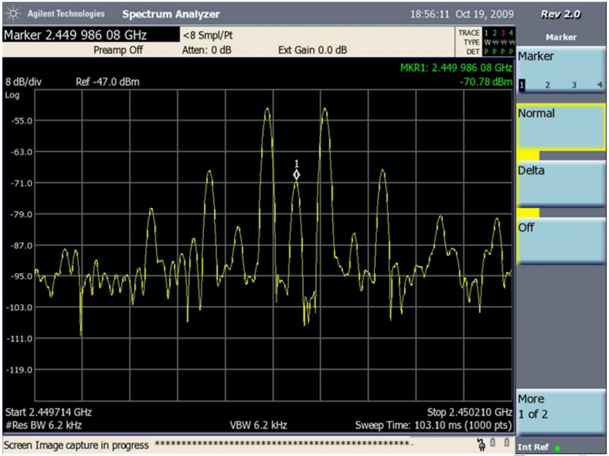
Complex Sinusoid Transmission

- Output of the baseband portion of the digital transmitter is: $s(t) = A(\cos(2\pi ft + \phi) + j\sin(2\pi ft + \phi))$



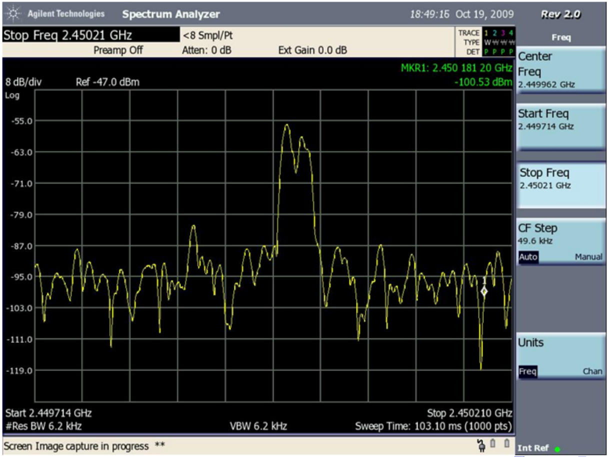
Real Sinusoid Transmission

- Output of the baseband portion of the digital transmitter is: $s(t) = A\sin(2\pi ft + \phi)$



Quadrature Phase Shift Keying Transmission

- Output of the baseband portion of the digital transmitter is a pulse-shaped QPSK modulated signal



Digital Television Transmissions Along Interstate 90

- Forty eight locations chosen across I-90 in the state of Massachusetts approximately two miles apart
- Spectrum measurements taken in a moving vehicle on June 30, 2009

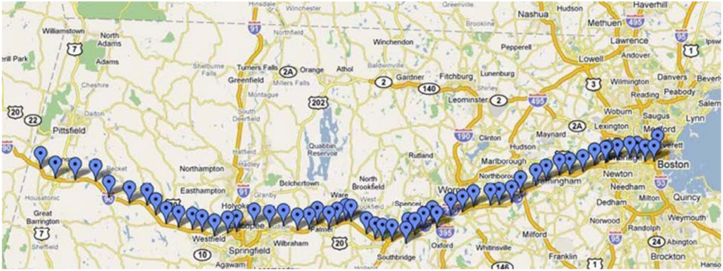


Figure : Locations close to I-90 between Boston, MA and Blandfield, MA over which spectrum measurements were collected.

Stationary DTV PSD Measurements

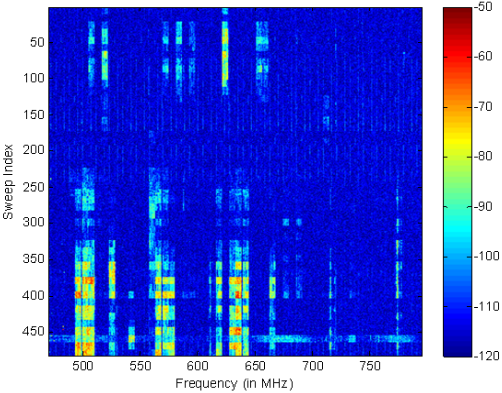


Figure : PSD plots for the TV frequencies in 470 806 MHz frequency range across 48 locations close to I-90 between Boston, MA and West Stockbridge, MA.

Moving DTV PSD Measurements

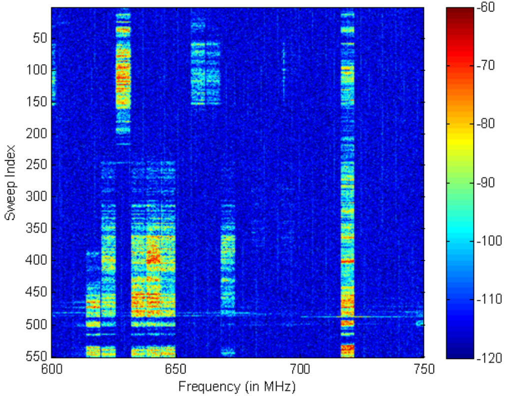


Figure : PSD plots for the TV frequencies in 600 750 MHz frequency range over 550 time sweeps close on I-90 between Boston, MA and West Stockbridge, MA in a vehicle moving at an average velocity of 60 mph.

U.S. DTV Transition – June 12, 2009

- DTV measurements taken at Bunker Hill National Monument in Charlestown, MA, USA

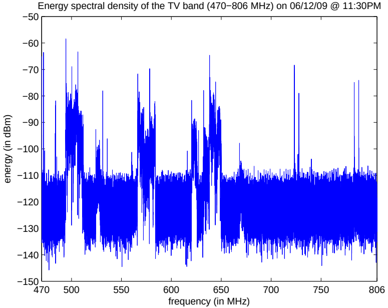


Figure : Before Midnight.

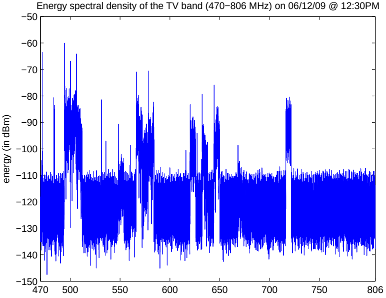


Figure : After Midnight.

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