Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 17

Choice of Analysis and Synthesis Filters

- We have just seen the general form for multicarrier modulation, which consists of multiplexing and demultiplexing data as well as employ *synthesis* and *analysis* filter banks
  - Different multicarrier implementations are mainly based on the choice of filters for the signal analysis and synthesis stages
  - Each of the N subchannels consist of bandwidth $\Delta f = W/N$, where W is the total transmission bandwidth
  - Division of data into independent subchannels yields an opportunity for a "divide-and-conquer" approach to data transmission
- **Question:** Can we implement multicarrier modulation efficiently?

OFDM Framework

- High-speed digital input, $d[m]$, is demultiplexed into N subcarriers using a commutator
- Data on each subcarrier is then modulated into an M-QAM symbol
  - For subcarrier k, we will rearrange $a_k[\ell]$ and $b_k[\ell]$ into real and imaginary components yielding $p_k[\ell] = a_k[\ell] + jb_k[\ell]$
- In order for the output of the IDFT block to be real, given N subcarriers, we must use a $2N$-point IDFT
  - Terminals $k = 0$ and $k = N$ are "don't care" inputs
  - For subcarriers $1 \leq k \leq N - 1$, the inputs are $p_k[\ell] = a_k[\ell] + jb_k[\ell]$
  - For subcarriers $N + 1 \leq k \leq 2N - 1$, the inputs are $p_k[\ell] = a_{2N-k}[\ell] + jb_{2N-k}[\ell]$

OFDM Framework

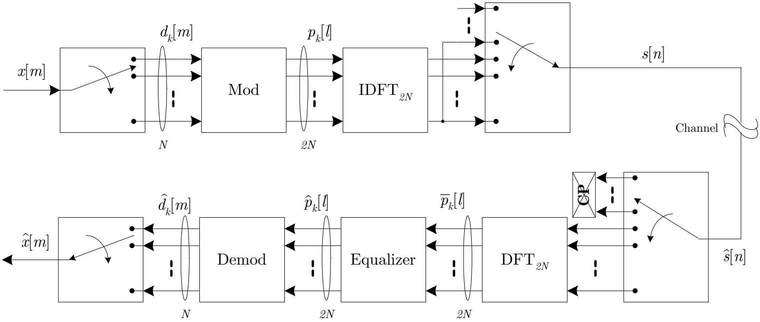

Figure : Orthogonal Frequency Division Multiplexing Transceiver.

OFDM Spectra

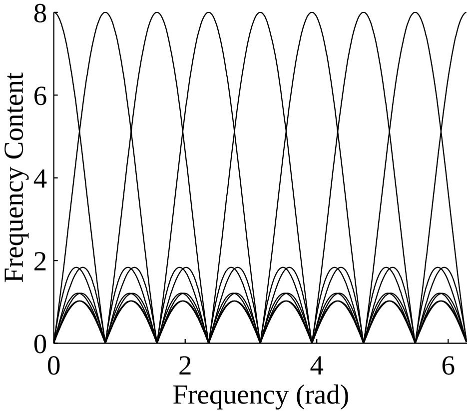

Figure : Frequency Response of OFDM Transmission Consisting of Superimposed Sinc Spectra.

OFDM Modulation and Demodulation

- The IDFT is then performed, yielding:

$$s[2\ell N + n] = \frac{1}{2N} \sum_{k=0}^{2N-1} p_k[\ell] e^{j(2\pi nk/2N)} \tag{1}$$

  where $2N$ consecutive samples of $s[n]$ constitute an OFDM symbol
  - Sum of N different QAM symbols
- At the receiver, the signal is demultiplexed into $2N$ subcarriers of data, $\hat{s}[n]$
- A $2N$-point DFT, defined as:

$$\bar{p}_k[\ell] = \sum_{n=0}^{2N-1} \hat{s}[2\ell N + n] e^{-j(2\pi nk/2N)}, \tag{2}$$

  is applied to the inputs, yielding the estimates of $p_k[\ell]$, $\bar{p}_k[\ell]$

OFDM Advantages

- What makes OFDM so special?
  - Suppose we select the symbol rate $1/T$ is each subcarrier to be equal to the separate frequency Δf of the adjacent subcarriers
  - We can then make the subcarriers *overlap* with each other while remaining orthogonal over the symbol interval T while being independent of the relative phase relationship between subcarriers
- How does this look in the frequency domain?

Subcarrier Equalization

- Multicarrier modulation lends itself to an efficient receiver implementation when dealing with frequency-selective fading channels
  - When N is very large for the same finite bandwidth, all the subcarriers become sufficiently narrow
  - Channel frequency response across each subcarrier can be approximated by a piecewise constant value represented using a complex gain
  - Consequently, equalization is performed using single tap equalizers per subcarrier such that its product with the subcarrier complex gain results in unity across each subcarrier
- Employing a single tap per subcarrier in order to mitigate the effects of the frequency-selective fading yields both hardware and complexity savings relative to most single carrier solutions

Multipath Propagation

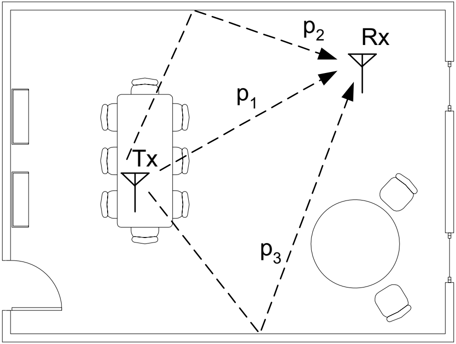

Figure : Example of Indoor Multipath Propagation.

Channel Response

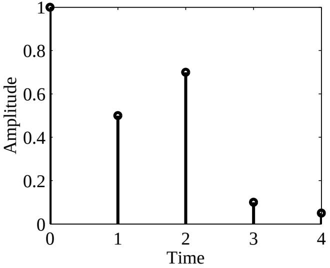

Figure : Channel Impulse Response.

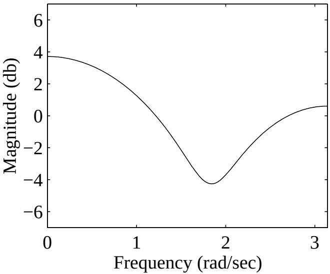

Figure : Channel Frequency Response.

Mathematical Setup

- Suppose the channel frequency response $C(f_k)$, $k = 0, 1, \ldots, N-1$, is equal to:

$$C(f_k) = C_k = |C_k| e^{j\phi_k}, \qquad k = 0, 1, \ldots, N-1 \qquad (3)$$

- Suppose that the received signal on the kth subcarrier is:

$$r_k(t) = \sqrt{\frac{2}{T}} |C_k| \left(A_{kc} \cos(2\pi f_k t + \phi_k) + A_{ks} \sin(2\pi f_k t + \phi_k) \right) + n_k(t)$$
$$= \text{Real}\left\{ \sqrt{\frac{2}{T}} C_k X_k e^{j2\pi f_k t} \right\} + n_k(t)$$

where:

- $n_k(t)$ is zero-mean Gaussian random variable
- C_k and ϕ_k are known at the receiver

Counteracting Frequency-Selective Fading

- Demodulate kth subcarrier of the received signal $r_k(t)$ using the basis functions $(0 \leq t \leq T)$:

$$\Phi_1(t) = \sqrt{\frac{2}{T}}\cos(2\pi f_k t + \phi_k),\ \Phi_2(t) = -\sqrt{\frac{2}{T}}\sin(2\pi f_k t + \phi_k)$$

- Sampling output at $t = kT$ yields:

$$\begin{aligned}\mathbf{y}_k &= (|C_k|A_{ki} + n_{kr}, |C_k|A_{kq} + n_{ki}) \\ &= |C_k|\mathbf{X}_k + \mathbf{n}_k\end{aligned}$$

where $\mathbf{n}_k = n_{kr} + jn_{ki}$ is additive noise

- To effectively equalize the received signal on subcarrier k:

$$\mathbf{Y}'_k = \mathbf{Y}_k/|C_k| = \mathbf{X}_k + \mathbf{n}'_k \tag{4}$$

Fast Fourier Transform Approach

- It is possible to devise a multicarrier modulation communication system based on OFDM using the *Fast Fourier Transform* (FFT) and its inverse (IFFT)
  - The IFFT replaces the synthesis filter bank while the FFT replaces the analysis filter bank
- How do we express this formulation in terms of IFFT and FFT modules?
  - Suppose we have complex-valued signal points X_k, $k = 0, 1, \ldots, \tilde{N} - 1$
  - Compute the inverse discrete Fourier transform (IDFT), which can be implemented efficiently using the IFFT:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi nk/N}, \;\; n = 0, 1, \ldots, N-1$$

QAM Implementation

- If we compute the $\tilde{N}$-point IDFT, we get a complex-valued time series that is not equivalent to $\tilde{N}$ QAM-modulated subcarriers
  - Create $N = 2\tilde{N}$ information symbols by defining:

$$X_{N-k} = X_k^*, \text{ for } k = 1, \ldots, \tilde{N} - 1$$
$$X_0 = \text{Real}\{X_0\}, \quad X_{\tilde{N}} = \text{Imag}\{X_0\}$$

  - We get x_n to be real-valued at the output of the IDFT (IFFT)

FFT Butterfly

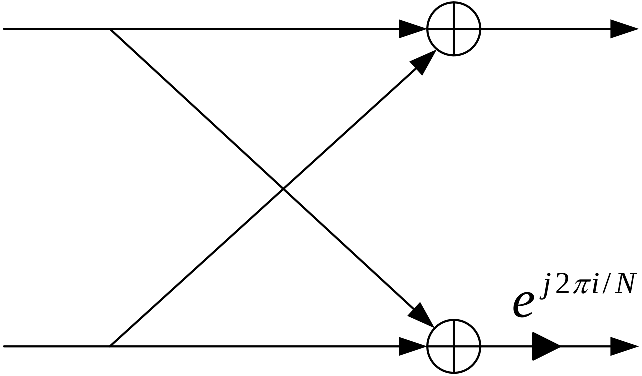

Figure : 2×2 Fast Fourier Transform Butterfly.