Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 04

Signal Types

- Analog signal: continuous in time and amplitude
  - Voltage, current, temperature
- Digital signal: discrete both in time and amplitude
  - Attendance of this class, digitized analog signals
- Discrete-time signal: discrete in time and continuous in amplitude
  - Hourly change of temperature in Worcester

Sampling Theory

- Sampling is a continuous to discrete-time conversion
- Most common sampling is periodic
  - $x[n] = x_c(nT) \quad -\infty < n < \infty$
  - T is the sampling period in second
  - $f_s = 1/T$ is the sampling frequency in Hz
  - Sampling frequency in radian-per-second $\Omega = 2\pi f_s \text{rad/sec}$
  - Use $[\cdot]$ for discrete-time and $(\cdot)$ for continuous-time signals
- This is the ideal case, not the practical, but close enough
  - In practice, it is implemented with an analog-to-digital converter
  - We get digital signals that are quantized in amplitude and time

Sampling Theory

- In general, sampling is not reversible
- Given a sampled signal, one can fit infinite continuous signals through the samples
- Fundamental issue in digital signal sampling
  - If we lose information during sampling, we cannot recover it
  - Under certain conditions, an analog signal can be sampled without loss so that it can be reconstructed perfectly

Representation of Sampling

- Mathematically convenient to represent in two stages
  - Impulse train modulator
  - Conversion of impulse train to a sequence

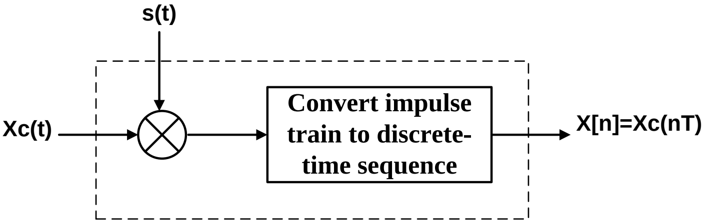

Figure : Conversion of Samples to Analog Waveforms.

Continuous-Time Fourier Transform

- Continuous-Time Fourier transform pair is defined as
  - $X_c(j\Omega) = \int_{-\infty}^{\infty} x_c(t) e^{-j\Omega t} dt$
  - $x_c(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_c(j\Omega) e^{j\Omega t} d\Omega$
- We write $x_c(t)$ as a weighted sum of complex exponentials
- Remember some Fourier Transform properties
  - $x(t) * y(t) \leftrightarrow X(j\Omega)Y(j\Omega)$
  - $x(t)y(t) \leftrightarrow X(j\Omega) * Y(j\Omega)$
  - $x(t)e^{j\Omega_0 t} \leftrightarrow X(j(\Omega - \Omega_0))$

Frequency Domain Representation of Sampling

- Modulate (multiply) continuous-time signal with pulse train:
  - $x_s(t) = x_c(t)s(t) = \sum_{n=-\infty}^{\infty} x_c(t)\delta(t - nT)$
    $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$
- Let's take the Fourier Transform of $x_s(t)$ and $s(t)$
  - $X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$
    $S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s)$
- Fourier transform of pulse train is again a pulse train
- Note that multiplication in time is convolution in frequency
- We represent frequency with $\Omega = 2\pi f$, hence $\Omega_s = 2\pi f_s$
  - $X_s(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$

Frequency Domain Representation of Sampling

- Convolution with pulse creates replicas at pulse location:
  - $X_s(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s))$
- This tells us that the impulse train modulator
  - Creates images of the Fourier transform of the input signal
  - Images are periodic with sampling frequency
  - If $\Omega_s < \Omega_N$, sampling maybe irreversible due to aliasing of images

Frequency Domain Representation of Sampling

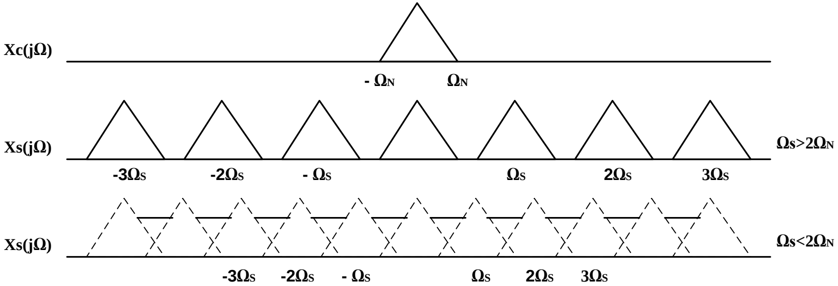

Figure : Signal Spectra (Top: Original, Middle: Sampled Without Aliasing, Bottom: Sampled With Aliasing).

Nyquist Sampling Theorem

- Let $x_c(t)$ be a bandlimited signal with
  - $X_s(j\Omega) = 0$ for $|\Omega| \geq \Omega_N$
- Then $x_c(t)$ is uniquely determined by its samples $x[n] = x_c(nT)$ if $\Omega_s = \frac{2\pi}{T} = 2\pi f_s \geq 2\Omega_N$
  - Ω_N is generally known as the Nyquist Frequency
- The minimum sampling rate that must be exceeded is known as the Nyquist Rate

USRP receive and transmit paths

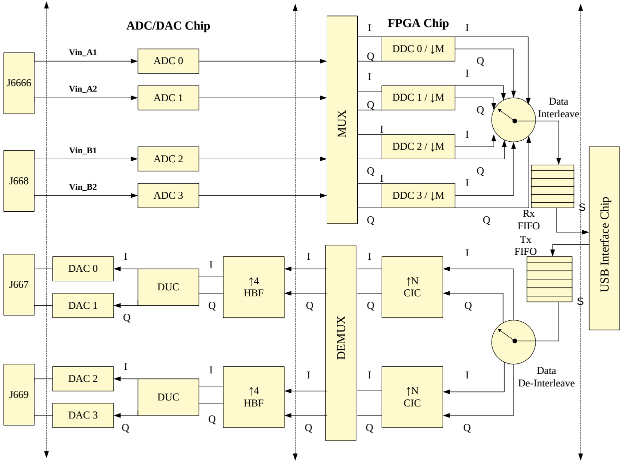

Figure : Schematic of Sampling on USRP2 Platform.

USRP Receive Path

- Main components on Rx path
  - Four analog-to-digital converter (ADC)
  - Four digital down converter (DDC)
- Signal processing on Rx path
  - IF band $\rightarrow$ ADC $\rightarrow$ DDC $\rightarrow$ USRP board $\rightarrow$ Baseband I/Q

DDC on Receive Path

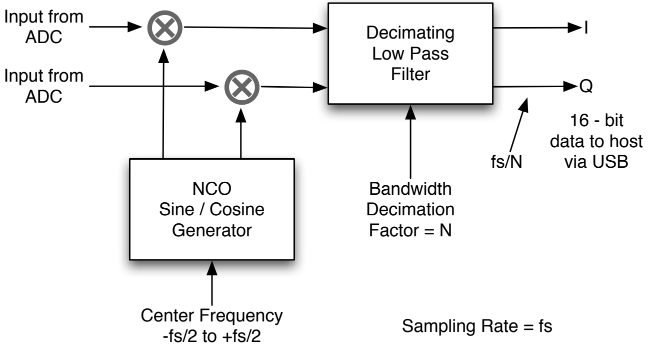

Figure : Schematic of Digital Down Conversion on USRP2 Platform.

Scenario 1: Four Users

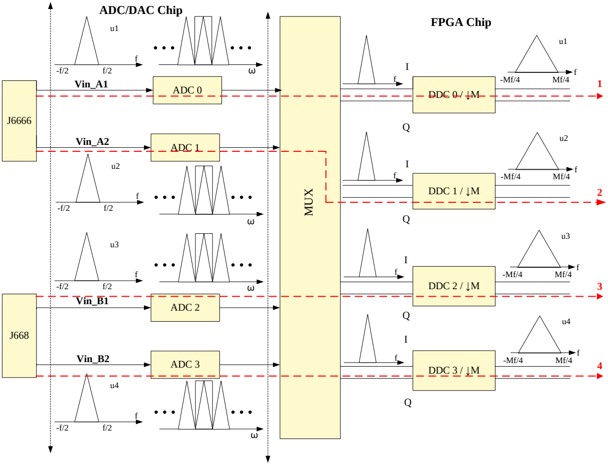

Figure : USRP2 Receiver Configuration for Sampling with Four Users.

Scenario 2: Two Users

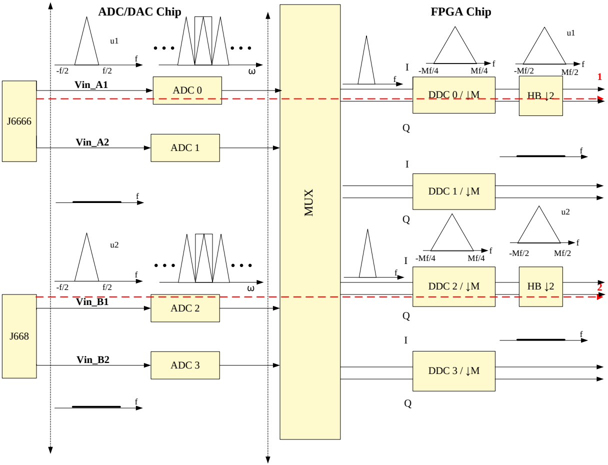

Figure : USRP2 Receiver Configuration for Sampling with Two Users.

USRP Transmit Path

- Main components on Tx path
  - Four digital-to-analog converter (DAC)
  - Four digital up converter (DUC)
- Signal processing on Tx path
  - Reverse to Rx path
  - Baseband I/Q $\rightarrow$ USRP board $\rightarrow$ DUC $\rightarrow$ DAC $\rightarrow$ IF band