Digital Communication Systems Engineering with Software-Defined Radio

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Lecture 07

Anatomy of a Typical Digital Communication System

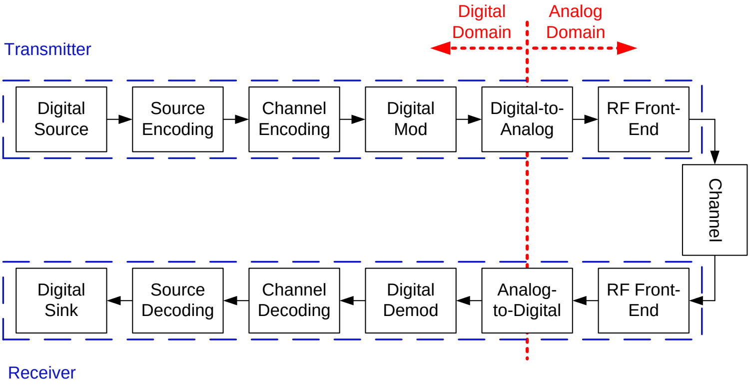

Figure : Generic representation of a digital communication transceiver.

Sending Messages

- Digital communications involves the transmission of a message $m(t)$ and producing a reconstructed version of the message $\hat{m}(t)$ at the output of the receiver
  - Our goal is to have $P(\hat{m}(t) \neq m(t))$ as small as needed for a particular application
  - Probability of Error: $Pe = P(\hat{m}(t) \neq m(t))$
- Various data applications possess different P_e requirements
  - Voice: $Pe \sim 10^{-3}$
  - Data: $Pe \sim 10^{-5} - 10^{-6}$
  - Fiber optics: $Pe \sim 10^{-9}$

Purpose of Source Encoder

- Given $\underline{u}$ = sequence of source symbols and $\underline{v}$ = sequence of source encoded symbols:
  - We should have $v_i \in \underline{v}$ as close to random as possible
  - Components of $\underline{v}$ are uncorrelated, i.e., unrelated
- Why do we perform source encoding?
  - No redundancy in $v_i \in \underline{v}$
  - Do not want to waste channel resources (e.g: power, bandwidth) in the transmission of "predictable symbols"
- Source encoder removes redundant information from the source symbols in order to realize efficient transmission
  - Note that the source needs to be *digital*

A Source Encoding Example

- Analog TV uses 6 MHz bands to transmit every channel
- Digital TV employing source encoding
  - This can be performed since information is digitally represented
  - Eight digitally encoded TV channels can be fit in one 6 MHz band

Why Perform Channel Coding?

- Channel coding is designed to correct channel transmission errors
  - Achieved via *controlled* introduction of redundancy
  - How is this different than the redundancy removed during source encoding?
- Each vector of source encoded outputs, v_l, $l = 1, 2, ..., 2^k$, are assigned a unique *codeword*
  - Take $v_l = (101010\ldots)$ and assign unique codeword $c_l \in \mathbb{C}$ where $\mathbb{C}$ is a *codebook*
- We have added $N - K = r$ controlled number of bits to the channel encoding process
- The *code rate* of a communications system is equal to the ratio of the number of information bits to the size of the codeword, i.e., Code Rate$=k/N$

Hamming Distance

- The *Hamming Distance* $d_H(c_i, c_j)$ between two codewords say c_i and c_j is equal to the number of components in which c_i and c_j are different
- We often are looking for the minimum Hamming Distances between codewords, i.e.:

$$d_{H,\min} = \min_{c_i, c_j \in \mathbb{C},\ i \neq j} d_H(c_i, c_j) \tag{1}$$

- Our goal is to maximize the minimum Hamming Distance in a given codebook
- For example:
  - $\{101, 010\} \rightarrow d_{H,min} = 3 \rightarrow$ GOOD!
  - $\{111, 101\} \rightarrow d_{H,min} = 1 \rightarrow$ POOR!

Decoding Spheres

- We can use *decoding spheres* (also known as *Hamming spheres*) to make decisions on received information
- Decoding spheres should not overlap $\rightarrow d_{H,\min} = 2t + 1$

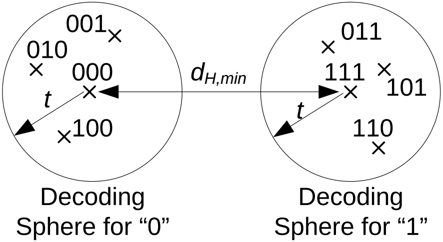

Figure : An Example of Decoding Spheres.

A Channel Encoding Example

- Example: A rate 1/3 repetition code with no source encoding would look like:

$$\begin{aligned} 1 &\rightarrow 111 = c_1 \text{ (1st codeword)} \\ 0 &\rightarrow 000 = c_2 \text{ (2nd codeword)} \\ \therefore C &= \{000, 111\} \end{aligned} \tag{2}$$

- What are the Hamming Distances for the following codeword pairs?
  - d_H(111,000)=3
  - d_H(111,101)=1

Shannon's Channel Coding Theorem (1948)

- Consider a channel with capacity C and we transmit at a fixed code rate K/N which is equal to R_c (a constant)
  - If we increase N, we must increase K to keep R_c equal to a constant
- There exists a code such that for $R_c = K/N < C$ as $N \to \infty$, we have $P_e \to 0$
  - Conversely, for $R_c = K/N \geq C$, no such code exists
- Hence, C is the limit in rate for reliable communications, i.e., C is the **absolute limit** that you cannot go any faster than this amount without causing errors

Shannon's Channel Capacity

- *Reliability* in digital communications is usually expressed as the Probability of Bit Error measured at the output of the receiver
- It would be nice to know what this capacity is given the transmission bandwidth, B, the received signal-to-noise ratio (SNR)
  - Shannon derived the *information capacity of the channel* to be equal to:
  $$C = B \log_2(1 + SNR) \qquad [\mathrm{b/s}] \tag{3}$$
  - The information capacity tells us the achievable data rate, but it does not tell us how to build a transceiver to achieve this

Why Is This Important?

- The information capacity of the channel is useful for the following reasons:
  - This expression provides us with a bound on the achievable data rate given bandwidth B and received SNR, employed in the ratio $\eta = R/C$, where R is the signalling rate and C is the channel capacity
    - As $\eta \to 1$, the system becomes more efficient
  - The capacity expression provides us with a basis for trade-off analysis between B and SNR
  - The capacity expression can be used for comparing the noise performance of one modulated scheme versus another

Additive White Gaussian Noise Channel

- In many instances, we model the communications channel as an additive white Gaussian noise (AWGN) channel
- The white Gaussian noise has an autocorrelation $R_n(\tau) = R_n(-\tau) = E\{n(t)n(t+\tau)\} = (N_0/2)\delta(t)$

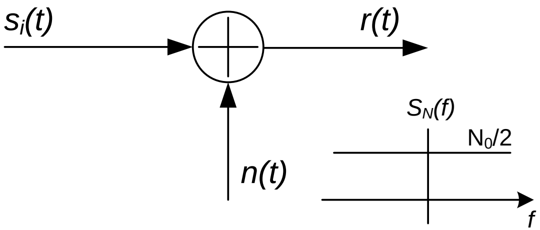

Figure : Additive White Gaussian Noise Channel Model.

Filtered AWGN Channel

- We know that the power spectral density of white Gaussian noise is equal to:

$$S_N(f) = \mathbb{F}\{R_n(\tau)\} = \int_{-\infty}^{\infty} R_n(\tau) e^{-j2\pi f \tau} d\tau = \frac{N_0}{2} \tag{4}$$

- When this noise is passed through an LTI system with impulse response $h(t)$, the output power spectral density will be defined by the Einstein-Wiener-Khinchin (EWK) Theorem, namely:

$$S_Y(f) = |H(f)|^2 S_N(f) \tag{5}$$