Digital Communication Systems Engineering with Software-Defined Radio

Di Pu, Alexander M. Wyglinski
Worcester Polytechnic Institute

Lecture 22

Advantages of Multicarrier Modulation

- Recall from Lecture 15 the following advantages of multicarrier modulation:
  - Multicarrier transmission not as severely affected by delay spread of channel
    - Due to longer symbol periods T_{mc}
    - Channel dispersion in single carrier transmissions much greater than T_{sc}
  - Separate subbands of the multicarrier scheme can more readily handle frequency-selective fading using "divide-and-conquer" approach
    - Simple equalization techniques applied per transmission band
    - Single carrier schemes usually require very complex equalization techniques to undo effects of channel
  - Multicarrier transmission suitable for high data rate communications due to its high robustness to error
    - Employed widely in numerous communication standards
    - Examples include: DSL modems, WiFi, WiMAX

Multicarrier Transmission

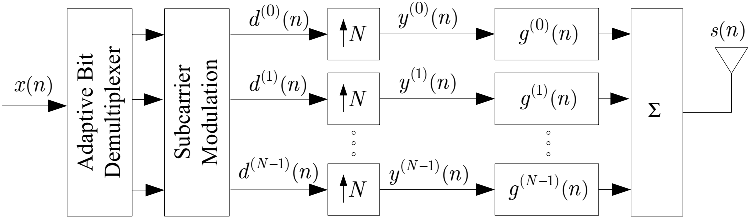

Figure : Schematic of a Generic Multicarrier Transmitter.

Multicarrier Reception

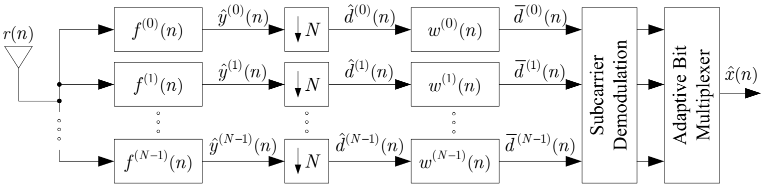

Figure : Schematic of a Generic Multicarrier Receiver.

Recall Shannon Capacity

- From Shannon, we know that the capacity of an ideal, bandlimited, AWGN channel is equal to:

$$C = W \log_2 \left(1 + \frac{\gamma}{W}\right) \tag{1}$$

where:

  - W is the channel bandwidth
  - γ is the ratio of the average transmit power with the noise power spectral density, i.e., signal-to-noise ratio (SNR)
  - C is the capacity in bits/second

Divide-and-Conquer Approach

- For a multicarrier system, we can break down the transmission channel into corresponding subchannels
- The subchannel bandwidth Δf can be made to be sufficiently small
- The capacity of subchannel i is equal to:

$$C_i = \Delta f \log_2\left(1 + \frac{\Delta f P(f_i)|C(f_i)|^2}{\Delta f S_{nn}(f_i)}\right) \tag{2}$$

  where:
  - Δf is the subchannel bandwidth
  - $P(f_i)$ is the power spectral density for the i^{th} subchannel
  - $|C(f_i)|^2$ is the magnitude square of the channel frequency response across the i^{th} subchannel
  - $S_{nn}(f_i)$ is the power spectral density of the noise

Putting It Together

- Under a *global* perspective, we can view each subchannel with its own noise and channel characteristics, which could be unique to the i^{th} subchannel
  - Notice how each subchannel may possess a different $P(f_i)$ and $|C(f_i)|^2$
- Therefore, we can synthesize all the subchannel capacity results into a single overall capacity of the system, namely:

$$C = \sum_{i=1}^{N} C_i = \Delta f \sum_{i=1}^{N} \log_2 \left(1 + \frac{P(f_i)|C(f_i)|^2}{S_{nn}(f_i)} \right) \tag{3}$$

An Infinite Number of Subchannels

- We can convert the overall capacity of the multicarrier system into an integral expression covering a continuum of subchannels across a specified bandwidth W:

$$C = \int_{W} \log_2 \left(1 + \frac{P(f)|C(f)|^2}{S_{nn}(f)} \right) df \tag{4}$$

- Note that we subject $P(f)$ to a *total power constraint*, namely:

$$\int_{W} P(f) df \leq P_{\text{avg}} \tag{5}$$

where P_{avg} is the available average power of the transmitter
  - Why do we need a total power constraint?
  - Is this a sufficient condition?

Maximizing Capacity

- The advantage of expressing the capacity of a system via a multicarrier approach is that we can also tailor (i.e., optimize) the operating parameters of the communication system in order to enhance performance
  - In this case, we can choose $P(f)$ such that C is maximized
  - Recall adaptive bit allocation from Lecture 18
- We can use Lagrange multipliers in order to maximize the integral expression:

$$\int_W \left(\log_2 \left(1 + \frac{P(f)|C(f)|^2}{S_{nn}(f)} \right) + \lambda P(f) \right) df \qquad (6)$$

  - Using *calculus of variations* and obeying the power constraint, we get:

$$\frac{1}{P(f) + S_{nn}(f)/|C(f)|^2} + \lambda = 0 \qquad (7)$$

Power Allocation

- Isolating for $P(f)$, we get the following result:

$$P(f) = \begin{cases} K - S_{nn}(f)/|C(f)|^2 & f \in W \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

  - This result is subject to a total power constraint based on P_{avg}

Interpretation

- We can interpret this resulting expression for $P(f)$ as follows:
  - Signal power is **high** when channel SNR $|C(f)|^2/S_{nn}(f)$ is **high**
  - Signal power is **low** when channel SNR $|C(f)|^2/S_{nn}(f)$ is **low**
- Graphically this looks like filling in the channel frequency response with transmission power
  - Optimal power distribution based on waterfilling interpretation

An Example

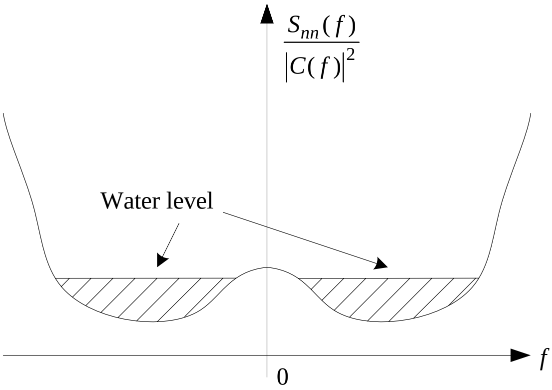

Figure : Schematic of Waterfilling Performed on a Frequency Selective Fading Channel.