Defining, Designing, and Evaluating Digital Communication Systems

A tutorial that emphasizes the subtle but straightforward relationships we encounter when transforming from data-bits to channel-bits to symbols to chips.

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The design of any digital communication system begins with a description of the channel (received power, available bandwidth, noise statistics, and other impairments such as fading), and a definition of the system requirements (data rate and error performance). Given the channel description, we need to determine design choices that best match the channel and meet the performance requirements. An orderly set of transformations and computations has evolved to aid in characterizing a system's performance. Once this approach is understood, it can serve as the format for evaluating most communication systems.

In subsequent sections, we shall examine the following four system examples, chosen to provide a representative assortment: a bandwidth-limited uncoded system, a power-limited uncoded system, a bandwidth-limited and power-limited coded system, and a direct-sequence spread-spectrum coded system. The term coded (or uncoded) refers to the presence (or absence) of error-correction coding schemes involving the use of redundant bits.

Two primary communications resources are the received power and the available transmission bandwidth. In many communication systems, one of these resources may be more precious than the other, and hence most systems can be classified as either bandwidth limited or power limited. In bandwidth-limited systems, spectrally-efficient modulation techniques can be used to save bandwidth at the expense of power; in power-limited systems, power-efficient modulation techniques can be used to save power at the expense of bandwidth. In both bandwidth- and power-limited systems, error-correction coding (often called channel coding) can be used to save power or to improve error performance at the expense of bandwidth. Recently, trellis-coded modulation (TCM) schemes have been used to improve the error performance of bandwidth-limited channels without *any* increase in bandwidth [1], but these methods are beyond the scope of this tutorial.

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The Bandwidth Efficiency Plane

Figure 1 shows the abscissa as the ratio of bit-energy to noise-power spectral density, E_b/N_0 (in decibels), and the ordinate as the ratio of throughput, R (in bits per second), that can be transmitted per hertz in a given bandwidth, W. The ratio R/W is called bandwidth efficiency, since it reflects how efficiently the bandwidth resource is utilized. The plot stems from the Shannon-Hartley capacity theorem [2-4], which can be stated as

$$C = W\log_2\left(1+\frac{S}{N}\right) \qquad (1)$$

where S/N is the ratio of received average signal power to noise power. When the logarithm is taken to the base 2, the capacity, C, is given in b/s. The capacity of a channel defines the maximum number of bits that can be reliably sent per second over the channel. For the case where the data (information) rate, R, is equal to C, the curve separates a region of practical communication systems from a region where such communication systems cannot operate reliably [3,4].

M-ary Signaling

Each symbol in an M-ary alphabet is related to a unique sequence of m bits, expressed as

$$M = 2^m \quad \text{or} \quad m = \log_2 M \qquad (2)$$

where M is the size of the alphabet. In the case of digital transmission, the term "symbol" refers to the member of the M-ary alphabet that is transmitted during each symbol duration, T_s. In order to transmit the symbol, it must be mapped onto an electrical voltage or current waveform. Because the waveform represents the symbol, the terms "symbol" and "waveform" are sometimes used interchangeably. Since one of M symbols or waveforms is transmitted during each symbol duration, T_s, the data rate, R in b/s, can be expressed as

$$R = \frac{m}{T_s} = \frac{\log_2 M}{T_s} \quad \text{bit/s} \tag{3}$$

Data-bit-time duration is the reciprocal of data rate. Similarly, symbol-time duration is the reciprocal of symbol rate. Therefore, from Equation (3), we write that the effective time duration, T_b, of each bit in terms of the symbol duration, T_s, or the symbol rate, R_s, is

$$T_b = \frac{1}{R} = \frac{T_s}{m} = \frac{1}{mR_s} \tag{4}$$

Then, using Equations (2) and (4) we can express the symbol rate, R_s, in terms of the bit rate, R, as follows.

$$R_s = \frac{R}{\log_2 M} \tag{5}$$

From Equations (3) and (4), any digital scheme that transmits $m = \log_2 M$ bits in T_s seconds using a bandwidth of W Hz operates at a bandwidth efficiency of

$$\frac{R}{W} = \frac{\log_2 M}{WT_s} = \frac{1}{WT_b} \quad \text{(bit/s)/Hz} \tag{6}$$

where T_b is the effective time duration of each data bit.

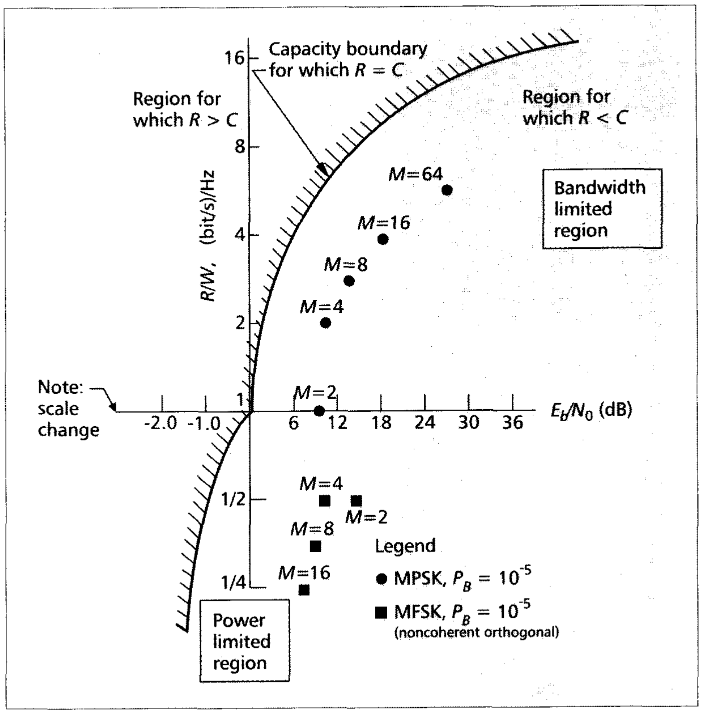
Bandwidth-Limited Systems

From Equation (6), the smaller the WT_b product, the more bandwidth efficient will be any digital communication system. Thus, signals with small WT_b products are often used with bandwidth-limited systems. For example, the new European digital mobile telephone system known as groupe special mobile (GSM) uses Gaussian minimum-shift keying (GMSK) modulation having a WT_b product equal to 0.3 Hz/(b/s), where W is the bandwidth of a Gaussian filter [5].

For uncoded bandwidth-limited systems, the objective is to maximize the transmitted information rate within the allowable bandwidth, at the expense of E_b/N_0 (while maintaining a specified value of bit-error probability, P_B). The operating points for coherent M-ary phase-shift keying (MPSK) at $P_B = 10^{-5}$ are plotted on the bandwidth-efficiency plane (Fig. 1). We assume Nyquist (ideal rectangular) filtering at baseband [6]. Thus, for MPSK, the required double-sideband (DSB) bandwidth at an intermediate frequency (IF) is related to the symbol rate as follows.

$$W = \frac{1}{T_s} = R_s \tag{7}$$

where T_s is the symbol duration, and R_s is the symbol rate. The use of Nyquist filtering results in the minimum required transmission bandwidth that yields zero intersymbol interference; such ideal filtering gives rise to the name Nyquist minimum bandwidth.



■ **Figure 1.** *Bandwidth-efficiency plane.*

From Equations (6) and (7), the bandwidth efficiency of MPSK modulated signals using Nyquist filtering can be expressed as

$$\frac{R}{W} = \log_2 M \quad \text{(bit/s)/Hz} \tag{8}$$

The MPSK points in Fig. 1 confirm the relationship shown in Equation (8). Note that MPSK modulation is a bandwidth-efficient scheme. As M increases in value, R/W also increases. MPSK modulation can be used for realizing an improvement in bandwidth efficiency at the cost of increased E_b/N_0. Although beyond the scope of this article, many highly bandwidth-efficient modulation schemes are under investigation [7].

Power-Limited Systems

Operating points for noncoherent orthogonal M-ary frequency-shift keying (MFSK) modulation at $P_B = 10^{-5}$ are also plotted (Fig. 1). For MFSK, the IF Nyquist minimum bandwidth is as follows [4]:

$$W = \frac{M}{T_s} = MR_s \tag{9}$$

where T_s is the symbol duration, and R_s is the symbol rate. With MFSK, the required transmission bandwidth is expanded M-fold over binary FSK since there are M different orthogonal waveforms, each requiring a bandwidth of $1/T_s$. Thus, from Equations (6) and (9), the bandwidth efficiency of noncoherent orthogonal MFSK signals using Nyquist filtering can be expressed as

$$\frac{R}{W} = \frac{\log_2 M}{M} \quad \text{(bit/s)/Hz} \tag{10}$$

The MFSK points in Fig. 1 confirm the relationship shown in Equation (10). Note that MFSK modulation is a bandwidth-expansive scheme. As M increases, R/W decreases. MFSK modulation can be

M	m	R (b/s)	R_s (symb/s)	MPSK minimum bandwidth (Hz)	MPSK R/W	MPSK E_b/N_0 (dB) $P_B = 10^{-5}$	Noncoherent orthog MFSK min. bandwidth (Hz)	MFSK R/W	MFSK E_b/N_0 (dB) $P_B = 10^{-5}$
2	1	9600	9600	9600	1	9.6	19,200	1/2	13.4
4	2	9600	4800	4800	2	9.6	19,200	1/2	10.6
8	3	9600	3200	3200	3	13.0	25,600	1/3	9.1
16	4	9600	2400	2400	4	17.5	38,400	1/4	8.1
32	5	9600	1920	1920	5	22.4	61,440	1/8	7.4

■ **Table 1.** *Symbol rate, Nyquist minimum bandwidth, bandwidth efficiency, and required E_b/N_0 for MPSK and noncoherent orthogonal MFSK signaling at 9600 b/s.*

used for realizing a reduction in required E_b/N_0 at the cost of increased bandwidth.

In Equations (7) and (8) for MPSK, and Equations (9) and (10) for MFSK, and for all the points plotted in Fig. 1, Nyquist (ideal rectangular) filtering has been assumed. Such filters are not realizable! For realistic channels and waveforms, the required transmission bandwidth must be increased to account for realizable filters.

In the examples that follow, we will consider radio channels that are disturbed only by additive white Gaussian noise (AWGN) and have no other impairments and, for simplicity, we will limit the modulation choice to constant-envelope types, i.e., either MPSK or noncoherent orthogonal MFSK. For an uncoded system, MPSK is selected if the channel is bandwidth limited, and MFSK is selected if the channel is power limited. When error-correction coding is considered, modulation selection is not so simple, because coding techniques can provide power-bandwidth tradeoffs more effectively than would be possible through the use of any M-ary modulation scheme considered in this article [8].

In the most general sense, M-ary signaling can be regarded as a waveform-coding procedure, i.e., when we select an M-ary modulation technique instead of a binary one, we in effect have replaced the binary waveforms with better waveforms — either better for bandwidth performance (MPSK), or better for power performance (MFSK). Even though orthogonal MFSK signaling can be considered a coded system, i.e., a first-order Reed-Muller code [9], we restrict our use of the term "coded system" to those traditional error-correction codes using redundancies, e.g., block codes and convolutional codes.

Nyquist Minimum Bandwidth Requirements for MPSK and MFSK Signaling

The basic relationship between the symbol (or waveform) transmission rate, R_s, and the data rate, R, was shown in Equation (5) to be

$$R_s = \frac{R}{\log_2 M}$$

Using this relationship together with Equations (7-10) and R = 9600 b/s, a summary of symbol rate, Nyquist minimum bandwidth, and bandwidth efficiency for MPSK and noncoherent orthogonal MFSK was compiled for M = 2, 4, 8, 16, and 32 (Table 1). Values of E_b/N_0 required to achieve a bit-error probability of 10^{-5} for MPSK and MFSK are also given for each value of M. These entries (which were computed using relationships that are presented later in this paper) corroborate the trade-offs shown in Fig. 1. As M increases, MPSK signaling provides more bandwidth efficiency at the cost of increased E_b/N_0, while MFSK signaling allows a reduction in E_b/N_0 at the cost of increased bandwidth.

Example 1: Bandwidth-limited Uncoded System

Suppose we are given a bandwidth-limited AWGN radio channel with an available bandwidth of W = 4000 Hz. Also, suppose that the link constraints (transmitter power, antenna gains, path loss, etc.) result in the received average signal-power to noise-power spectral density, S/N_0 being equal to 53 dB-Hz. Let the required data rate, R, be equal to 9600 b/s, and let the required bit-error performance, P_B, be at most 10^{-5}. The goal is to choose a modulation scheme that meets the required performance. In general, an error-correction coding scheme may be needed if none of the allowable modulation schemes can meet the requirements. However, in this example, we shall find that the use of error-correction coding is not necessary.

Solution to Example 1

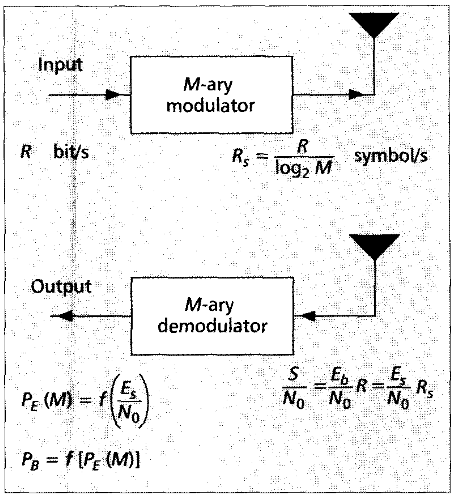
For any digital communication system, the relationship between received S/N_0 and received bit-energy to noise-power spectral density, E_b/N_0, is as follows [4].

$$\frac{S}{N_0} = \frac{E_b}{N_0} R \tag{11}$$

Solving for E_b/N_0 in decibels, we obtain

$$\begin{aligned} \frac{E_b}{N_0}(\text{dB}) &= \frac{S}{N_0}(\text{dB-Hz}) - R\ (\text{dB-bit/s}) \\ &= 53\,\text{dB-Hz} - (10 \times \log_{10} 9600)\ \text{dB-bit/s} \\ &= 13.2\,\text{dB (or 20.89)} \end{aligned} \tag{12}$$

Since the required data rate of 9600 b/s is much larger than the available bandwidth of 4000 Hz, the channel is bandwidth limited. We therefore select MPSK as our modulation scheme. We have confined the possible modulation choices to be constant-envelope types; without such a restriction, we would be able to select a modulation type with greater bandwidth-efficiency. To conserve power, we compute the *smallest possible* value of M such that the MPSK minimum bandwidth does not exceed the available bandwidth of 4000 Hz.



■ **Figure 2.** *Basic modulator/demodulator (MODEM) without channel coding.*

Table 1 shows that the smallest value of M meeting this requirement is $M = 8$. Next, we determine whether the required bit-error performance of $P_B \leq 10^{-5}$ can be met by using 8-PSK modulation alone, or whether it is necessary to use an error-correction coding scheme. Table 1 shows that 8-PSK alone will meet the requirements, since the required E_b/N_0 listed for 8-PSK is less than the received E_b/N_0 derived in Equation (12). Let us imagine that we do not have Table 1, however, and evaluate whether or not error-correction coding is necessary.

Figure 2 shows a basic modulator/demodulator (MODEM) block diagram that summarizes the functional details of this design. At the modulator, the transformation from data bits to symbols yields an output symbol rate, R_s, that is a factor $\log_2 M$ smaller than the input data-bit rate, R, as is seen in Equation (5). Similarly, at the input to the demodulator, the symbol-energy to noise-power spectral density E_s/N_0 is a factor $\log_2 M$ larger than E_b/N_0, since each symbol is made up of $\log_2 M$ bits. Because E_s/N_0 is larger than E_b/N_0 by the same factor that R_s is smaller than R, we can expand Equation (11), as follows.

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_s}{N_0} R_s \tag{13}$$

The demodulator receives a waveform (in this example, one of $M = 8$ possible phase shifts) during each time interval T_s. The probability that the demodulator makes a symbol error, $P_E(M)$, is well approximated by the following equation [10].

$$P_E(M) \cong 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right] \quad \text{for } M > 2 \tag{14}$$

where $Q(x)$, sometimes called the complementary error function, represents the probability under the tail of a zero-mean unit-variance Gaussian density function. It is defined as follows [11].

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) du \tag{15}$$

A good approximation for $Q(x)$, valid for $x > 3$, is given by the following equation [12].

$$Q(x) \cong \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{16}$$

In Fig. 2 and all the figures that follow, rather than show explicit probability relationships, the generalized notation f(x) has been used to indicate some functional dependence on x.

A traditional way of characterizing communication efficiency in digital systems is in terms of the received E_b/N_0 in decibels. This E_b/N_0 description has become standard practice, but recall that there are no bits at the input to the demodulator; there are only waveforms that have been assigned bit meanings. The received E_b/N_0 represents a bit-apportionment of the arriving waveform energy.

To solve for $P_E(M)$ in Equation (14), we need to compute the ratio of received symbol-energy to noise-power spectral density, E_s/N_0. Since from Equation (12)

$$\frac{E_b}{N_0} = 13.2 \text{ dB (or 20.89)}$$

and because each symbol is made up of $\log_2 M$ bits, we compute the following using $M = 8$.

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_b}{N_0} = 3 \times 20.89 = 62.67 \tag{17}$$

Using the results of Equation (17) in Equation (14), yields the symbol-error probability, $P_E = 2.2 \times 10^{-5}$. To transform this to bit-error probability, we use the relationship between bit-error probability P_B, and symbol-error probability P_E, for multiple-phase signaling [9], as follows:

$$P_B \cong \frac{P_E}{\log_2 M} = \frac{P_E}{m} \quad (\text{for } P_E \ll 1) \tag{18}$$

which is a good approximation when Gray coding is used for the bit-to-symbol assignment [10]. This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. No error-correction coding is necessary and 8-PSK modulation represents the design choice to meet the requirements of the bandwidth-limited channel (as we had predicted by examining the required E_b/N_0 values in Table 1).

Example 2: Power-limited Uncoded System

Now, suppose that we have exactly the same data rate and bit-error probability requirements as in Example 1, but let the available bandwidth, W, be equal to 45 kHz, and the available S/N_0 be equal to 48 dB-Hz. The goal is to choose a modulation or modulation/coding scheme that yields the required performance. We shall again find that error-correction coding is not required.

Solution to Example 2

The channel is clearly not bandwidth limited since the available bandwidth of 45 kHz is more than adequate for supporting the required data rate of 9600 b/s. We find the received E_b/N_0 from Equation (12) as follows.

$$\frac{E_b}{N_0}(\text{dB}) = 48 \text{ dB-Hz} - (10 \times \log_{10} 9600) \text{ dB-bit/s} = 8.2 \text{ dB (or 6.61)} \tag{19}$$

F*or an uncoded system, we select MPSK if the channel is bandwidth limited, and we select MFSK if the channel is power limited.*

The code should be as simple as possible. Generally, the shorter the code, the simpler will be its implementation.

Since there is abundant bandwidth but a relatively small E_b/N_0 for the required bit-error probability, we consider that this channel is power limited and choose MFSK as the modulation scheme. To conserve power, we search for the *largest possible* M such that the MFSK minimum bandwidth is not expanded beyond our available bandwidth of 45 kHz. A search results in the choice of $M = 16$ (Table 1). Next, we determine whether the required error performance of $P_B \leq 10^{-5}$ can be met using 16-FSK alone, i.e., without error-correction coding. Table 1 shows that 16-FSK alone meets the requirements, since the required E_b/N_0 for 16-FSK is less than the received E_b/N_0 derived in Equation (19). Let us imagine again that we do not have Table 1 and evaluate whether or not error-correction coding is necessary.

The block diagram in Fig. 2 summarizes the relationships between symbol rate R_s and bit rate R, and between E_s/N_0 and E_b/N_0, which is identical to each of the respective relationships in Example 1. The 16-FSK demodulator receives a waveform (one of 16 possible frequencies) during each symbol time interval T_s. For noncoherent orthogonal MFSK, the probability that the demodulator makes a symbol error, $P_E(M)$, is approximated by the following upper bound [13].

$$P_E(M) \leq \frac{M-1}{2}\exp\left(-\frac{E_s}{2N_0}\right) \tag{20}$$

To solve for $P_E(M)$ in Equation (20), we compute E_s/N_0, as in Example 1. Using the results of Equation (19) in Equation (17), with $M = 16$, we get

$$\begin{aligned}\frac{E_s}{N_0} &= (\log_2 M)\frac{E_b}{N_0} \\ &= 4 \times 6.61 = 26.44\end{aligned} \tag{21}$$

Next, using the results of Equation (21) in Equation (20) yields the symbol-error probability, $P_E = 1.4 \times 10^{-5}$. To transform this to bit-error probability, P_B, we use the relationship between P_B and P_E for orthogonal signaling [13], given by

$$P_B = \frac{2^{m-1}}{2^m - 1}P_E \tag{22}$$

This last computation yields $P_B = 7.3 \times 10^{-6}$, which meets the required bit-error performance. We can meet the given specifications for this power-limited channel by using 16-FSK modulation, without any need for error-correction coding (as we had predicted by examining the required E_b/N_0 values in Table 1).

Example 3: Bandwidth-limited and Power-limited Coded System

We start with the same channel parameters as in Example 1 ($W = 4000$ Hz, $S/N_0 = 53$ dB-Hz, and $R = 9600$ b/s), with one exception. In this example, we specify that P_B must be at most 10^{-9}. Table 1 shows that the system is both bandwidth limited and power limited, based on the available bandwidth of 4000 Hz and the available E_b/N_0 of 13.2 dB, from Equation (12). (8-PSK is the only possible choice to meet the bandwidth constraint; however, the available E_b/N_0 of 13.2 dB is certainly insufficient to meet the required P_B of 10^{-9}). For this small value of P_B, we need to consider the performance improvement that error-correction coding can provide within the available bandwidth. In general, one can use convolutional codes or block codes.

n	k	t
7	4	1
15	11	1
	7	2
	5	3
31	26	1
	21	2
	16	3
	11	5
63	57	1
	51	2
	45	3
	39	4
	36	5
	30	6
127	120	1
	113	2
	106	3
	99	4
	92	5
	85	6
	78	7
	71	9
	64	10

■ **Table 2.** *BCH codes (partial catalog).*

The Bose, Chaudhuri, and Hocquenghem (BCH) codes form a large class of powerful error-correcting cyclic (block) codes [14]. To simplify the explanation, we shall choose a block code from the BCH family. Table 2 presents a partial catalog of the available BCH codes in terms of n, k, and t, where k represents the number of information (or data) bits that the code transforms into a longer block of n coded bits (or channel bits), and t represents the largest number of incorrect channel bits that the code can correct within each n-sized block. The rate of a code is defined as the ratio k/n; its inverse represents a measure of the code's redundancy [14].

Solution to Example 3

Since this example has the same bandwidth-limited parameters given in Example 1, we start with the same 8-PSK modulation used to meet the stated bandwidth constraint. However, we now employ error-correction coding so that the bit-error probability can be lowered to $P_B \leq 10^{-9}$.

To make the optimum code selection from Table 2, we are guided by the following goals:

- The output bit-error probability of the combined modulation/coding system must meet the system error requirement.
- The rate of the code must not expand the required transmission bandwidth beyond the available channel bandwidth.
- The code should be as simple as possible. Generally, the shorter the code, the simpler will be its implementation.

The uncoded 8-PSK minimum bandwidth requirement is 3200 Hz (Table 1) and the allowable channel bandwidth is 4000 Hz, so the uncoded signal bandwidth can be increased by no more than a factor of 1.25 (i.e., an expansion of 25 percent). The very first step in this (simplified) code selection example is to eliminate the candidates in Table 2 that would expand the bandwidth by more than 25 percent. The remaining entries form a much reduced set of "bandwidth-compatible" codes (Table 3).

A column designated "Coding Gain, G" has been added for MPSK at $P_B = 10^{-9}$ (Table 3). Coding gain in decibels is defined as follows.

$$G\ (\text{dB}) = \left(\frac{E_b}{N_0}\right)_{uncoded} (\text{dB}) - \left(\frac{E_b}{N_0}\right)_{coded} (\text{dB}) \quad (23)$$

G can be described as the reduction in the required E_b/N_0 (in decibels) that is needed due to the error-performance properties of the channel coding. G is a function of the modulation type and bit-error probability, and it has been computed for MPSK at $P_B = 10^{-9}$ (Table 3). For MPSK modulation, G is relatively independent of the value of M. Thus, for a particular bit-error probability, a given code provides about the same coding gain when used with any of the MPSK modulation schemes. Coding gains were calculated using a procedure outlined in the "Calculating Coding Gain" section below.

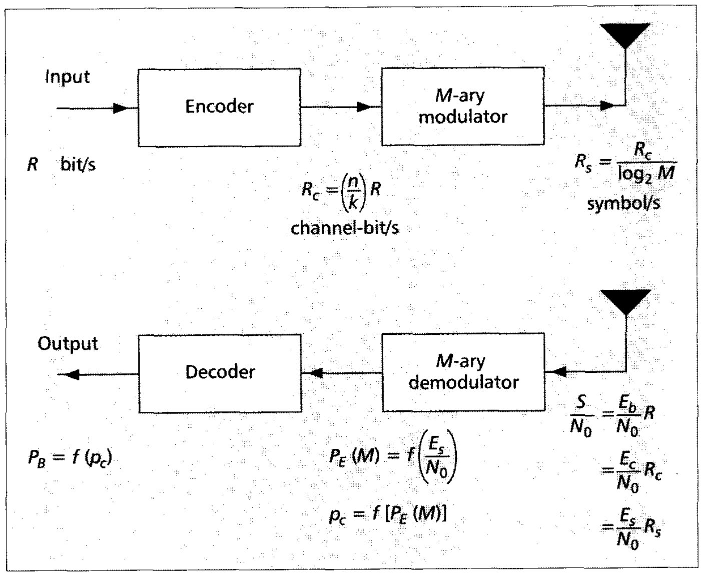
A block diagram summarizes this system which contains both modulation and coding (Fig. 3). The introduction of encoder/decoder blocks brings about additional transformations. The relationships that exist when transforming from R b/s to R_c channel-b/s to R_s symbol/s are shown at the encoder/modulator. Regarding the channel-bit rate, R_c, some authors prefer the units of channel-symbol/s (or code-symbol/s). The benefit is that error-correction coding is often described more efficiently with nonbinary digits. We reserve the term "symbol" for that group of bits mapped onto an electrical waveform for transmission, and we designate the units of R_c to be channel-b/s (or coded-b/s).

We assume that our communication system cannot tolerate any message delay, so the channel-bit rate, R_c, must exceed the data-bit rate, R, by the factor n/k. Further, each symbol is made up of $\log_2 M$ channel bits, so the symbol rate, R_s, is less than R_c by the factor $\log_2 M$. For a system containing both modulation and coding, we summarize the rate transformations as follows.

$$R_c = \left(\frac{n}{k}\right) R \quad (24)$$

$$R_s = \frac{R_c}{\log_2 M} \quad (25)$$

At the demodulator/decoder in Fig. 3, the transformations among data-bit energy, channel-bit energy, and symbol energy are related (in a reciprocal fashion) by the same factors as shown among the rate transformations in Equations (24) and (25). Since the encoding transformation has replaced k data bits with n channel bits, then the ratio of channel-bit energy to noise-power spectral density, E_c/N_0, is computed by decrementing the value of E_b/N_0 by the factor k/n. Also, since each transmission symbol is made up of $\log_2 M$ channel bits, then E_s/N_0, which is needed in Equation (14) to solve for P_E, is computed by incrementing E_c/N_0 by the factor $\log_2 M$. For a system containing both modulation and coding, we summarize the energy to noise-power spectral density transformations, as follows.



■ **Figure 3.** *MODEM with channel coding.*

n	k	t	Coding Gain, G (dB) MPSK, $P_B = 10^{-9}$
31	26	1	2.0
63	57	1	2.2
	51	2	3.1
127	120	1	2.2
	113	2	3.3
	106	3	3.9

■ **Table 3.** *Bandwidth-compatible BCH codes.*

$$\frac{E_c}{N_0} = \left(\frac{k}{n}\right) \frac{E_b}{N_0} \quad (26)$$

$$\frac{E_s}{N_0} = (\log_2 M) \frac{E_c}{N_0} \quad (27)$$

Using Equations (24) through (27), we can now expand the expression for S/N_0 in Equation (13), as follows (Appendix A).

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s \quad (28)$$

As before, a standard way of describing the link is in terms of the received E_b/N_0 in decibels. However, there are no data bits at the input to the demodulator, and there are no channel bits; there are only waveforms that have bit meanings, and thus the waveforms can be described in terms of bit-energy apportionments.

Since S/N_0 and R were given as 53 dB-Hz and 9600 b/s, respectively, we find as before, from Equation (12), that the received E_b/N_0 = 13.2 dB. The received E_b/N_0 is fixed and independent of n, k, and t (Appendix A). As we search Table 3 for the ideal code to meet the specifications, we can iteratively repeat the computations suggested in Fig. 3. It might be useful to program on a PC (or cal-

For error-performance improvement due to coding, the decoder must provide enough error correction to more than compensate for the poor performance of the demodulator.

culator) the following four steps as a function of n, k, and t. Step 1 starts by combining Equations (26) and (27).

Step 1:

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_c}{N_0} = (\log_2 M)\left(\frac{k}{n}\right)\frac{E_b}{N_0} \quad (29)$$

Step 2:

$$P_E(M) \cong 2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right] \quad (30)$$

which is the approximation for symbol-error probability, P_E, rewritten from Equation (14). At each symbol-time interval, the demodulator makes a symbol decision, but it delivers a channel-bit sequence representing that symbol to the decoder. When the channel-bit output of the demodulator is quantized to two levels, 1 and 0, the demodulator is said to make hard decisions. When the output is quantized to more than two levels, the demodulator is said to make soft decisions [4]. Throughout this paper, we assume hard-decision demodulation.

Now that we have a decoder block in the system, we designate the channel-bit-error probability out of the demodulator and into the decoder as p_c, and we reserve the notation P_B for the bit-error probability out of the decoder. We rewrite Equation (18) in terms of p_c as follows.

Step 3:

$$p_c \cong \frac{P_E}{\log_2 M} = \frac{P_E}{m} \quad (31)$$

relating the channel-bit-error probability to the symbol-error probability out of the demodulator, assuming Gray coding, as referenced in Equation (18).

For traditional channel-coding schemes and a given value of received S/N_0, the value of E_s/N_0 with coding will always be less than the value of E_s/N_0 without coding. Since the demodulator with coding receives less E_s/N_0, it makes more errors! When coding is used, however, the system error-performance doesn't only depend on the performance of the demodulator, it also depends on the performance of the decoder. For error-performance improvement due to coding, the decoder must provide enough error correction to more than compensate for the poor performance of the demodulator.

The final output decoded bit-error probability, P_B, depends on the particular code, the decoder, and the channel-bit-error probability, p_c. It can be expressed by the following approximation [15].

Step 4:

$$P_B \cong \frac{1}{n}\sum_{j=t+1}^{n} j\binom{n}{j} p_c^j (1-p_c)^{n-j} \quad (32)$$

where t is the largest number of channel bits that the code can correct within each block of n bits. Using Equations (29) through (32) in the above four steps, we can compute the decoded bit-error probability, P_B, as a function of n, k, and t for each of the codes listed in Table 3. The entry that meets the stated error requirement with the largest possible code rate and the smallest value of n is the double-error correcting (63, 51) code. The computations are

Step 1:

$$\frac{E_s}{N_0} = 3\left(\frac{51}{63}\right)20.89 = 50.73$$

where M = 8, and the received E_b/N_0 = 13.2 dB (or 20.89).

Step 2:

$$P_E \cong 2Q\left[\sqrt{101.5} \times \sin\left(\frac{\pi}{8}\right)\right]$$

$$= 2Q(3.86) = 1.2 \times 10^{-4}$$

Step 3:

$$p_c \cong \frac{1.2\times10^{-4}}{3} = 4\times10^{-5}$$

Step 4:

$$P_B \cong \frac{3}{63}\binom{63}{3}(4\times10^{-5})^3\ (1-4\times10^{-5})^{60}$$
$$+\frac{4}{63}\binom{63}{4}(4\times10^{-5})^4\ (1-4\times10^{-5})^{59} + \ldots$$
$$= 1.2\times10^{-10}$$

where the bit-error-correcting capability of the code is t = 2. For the computation of P_B in Step 4, we need only consider the first two terms in the summation of Equation (32) since the other terms have a vanishingly small effect on the result. Now that we have selected the (63, 51) code, we can compute the values of channel-bit rate, R_c, and symbol rate, R_s, using Equations (24) and (25), with M = 8.

$$R_c = \left(\frac{n}{k}\right)R = \left(\frac{63}{51}\right)9600 \cong 11{,}859 \text{ channel-bit / s}$$

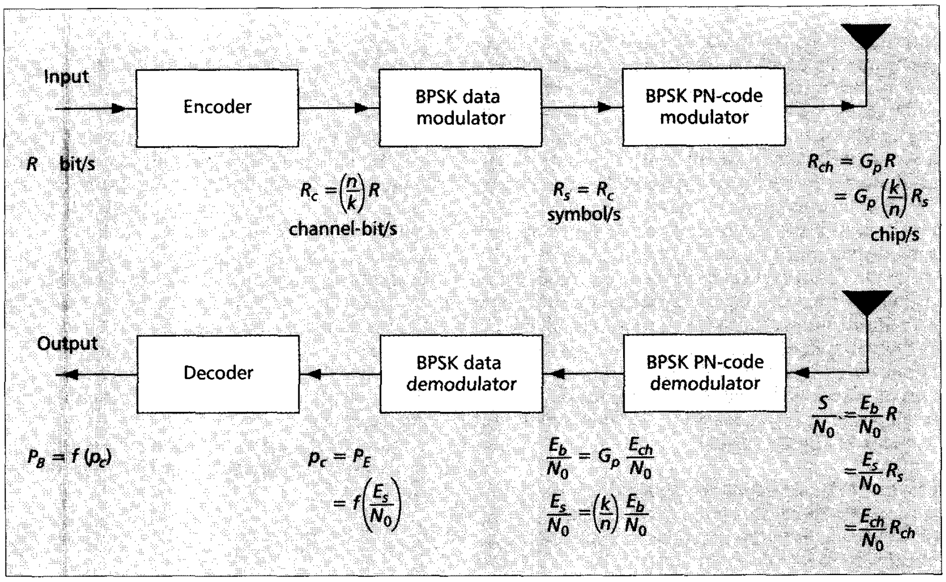
$$R_s = \frac{R_c}{\log_2 M} = \frac{11859}{3} = 3953 \text{ symbol / s}$$

Calculating Coding Gain

Perhaps a more direct way of finding the simplest code that meets the specified error performance is to first compute how much coding gain, G, is required in order to yield $P_B = 10^{-9}$ when using 8-PSK modulation alone; then we can simply choose the code that provides this performance improvement (Table 3). First, we find the uncoded E_s/N_0 that yields an error probability of $P_B = 10^{-9}$ by writing from Equations (18) and (31) the following.

$$P_B \cong \frac{P_E}{\log_2 M} \cong \frac{2Q\left[\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right]}{\log_2 M} = 10^{-9} \quad (33)$$

At this low value of bit-error probability, it is valid to use Equation (16) to approximate $Q(x)$ in Equation (33). By trial-and-error (on a programmable calculator), we find that the uncoded E_s/N_0 = 120.67 = 20.8 dB, and since each symbol is made up of $\log_2 8$ = 3 bits, the required $(E_b/N_0)_{uncoded}$ = 120.67/3 = 40.22 = 16 dB. From the given parameters and Equation (12), we know that the received $(E_b/N_0)_{coded}$ = 13.2 dB. Using Equation (23), the



■ **Figure 4.** *Direct-sequence spread-spectrum MODEM with channel coding.*

For this spread-spectrum example, it is useful to characterize the modulation process in two separate steps.

required coding gain to meet the bit-error performance of $P_B = 10^{-9}$ is

$$G\ (\text{dB}) = \left(\frac{E_b}{N_0}\right)_{uncoded}(\text{dB}) - \left(\frac{E_b}{N_0}\right)_{coded}(\text{dB})$$
$$= 16\ \text{dB} - 13.2\ \text{dB} = 2.8\ \text{dB}$$

To be precise, each of the E_b/N_0 values in the above computation must correspond to exactly the same value of bit-error probability (which they do not). They correspond to $P_B = 10^{-9}$ and $P_B = 1.2 \times 10^{-10}$, respectively. However, at these low probability values, even with such a discrepancy, this computation still provides a good approximation of the required coding gain. In searching Table 3 for the simplest code that will yield a coding gain of at least 2.8 dB, we see that the choice is the (63, 51) code, which corresponds to the same code choice that we made earlier.

Example 4: Direct Sequence (DS) Spread Spectrum Coded System

Spread-spectrum systems are not usually classified as being bandwidth- or power-limited. However, they are generally perceived to be power-limited systems because the bandwidth occupancy of the information is much larger than the bandwidth that is intrinsically needed for the information transmission. In a direct-sequence spread-spectrum (DS/SS) system, spreading the signal bandwidth by some factor permits lowering the signal-power spectral density by the same factor (the total average signal power is the same as before spreading). The bandwidth spreading is typically accomplished by multiplying a relatively narrowband data signal by a wideband spreading signal. The spreading signal or spreading code is often referred to as a pseudorandom code, or PN code.

Processing Gain — A typical DS/SS radio system is often described as a two-step BPSK modulation process. In the first step, the carrier wave is modulated by a bipolar data waveform having a value +1 or −1 during each data-bit duration; in the second step, the output of the first step is multiplied (modulated) by a bipolar PN-code waveform having a value +1 or −1 during each PN-code-bit duration. In reality, DS/SS systems are usually implemented by first multiplying the data waveform by the PN-code waveform and then making a single pass through a BPSK modulator. For this example, however, it is useful to characterize the modulation process in two separate steps — the outer modulator/demodulator for the data, and the inner modulator/demodulator for the PN code (Fig. 4).

A spread-spectrum system is characterized by a processing gain, G_p, that is defined in terms of the spread-spectrum bandwidth, W_{ss}, and the data rate, R, as follows [16].

$$G_p = \frac{W_{ss}}{R} \tag{34}$$

For a DS/SS system, the PN-code bit has been given the name "chip," and the spread-spectrum signal bandwidth can be shown to be about equal to the chip rate. Thus, for a DS/SS system, the processing gain in Equation (34) is generally expressed in terms of the chip rate, R_{ch}, as follows.

$$G_p = \frac{R_{ch}}{R} \tag{35}$$

Some authors define processing gain to be the ratio of the spread-spectrum bandwidth to the symbol rate. This definition separates the system performance due to bandwidth spreading from the performance due to error-correction coding. Since we ultimately want to relate all of the coding mechanisms relative to the information source, we shall conform to the most usually accepted definition for processing gain, as expressed in Equations (34) and (35).

A spread-spectrum system can be used for interference rejection and multiple access (allowing multiple users to access a communications resource simultaneously). The benefits of DS/SS signals are best achieved when the processing gain is very large; in other words, the chip rate of the spreading (or PN) code is much larger than the data

Received power is the same, whether computed on the basis of data-bits, channel-bits, symbols, or chips.

rate. In such systems, the large value of G_p allows the signaling chips to be transmitted at a power level well below that of the thermal noise. We will use a value of $G_p = 1000$. At the receiver, the despreading operation correlates the incoming signal with a synchronized copy of the PN code, and thus accumulates the energy from multiple (G_p) chips to yield the energy per data bit. The value of G_p has a major influence on the performance of the spread-spectrum system application. However, the value of G_p has no effect on the received E_b/N_0. In other words, spread spectrum techniques offer no error-performance advantage over thermal noise. For DS/SS systems, there is no disadvantage either! Sometimes such spread-spectrum radio systems are employed only to enable the transmission of very small power-spectral densities, and thus avoid the need for FCC licensing [17].

Channel Parameters for Example 4 — Consider a DS/SS radio system that uses the same (63, 51) code as in the previous example. Instead of using MPSK for the data modulation, we shall use BPSK. Also, we shall use BPSK for modulating the PN-code chips. Let the received S/N_0 = 48 dB-Hz, the data rate R = 9600 b/s, and the required $P_B \leq 10^{-6}$. For simplicity, assume that there are no bandwidth constraints. Our task is simply to determine whether or not the required error performance can be achieved using the given system architecture and design parameters. In evaluating the system, we will use the same type of transformations used in previous examples.

Solution to Example 4

A typical DS/SS system can be implemented more simply than the one shown in Fig. 4. The data and the PN code would be combined at baseband, followed by a single pass through a BPSK modulator. We assume the existence of the individual blocks in Fig. 4, however, because they enhance our understanding of the transformation process. The relationships in transforming from data bits, to channel bits, to symbols, and to chips (Fig. 4) have the same pattern of subtle but straightforward transformations in rates and energies as previous relationships (Figs. 2-3). The values of R_c, R_s, and R_{ch} can now be calculated immediately since the (63,51) BCH code has already been selected. From Equation (24)

$$R_c = \left(\frac{n}{k}\right) R = \left(\frac{63}{51}\right) 9600 \cong 11{,}859 \text{ channel-bit/s}$$

Since the data modulation considered here is BPSK,

$$R_s = R_c \cong 11{,}859 \text{ symbol/s}$$

and from Equation (35), with an assumed value of $G_p = 1000$,

$$R_{ch} = G_p R = 1000 \times 9600 = 9.6 \times 10^6 \text{ chip/s}$$

Since we have been given the same S/N_0 and the same data rate as in Example 2, we find the value of received E_b/N_0 from Equation (19) to be 8.2 dB (or 6.61). At the demodulator, we can now expand the expression for S/N_0 in Equation (28) and Appendix A, as follows.

$$\frac{S}{N_0} = \frac{E_b}{N_0} R = \frac{E_c}{N_0} R_c = \frac{E_s}{N_0} R_s = \frac{E_{ch}}{N_0} R_{ch} \qquad (36)$$

Corresponding to each transformed entity (data bit, channel bit, symbol, or chip) there is a change in rate, and similarly a reciprocal change in energy-to-noise spectral density for that received entity. Equation (36) is valid for any such transformation when the rate and energy are modified in a reciprocal way. There is a kind of *conservation of power (or energy)* phenomenon in the transformations. The total received average power (or total received energy per symbol duration) is fixed regardless of how it is computed — on the basis of data-bits, channel-bits, symbols, or chips.

The ratio E_{ch}/N_0 is much less in value than E_b/N_0. This can seen from Equations (36) and (35), as follows.

$$\frac{E_{ch}}{N_0} = \frac{S}{N_0}\left(\frac{1}{R_{ch}}\right) = \frac{S}{N_0}\left(\frac{1}{G_p R}\right) = \left(\frac{1}{G_p}\right)\frac{E_b}{N_0} \qquad (37)$$

But, even so, the despreading function (when properly synchronized) accumulates the energy contained in a quantity G_p of the chips, yielding the same value, E_b/N_0 = 8.2 dB, as was computed earlier from Equation (19). Thus, the DS spreading transformation has no effect on the error performance of an AWGN channel [4], and the value of G_p has no bearing on the value of P_B in this example. From Equation (37), we can compute

$$\begin{aligned} \frac{E_{ch}}{N_0}\ (\text{dB}) &= \frac{E_b}{N_0}\ (\text{dB}) - G_p\ (\text{dB}) \\ &= 8.2 \text{ dB} - (10 \times \log_{10} 1000) \text{ dB} \\ &= -21.8 \text{ dB} \end{aligned} \qquad (38)$$

The chosen value of processing gain (G_p = 1000) enables the DS/SS system to operate at a value of chip energy *well below the thermal noise*, with the same error performance as without spreading.

Since BPSK is the data modulation selected in this example, each message symbol therefore corresponds to a single channel bit, and we can write

$$\frac{E_s}{N_0} = \frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} = \left(\frac{51}{63}\right) \times 6.61 = 5.35 \qquad (39)$$

where the received E_b/N_0 = 8.2 dB (or 6.61). Out of the BPSK data demodulator, the symbol-error probability, P_E, (and the channel-bit error probability, p_c) is computed as follows [4].

$$p_c = P_E = Q\left(\sqrt{\frac{2E_c}{N_0}}\right) \qquad (40)$$

Using the results of Equation (39) in Equation (40) yields

$$p_c = Q(3.27) = 5.8 \times 10^{-4}$$

Finally, using this value of p_c in Equation (32) for the (63, 51) double-error correcting code yields the output bit-error probability of $P_B = 3.6 \times 10^{-7}$. We can therefore verify that, for the given architecture and design parameters of this example, the system does in fact achieve the required error performance.

Conclusion

The goal of this tutorial has been to review fundamental relationships in defining, designing, and evaluating digital communication system performance. First, we examined the concept of bandwidth-limited and power-limited systems and how such conditions influence the design when the choices are confined to MPSK and MFSK modulation. Most important, we focused on the definitions and computations involved in transforming from data bits to channel bits to symbols to chips. In general, most digital communication systems share these concepts; thus, understanding them should enable one to evaluate other such systems in a similar way.

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Appendix A

Received E_b/N_0 Is Independent of the Code Parameters

Starting with the basic concept that the received average signal power, S, is equal to the received symbol or waveform energy, E_s, divided by the symbol-time duration, T_s (or multiplied by the symbol rate, R_s), we write

$$\frac{S}{N_0} = \frac{E_s/T_s}{N_0} = \frac{E_s}{N_0} R_s \tag{A1}$$

where N_0 is noise-power spectral density.
Using Equations (27) and (25), rewritten below,

$$\frac{E_s}{N_0} = (\log_2 M)\frac{E_c}{N_0} \quad \text{and} \quad R_s = \frac{R_c}{\log_2 M}$$

let us make substitutions into Equation (A1), which yields

$$\frac{S}{N_0} = \frac{E_c}{N_0} R_c \tag{A2}$$

Next, using Equations (26) and (24), rewritten below,

$$\frac{E_c}{N_0} = \left(\frac{k}{n}\right)\frac{E_b}{N_0} \quad \text{and} \quad R_c = \left(\frac{n}{k}\right) R$$

let us now make substitutions into Equation (A2), which yields the relationship expressed in Equation (11).

$$\frac{S}{N_0} = \frac{E_b}{N_0} R \tag{A3}$$

Hence the received E_b/N_0 is only a function of the received S/N_0 and the data rate, R. It is independent of the code parameters, n, k, and t. These results are summarized in Fig. 3.

Biography

BERNARD SKLAR received a B.S. in math and science from the University of Michigan, an M.S. in electrical engineering from the Polytechnic Institute of Brooklyn, and a Ph.D. in engineering from the University of California, Los Angeles. He has more than 35 years experience in a wide variety of technical development positions at Republic Aviation Corp., Hughes Aircraft Co., Litton Industries, Inc., and The Aerospace Corporation. Currently, he is the head of advanced systems at Communications Engineering Services, a consulting company that he founded in 1984; an adjunct professor at the University of Southern California; and a visiting professor at the University of California at Los Angeles, where he teaches communications. He is the author the book, *Digital Communications*. He is a Fellow of the Institute for the Advancement of Engineering, and a past Chairman of the Los Angeles Council IEEE Education Committee.