

# Digital Communication Systems Engineering with Software-Defined Radio

Di Pu, Alexander M. Wyglinski  
Worcester Polytechnic Institute

## Lecture 16

# Extending Single Carrier to Multicarrier

- ▶ Multicarrier modulation can be viewed as the simultaneous transmission of several low data rate single carrier signals summed together
  - ▶ These signals, referred to as *subcarriers* are kept separate in the frequency domain
- ▶ In this lecture, we begin with the development of an *orthogonal quadrature amplitude modulation* (OQAM) multicarrier framework
  - ▶ Multiple QAM signals modulated to different carrier frequencies and summed together
- ▶ We then extend this result in the subsequent lecture to show it can be implemented using a *discrete Fourier transform* (DFT) and its inverse (IDFT)
  - ▶ This is referred to as *orthogonal frequency division multiplexing* (OFDM)

# Rectangular $M$ -ary QAM Revisited

- ▶  $D$  bits are taken from the input bit stream  $d[m]$
- ▶ Used to select one of  $2^D$  combinations of amplitudes for the two carriers,  $a[\ell]$  and  $b[\ell]$
- ▶ Resulting  $M$ -ary QAM signal is equal to:

$$s[n] = a'[n] \cos(\omega_k n) + b'[n] \sin(\omega_k n) \quad (1)$$

where

- ▶  $a'[n]$  and  $b'[n]$  piecewise constant signals
- ▶  $\omega_k = 2\pi k/2N$  is the carrier frequency
- ▶  $2N$  is the period of the symbol

# Rectangular $M$ -ary QAM Revisited

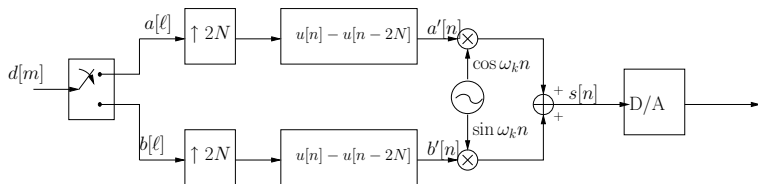


Figure : Rectangular  $M$ -ary QAM Transmitter.

# Rectangular $M$ -ary QAM Revisited

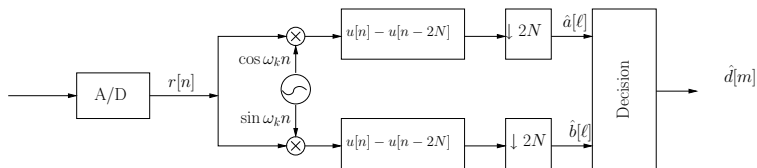


Figure : Rectangular  $M$ -ary QAM Receiver.

# Sample $M$ -ary QAM Signal Constellations

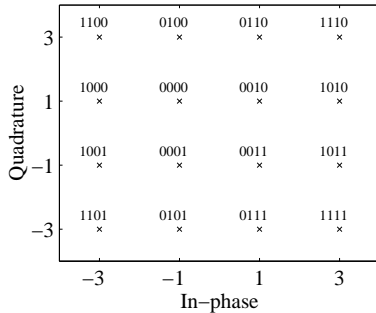


Figure : Rectangular 16-QAM Signal Constellation with Gray Coding.

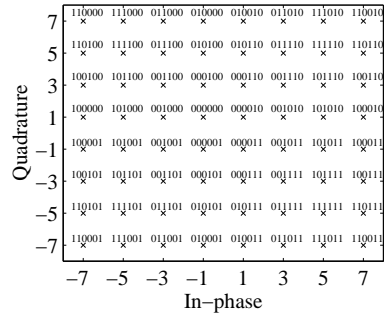


Figure : Rectangular 64-QAM Signal Constellation with Gray Coding.

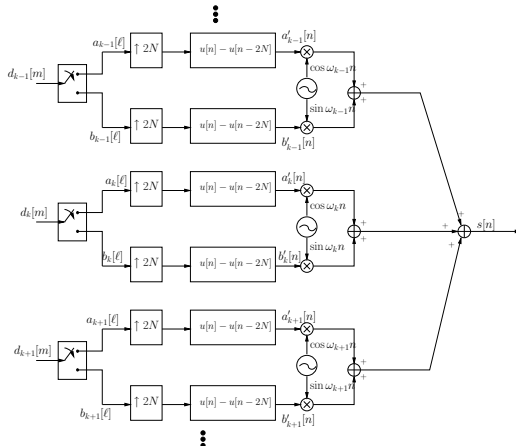
# Rectangular $M$ -ary QAM Demodulation

$$\begin{aligned}\hat{a}[\ell] &= \sum_{n=2\ell N}^{2\ell N+2N-1} r[n] \cos(\omega_k n) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} a'[n] \cos(\omega_k n) \cos(\omega_k n) + b'[n] \sin(\omega_k n) \cos(\omega_k n) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} a'[n] \cos\left(\frac{2\pi kn}{2N}\right) \cos\left(\frac{2\pi kn}{2N}\right) \\&\quad + b'[n] \sin\left(\frac{2\pi kn}{2N}\right) \cos\left(\frac{2\pi kn}{2N}\right) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} \frac{a'[n]}{2}\end{aligned}$$

# Rectangular $M$ -ary QAM Demodulation

$$\begin{aligned}\hat{b}[\ell] &= \sum_{n=2\ell N}^{2\ell N+2N-1} r[n] \sin(\omega_k n) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} a'[n] \cos(\omega_k n) \sin(\omega_k n) + b'[n] \sin(\omega_k n) \sin(\omega_k n) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} a'[n] \cos\left(\frac{2\pi kn}{2N}\right) \sin\left(\frac{2\pi kn}{2N}\right) \\&\quad + b'[n] \sin\left(\frac{2\pi kn}{2N}\right) \sin\left(\frac{2\pi kn}{2N}\right) \\&= \sum_{n=2\ell N}^{2\ell N+2N-1} \frac{b'[n]}{2}\end{aligned}$$

# Orthogonally Multiplexed Quadrature Amplitude Modulation



# Equivalence Between OQAM and OFDM

- ▶ *Orthogonal frequency division multiplexing* (OFDM) is an efficient type of multicarrier modulation
  - ▶ Employs discrete Fourier transform (DFT) and inverse DFT (IDFT) to modulate and demodulate data streams
- ▶ Carriers used in OQAM transmission are sinusoidal function of  $2\pi kn/2N$ 
  - ▶ A  $2N$ -point DFT or IDFT can carry out the same modulation
  - ▶ It contains also summations of terms of the form  $e^{\pm 2\pi kn/2N}$