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Physica B 241–243 (1998) 157–160

PHYSICA B

# Schroedinger cat-like neutron states and Wigner function formulation of interferometry and spin-echo experiments

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**Abstract**

The formalism of quantum optics can be adapted to advanced neutron optics. Measurements and calculations have identified a characteristic coherent and squeezed state behaviour for neutron states and a transfer of coherence features between ordinary and momentum space. In certain cases, the neutron occupies two well-separated regions in ordinary space which are coupled in momentum space giving a clear fingerprint of a Schroedinger cat-like quantum state. The visualization of such states can be achieved by a Wigner function representation where it becomes obvious that wave function tomography becomes feasible. Hence, spin-echo experiments are Ramse–Bordé interference experiments in the longitudinal direction; they can be described by Wigner functions as well. Additionally, it will be shown that Schroedinger cat-like states are rather fragile against any dissipative force from the environment. © 1998 Elsevier Science B.V. All rights reserved.

*Keywords:* Neutron interferometry; Spin-echo systems; Quantum optics

## 1. Introduction

Recent neutron interference experiments have shown that Schroedinger cat-like states can be produced when phase shifts larger than the coherence lengths are applied [1]. In these cases the appearing interference phenomena are exchanged between ordinary and momentum space and the neutron occupies two separate regions in space [2]. These highly nonclassical states are made by the power of the quantum mechanical superposition principle and exhibit typical squeezing phenomena and they are notoriously fragile to any kind of dephasing effects [3,4]. The features of any quantum state as they exist in split beam and spin-echo systems can be depicted properly by using Wigner quasi-distribution functions which combine the spatial and momentum distributions [5,6].

## 2. Basic relations

The Wigner function follows from the wave function describing the beam [5,6]

$$W_x(x,k) = \frac{1}{2\pi}\int e^{ik\Delta}\psi^*\left(x+\frac{\Delta}{2}\right)\psi\left(x-\frac{\Delta}{2}\right)d\Delta, \quad (1)$$

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0921-4526/98/$19.00 
*PII* S0921-4526(97)00537-1

and has the unique feature that integration over one variable gives the distribution function of the other one

$$\int W(x,k)\,\mathrm{d}k = |\psi(x)|^2 \quad \text{and} \quad \int W(x,k)\,\mathrm{d}x = |\psi(k)|^2. \tag{2}$$

Wave packets with an amplitude function $a(k)$ have to be used to describe the wave functions properly, where $|\psi(k)|^2 \propto |a(k)|^2 \propto g(k)$ represents the measurable momentum distribution. The spatial packet distribution follows from an interference experiment where the interference pattern or beam polarization is measured as a function of the phase shift between coherently overlapping beams. This behaviour is described by the coherence function which is given as the auto-correlation function of the wave function

$$\Gamma(\Delta) \propto \langle \psi(0)\psi(\Delta)\rangle \propto \int g(k)\mathrm{e}^{\mathrm{i}k\Delta}\,\mathrm{d}k, \tag{3}$$

whose characteristical length defines the coherence length $\Delta^{\mathrm{c}}$ of the beam. For Gaussian packets the momentum widths $\delta k$ and the coherence length are correlated according to the minimum uncertainty relation ($\delta k\,\Delta^{\mathrm{c}} = \frac{1}{2}$). The spatial shift of the wave trains can be calculated from the index of refraction and reads for nuclear and magnetic (spin-echo) phase shifts as

$$\Delta = \frac{2\pi N b_{\mathrm{c}} D}{k^2} \quad \text{and} \quad \Delta_m = \frac{\mu m B D}{\hbar^2 k^2}, \tag{4}$$

where $N$ denotes the particle density, $b_{\mathrm{c}}$ the coherent scattering length, $\mu$ and $m$ the magnetic moment and the mass of the neutron, $B$ the strength of the magnetic field, and $D$ the length of the sample or of the magnetic field, respectively.

## 3. Results

The results for split-beam interference systems have been reported elsewhere [4] and the predicted modulation of the momentum distribution has been observed experimentally [1]. Thus, the momentum distribution and the spatial distribution of the wave packets can be measured which permits a full reconstruction of the Wigner function.

Within a spin-echo system [7] where the neutrons propagate along the $y$-axis and the precession field lie along the $z$-axis, an initial polarization in the $x$-direction dephases gradually due to the different Larmor rotation angles of the different $k$-components of the wave packet and the related Zeeman splitting $\Delta k$.

$$\psi \propto a_+(k)|+\rangle + a_-(k)|-\rangle \tag{5}$$

and

$$k_+ - k_- = \Delta k = \frac{\mu m B}{\hbar^2 k}. \tag{6}$$

Using Eq. (1) one gets for Gaussian incident wave packets the Wigner function in the form [8]

$$W_s = \frac{1}{4\pi}\{A_1 + A_{-1} + 2A_0 \cos[2k\Delta k|y_1 - y_2|/k_0]\} \\ \exp[-(k-k_0)^2/2\delta k^2], \tag{7}$$

where

$$A_\varepsilon = \exp\left\{-2\delta k^2\left[y - |y_1 - y_2|\left(1 + \varepsilon\frac{\Delta k}{k}\right)\right]^2\right\}.$$

This function is shown in Fig. 1. The appearence of separated Schroedinger cat-like states and their retrieval in the second part of the spin-echo system is visible. The wiggles (smile) of the Wigner function appearing between the Schroedinger cat-like states indicate the nonclassical feature of these states, and their disappearance when field fluctuations are taken into account indicates a transition to a mixed state which equals a depolarized beam in this case.

## 4. Discussion

We have shown that neutron interference and spin-echo experiments can be described properly by the Wigner function formalism. This formalism gives a new insight to the changes of the spatial and momentum distributions in such experiments. It should be mentioned that the momentum distribution of the + and − component of the beam

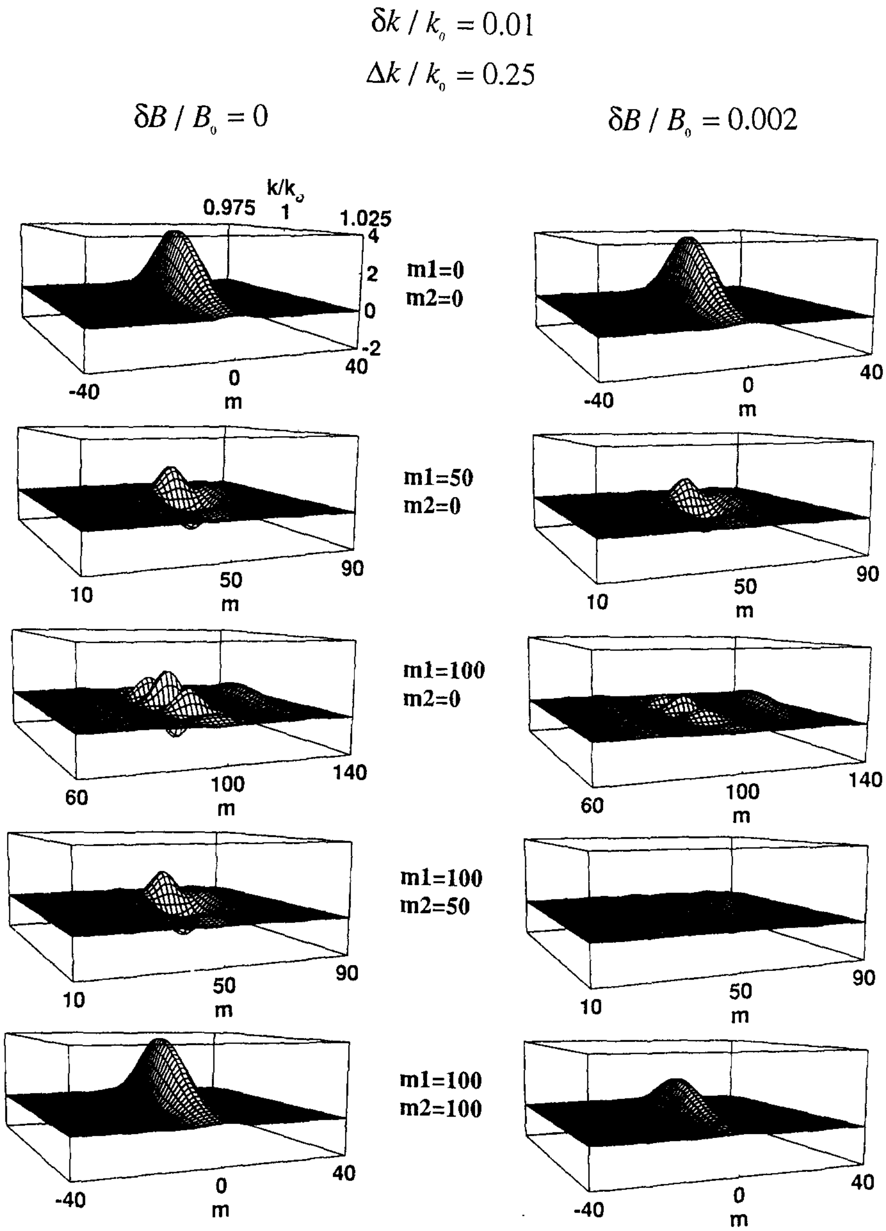


Fig. 1. Calculated Wigner functions along a spin-echo system without (left) and with (right) fluctuations of the precession field. In this figure $m$ denotes the number of Larmor precessions ($m = y\,\Delta k$).

hitting the sample strongly changes when the magnetic field is varied. In existing spin-echo systems $\Delta k/k_0$ is much smaller and $\delta k/k_0$ is much larger than the values used in Fig. 1. The spatial separation of the Schroedinger cat-like states can reach values of 0.15 μm which are quite large compared to the coherence lengths of these beams which are in the order of 4 nm. Additional features appear when the Wigner functions for zero-field spin-echo systems are calculated [9,10].

## Acknowledgements

This work was supported by the Austrian Fonds zur Förderung der Wissenschaftlichen Forschung, project No. P10491-PHY.

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