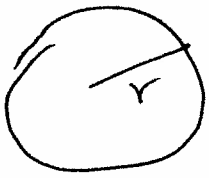
Lecture

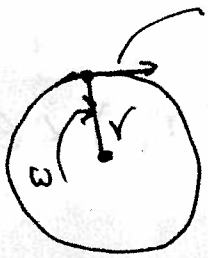


Circumference of a circle

$= 2\pi r$

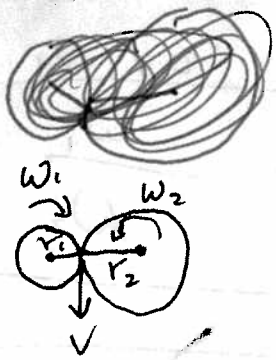
$\text{Arc} \cong r \cdot \Delta\theta$ if $\Delta\theta \approx 0$

$$\text{angular velocity} = \frac{\Delta\theta}{\Delta t} = \omega$$



V = tangential velocity
(translational velocity)

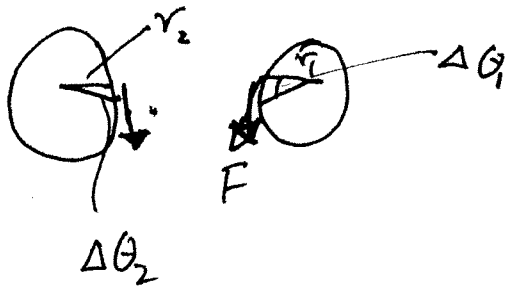
$= r\omega$



$V = r_1\omega_1 = r_2\omega_2$

$$\boxed{\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}}$$

Cons. of energy



$$\text{Work}_{\text{on}_2} = F \cdot \overbrace{(r_2 \Delta\theta_2)}^{x_2}$$

$$\text{Work}_{\text{on}_1} = F \cdot \overbrace{(r_1 \Delta\theta_1)}^{x_1}$$

$$\text{Work}_{\text{on}_2} = \text{Work}_{\text{on}_1}$$

$$(F r_2)\ \Delta\theta_2 = (F r_1)\ \Delta\theta_1$$

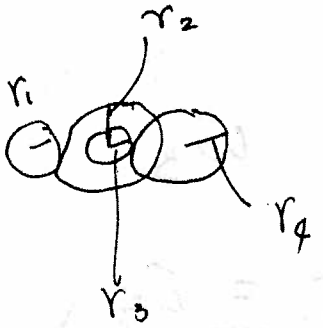
$$\tau_2\ \Delta\theta_2 = \tau_1\ \Delta\theta_1$$

$$\tau_2 \frac{\Delta\theta_2}{\Delta t} = \tau_1 \frac{\Delta\theta_1}{\Delta t}$$

$$\tau_2\ \omega_2 = \tau_1\ \omega_1$$

$$\boxed{\frac{\tau_2}{\tau_1} = \frac{\omega_1}{\omega_2}}$$

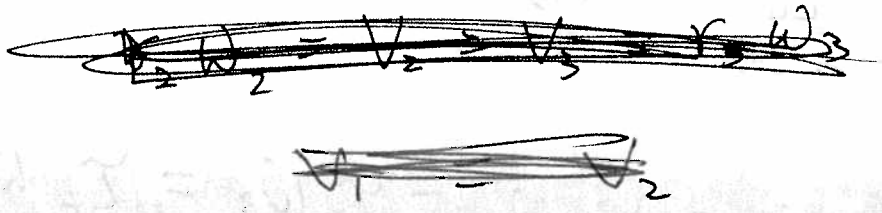
$$\boxed{\frac{\tau_2}{\tau_1} = \frac{r_2}{r_1}}$$



Technique for improving gear ratio

$$\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1} \Rightarrow \omega_1 = \left(\frac{r_2}{r_1}\right)\omega_2$$

$$\frac{\omega_3}{\omega_4} = \frac{r_4}{r_3} \Rightarrow \omega_3 = \frac{r_4}{r_3}\omega_4$$



$$\omega_1 = \left(\frac{r_2}{r_1}\right)\left(\frac{r_4}{r_3}\right)\omega_4$$

$$\frac{\omega_1}{\omega_4} = \left(\frac{r_2}{r_1}\right)\left(\frac{r_4}{r_3}\right)$$

key: $\boxed{\omega_2 = \omega_3 \Rightarrow \tau_2 = \tau_3}$

$$\frac{\tau_1}{\tau_2} = \frac{r_1}{r_2} = \frac{\omega_2}{\omega_1}$$

$$\frac{\tau_3}{\tau_4} = \frac{r_4}{r_3} = \frac{\omega_3}{\omega_4}$$

Cons. of energy

$$\tau_1 \omega_1 = \tau_2 \omega_2 = \tau_3 \omega_3 = \tau_4 \omega_4$$

$$\frac{\tau_1}{\tau_2} = \frac{\omega_2}{\omega_1} \Rightarrow \omega_2 = \frac{\tau_1 \omega_1}{\tau_2}$$

$$\frac{\tau_3}{\tau_4} = \frac{\omega_4}{\omega_3} \Rightarrow \tau_4 = \frac{\tau_3 \omega_3}{\omega_4}$$

$$\tau_4 = \frac{\tau_3}{\omega_4}\left(\frac{\tau_1 \omega_1}{\tau_2}\right)$$

$$= \tau_1 \left(\frac{\omega_1}{\omega_4}\right)$$

$$\frac{\tau_4}{\tau_1} = \left(\frac{r_2}{r_4}\right)$$