a.) Use energy balance to find the velocity at B:

$$mgh = \frac{1}{2}mv_b^2$$

$$gh = \frac{1}{2}v_b^2$$

$$v_b = (2gh)^{\frac{1}{2}} \qquad (10pts)$$

b.) There are (at least) two analogous ways to find t. The direction of motion is parallel to the length L, such that only a component of the acceleration g acts in the direction of motion. You can confirm this by drawing a free body diagram with the x-axis aligned along L, and the y-axis aligned in the direction of the normal force, N, revealing that the net acceleration comes from the component of gravity in the +L direction.

Solution 1:

$$l = \frac{(v_f - v_i)}{2} * t$$

$$l = \frac{v_f}{2} * t$$

$$t = \frac{2l}{v_f} = \frac{2l}{(2gh)^{\frac{1}{2}}} \qquad (5pts)$$

Solution 2:

$$a = g\frac{h}{l}$$

$$l = l_0 + v_o t + \frac{1}{2}at^2$$

$$l = \frac{1}{2}\left(\frac{gh}{l}\right)t^2$$

$$t = \left(\frac{2l^2}{gh}\right)^{\frac{1}{2}} = \frac{2l}{(2gh)^{\frac{1}{2}}} \qquad (5pts)$$

c.) First, find the velocity at c. Because the tube is oriented vertically, there is additional gravitational potential energy that we must consider:

$$\frac{1}{2}mgh + \frac{1}{2}mgp = \frac{1}{2}mv_c^2$$

$$v_c = (gh + gp)^{\frac{1}{2}}$$

$$a_n = \frac{v_c^2}{p}$$

$$a_t = g$$

$$a_{magnitude} = (a_n^2 + a_t^2)^{\frac{1}{2}}$$

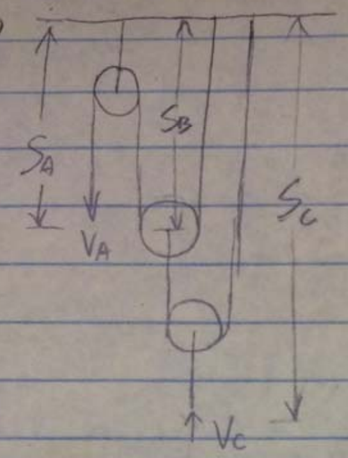
$$a_{magnitude} = \left(\left(g\frac{h}{p} + g\right)^2 + g^2\right)^{\frac{1}{2}}$$

$$a_{direction} = \tan^{-1}\left(\frac{a_t}{a_n}\right)$$

$$a_{direction} = \tan^{-1}\left(\frac{g}{g\frac{h}{p} + g}\right) = \tan^{-1}\left(\frac{1}{1 + \frac{h}{p}}\right)$$

Solution:

(a)

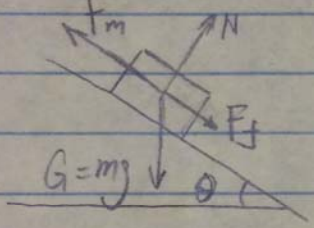


$$\begin{cases} S_A + 2S_B = l_1 \\ S_C + (S_C - S_B) = l_2 \end{cases}$$

$$\therefore \begin{cases} V_A + 2V_B = 0 \\ 2V_C - V_B = 0 \end{cases}$$

$$\Rightarrow |V_C| = \frac{1}{4}|V_A|$$

$$F_m = 4F$$



$$\begin{cases} F_f = N \cdot \mu_0 \\ N = mg \cdot \cos\theta \\ F_m = 4F \end{cases}$$

$$\Sigma F_x = m \cdot a_x$$

$$F_m - F_f - mg \cdot \sin\theta = m a_x$$

$$a_x = \frac{1}{m}(4F - mg\cos\theta \cdot \mu_0 - mg\sin\theta)$$

$$a\,ds = v\,dv$$

$$\int_0^l a\,ds = \int_0^v v\,dv$$

$$al = \frac{1}{2} \cdot v^2$$

$$v = \sqrt{2al}$$

$$= \sqrt{\frac{2l}{m}(4F - mg\cos\theta\mu_0 - mg\sin\theta)}$$

(b).

$$\Sigma U_{1-2} = W_F - W_{F_f} - W_G$$
$$= 4F \cdot l - mg\cos\theta \cdot \mu_0 \cdot l - mg \cdot \sin\theta \cdot l$$

$$\Sigma U_{1-2} = \frac{1}{2} \cdot m \cdot V_2^2 - \frac{1}{2} \cdot m \cdot V_1^2$$

$$4Fl - mg\cos\theta \cdot \mu_0 \, l - mg\sin\theta \cdot l = \frac{1}{2} \cdot m V^2$$

$$V = \sqrt{\frac{2l}{m}(4F - mg\cos\theta\mu_0 - mg\sin\theta)}$$

(c). $P = F_x \cdot V$

$$= (F_m - F_f - mg \cdot \sin\theta) \cdot V$$
$$= (4F - mg\cos\theta \cdot \mu_0 - mg\sin\theta) \cdot \sqrt{\frac{2l}{m}(4F - mg\cos\theta\mu_0 - mg\sin\theta)}$$

(d). $\varepsilon = \dfrac{\text{power out}}{\text{power input}}$

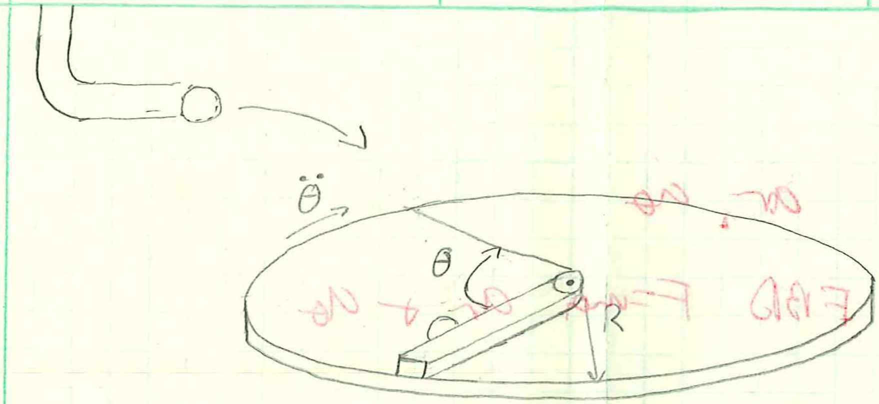
power output $= \frac{1}{2} \cdot m V^2$

$$= \frac{1}{2} m \frac{2l}{m}(4F - mg\cos\theta\,\mu_0 - mg\sin\theta)$$
$$= l \cdot (4F - mg\cos\theta \cdot \mu_0 - mg\sin\theta)$$

power input $= F_m \cdot l$

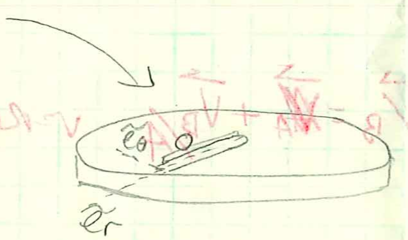
$$= 4F \cdot l$$

$$\varepsilon = \frac{l(4F - mg\cos\theta\,\mu_0 - mg\sin\theta)}{4Fl}$$
$$= 1 - \frac{mg(\cos\theta\,\mu_0 + \sin\theta)}{4F}$$



$$\frac{8}{10} = \frac{x}{15}$$

$$\frac{8 \cdot 15}{10} = \frac{8 \cdot 3}{2} = 12$$



$\ddot{\theta} = \text{const.}$

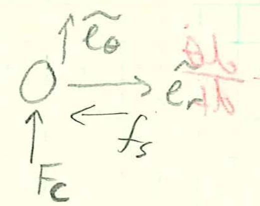
$\Sigma F_r = m a_r$

$-f_s = m(\cancel{\ddot{R}}^{\,0} - R\dot{\theta}^2)$

$-\mu_s F_c = m(-R\dot{\theta}^2)$

$F_c = \dfrac{mR\dot{\theta}^2}{\mu_s}$

FBD



(F_c is force from Arm C.)

$\Sigma F_\theta = m a_\theta$

$F_c = m(R\ddot{\theta} + \cancel{2\dot{R}\dot{\theta}}^{\,0})$

$\dfrac{\cancel{m}\cancel{R}\dot{\theta}^2}{\mu_s} = \cancel{m}(\cancel{R}\ddot{\theta})$

$\dfrac{\dot{\theta}^2}{\mu_s} = \ddot{\theta}$

$\boxed{\dot{\theta} = \sqrt{\mu_s \ddot{\theta}}}$

$\sqrt{\mu_s \ddot{\theta}} = \ddot{\theta} t$

$t = \sqrt{\dfrac{\mu_s}{\ddot{\theta}}}$

$\ddot{\theta} = c_1$

$\dot{\theta} = \ddot{\theta} t$

$\int c_1 t\, dt = \int d\theta$

$\dfrac{c_1 t^2}{2} = \theta$

Kinematic Relations

$v^2 = \cancel{v_0^2}^{\,0} + 2a_c(s - \cancel{s_0}^{\,0})$

$(R\dot{\theta})^2 = 2(R\ddot{\theta})(R\theta)$

$\cancel{R^2}\dot{\theta}^2 = 2\cancel{R^2}\ddot{\theta}\theta$

$\mu_s \ddot{\theta} = 2\ddot{\theta}\theta$

$\boxed{\theta = \dfrac{\mu_s}{2}}$

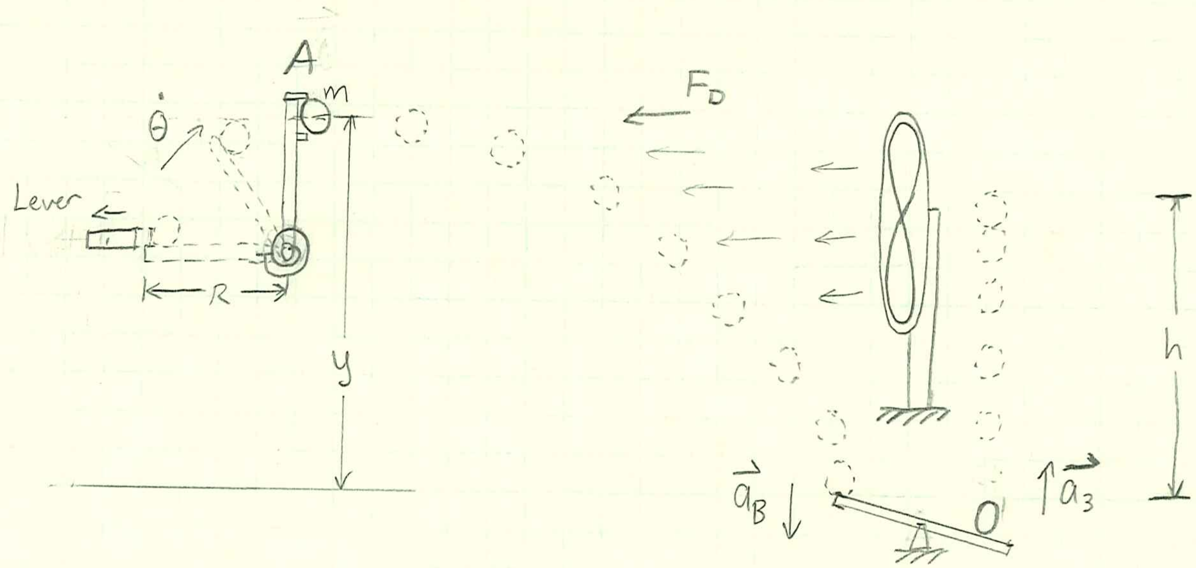
$\ddot{\theta} = \dfrac{d\dot{\theta}}{dt}$

$\int \ddot{\theta}\, dt = \int d\dot{\theta}$

$\ddot{\theta} t = \dot{\theta} + \cancel{c_1}^{\,0}$

$t = \dfrac{\dot{\theta}}{\ddot{\theta}} = \dfrac{\sqrt{\mu_s \ddot{\theta}}}{\ddot{\theta}}$

$t = \sqrt{\dfrac{\mu_s}{\ddot{\theta}}}$



$\vec{a}_2 = -a_{2y}\,\hat{j}$

$\vec{V}_1 = V_{1x}\,\hat{i}$

$\vec{a}_3 = -\alpha\,\vec{a}_2$ where $0 < \alpha < 1$

Find: If F_D is entirely horizontal and is proportional to the velocity of mass m ($F_D = kv$), find the constant k in terms of m, g, y where g is the grav. accel.

Given: $\dot{\theta}$ = constant

✓ y, R

✓ $\vec{V}_A$ is entirely horizontal

✓ $\vec{V}_B = v_{By}\,\hat{j} + e^{-1} V_A\,\hat{i}$

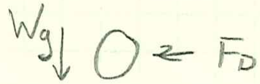
$\vec{F}_D$ is entirely horizontal

$\vec{F}_D = kv$

k = constant

Problem: A Lever has release a spring-loaded arm with a small spherical mass m on it.
The mass m, is released from the arm at point A with a purely horizontal velocity V_A.
A fan pushes air opposing the flight direction of the mass.
The resulting air drag force F_D acts on the mass until the mass lands on a board balanced on a fulcrum at point B.
The acceleration of the mass at B is entirely vertical.

FBD



$$\Sigma F_x = -F_D = ma_x$$

$$a_x = \frac{-F_D}{m} = \frac{-kv}{m}$$

$$a_x = \frac{dv}{dt}$$

$$\int_0^t dt = \int_{V_A}^{V_B} \frac{dv}{a_x} = \int_{V_A}^{V_B} \frac{dv}{-\frac{k}{m}v}$$

$$t = -\frac{m}{k} \ln(v) \Big|_{V_A}^{V_B}$$

$$t = -\frac{m}{k} \ln\left(\frac{V_B}{V_A}\right) \longrightarrow$$

$$t = -\frac{m}{k}(-1) = \frac{m}{k}$$

$$S_y = S_{oy} + \cancel{V_{oy}}^{0} + \frac{1}{2} a_c t^2$$

$$S_o - S_{oy} = -\frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2(S_y - S_{oy})}{-g}} = \sqrt{\frac{2y}{g}}$$

$$\downarrow$$

$$-\frac{m}{k} \ln\left(\frac{V_B}{V_A}\right) = \sqrt{\frac{2y}{g}}$$

$$k = -m\sqrt{\frac{g}{2y}} \ln\left(\frac{V_B}{V_A}\right) \qquad V_B = e^{-1} V_A$$

$$k = -m\sqrt{\frac{g}{2y}}(-1)$$

$$\boxed{k = m\sqrt{\frac{g}{2y}}}$$