## Vector Definition

Any quantity which got both magnitude and direction are called vector quantity. Examples are Displacement(an airplane has flown 200 km to the south west) and velocity (a car moving with velocity $62 \mathrm{~km} / \mathrm{h}$ to the north). Other familiar vector quantity is force.

A vector is characterized by an absolute value(magnitude) and a direction. The vector, as a mathematical object, is defined as a directed line segment. Displacement, velocity acceleration, force momentum, angular momentum are a few examples of vector quantities. A vector is geometrically represented by an arrow. Length of the arrow is proportional to the magnitude of the vector; head of the arrow gives the sense of direction. for example $\mathrm{a}^{\rightarrow}$.


The net effect of the two displacements i.e., $A$ to $B$ and $B$ to $C$ is the same as a displacement from $A$ to $C$. Therefore, we speak of $A C$ as the sum or resultant of the displacements AB and BC . Notice that this sum is not an algebraic sum and that a number alone cannot uniquely specify it.

## Scalar

The physical quantity which has only magnitude but no direction is called scalar quantity or scalars. Mass, length, time, distance covered, speed, temperature, work, etc. are few examples of scalar quantity. The scalars can be added, subtracted, multiplied and divided by ordinary laws of algebra. The scalar is specified by mere number and unit, where number represents its magnitude. A scalar may be positive or negative. A scalar can be represented by a single letter.

## Unit Vector

It is a vector having unit magnitude. It is used to denote the direction of a given vector. The Unit Vector Notation is $\mathrm{a}^{\wedge}$. Any vector $\mathrm{a}^{\rightarrow}$ can be expressed in terms of its unit vector a in the following way $\mathrm{a}{ }^{\vec{\prime}}=\mathrm{aa}{ }^{\wedge}$ where $\mathrm{a}=\left|\left|\mathrm{a}^{\vec{~}}\right|\right|$, the magnitude of the vector.

Here, $\mathrm{a}^{\wedge}$ is in the same direction as $\mathrm{a}^{\rightarrow} . \mathrm{a}^{\wedge}$ is read as 'a hat ' or 'a cap '.

$\therefore$, if a given vector is divided by its magnitude, we get a unit vector.
The three rectangular unit vectors $\mathrm{i}^{\vec{\prime}}, \mathrm{j} \overrightarrow{, k} \overrightarrow{\text { are illustrated in the figure. Basically }}$ these unit vectors $\mathrm{i} \rightarrow, \mathrm{j} \overrightarrow{\mathrm{k}}, \mathrm{k}$ are used to specify the positive $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ direction respectively.


Note that $\mathrm{i}^{\vec{\prime}}, \mathrm{j} \overrightarrow{ }, \mathrm{k} \overrightarrow{ }$ need not be located at the origin. Like all vectors, they can be translated anywhere in the coordinate space, as long as their directions with respect to the coordinate axes are not changed.

## Example :



## Resolution of a vector into its vector components

The vector $\mathrm{a}^{\overrightarrow{ }}$, as shown in the figure, is expressed in terms of its components and unit vectors as, $a^{\rightarrow}=i^{\rightarrow} a_{x}+j^{\rightarrow} a_{y}$ where $a_{x}, a_{y}$ are the magnitudes of 'a' along $X, Y$ direction respectively.

## Zero Vector

Zero Vector or null vector is a vector which has zero magnitude and an arbitrary direction. It is represented by $0^{\vec{~}}$. If a vector is multiplied by zero, the result is a zero vector.

$$
\text { If } \mathrm{a}^{\overrightarrow{ }}=-\mathrm{b} \rightarrow \text {, then } \mathrm{a}^{\overrightarrow{2}}+\mathrm{b}^{\vec{\prime}}=0^{\vec{~}}
$$

It is important to note that we cannot take the above result to be a number, the result has to be a vector and here lies the importance of the zero or null vector. The physical meaning of $0 \overrightarrow{ }$ can be understood from the following examples.

- The position vector of the origin of the coordinate axes is a zero vector.
- The displacement of a stationary particle from time $t$ to time $t l$ is zero.
- The displacement of a ball thrown up and received back by the thrower is a zero vector.
- The velocity vector of a stationary body is a zero vector.
- The acceleration vector of a body in uniform motion is a zero vector.


## Properties of the zero vector

- When a zero vector is added to another vector $\mathbf{a} \overrightarrow{ }$, the result is the vector $\mathrm{a} \overrightarrow{ }$ only.

$$
\mathbf{a}^{\overrightarrow{2}}+\mathbf{0}^{\vec{\prime}}=\mathbf{a}^{\vec{a}}
$$

- Similarly, when a zero vector is subtracted from a vector $\mathrm{a}^{\vec{\prime}}$, the result is the vector $\mathrm{a}^{\overrightarrow{2}}$.

$$
\mathbf{a}^{\vec{\rightarrow}}-\mathbf{0}^{\vec{\prime}}=\mathbf{a}^{\vec{~}}
$$

- When a zero vector is multiplied by a non-zero scalar, the result is a zero vector.

$$
\text { Now, } \mu 0 \rightarrow=0 \vec{~}
$$

- When a vector, say $\mathrm{b}^{\overrightarrow{ }}$ is multipliedby zero, the result is a zero vector.

$$
\mathbf{0}\left(\mathbf{b}^{\vec{r}}\right)=\mathbf{0}^{\vec{~}}
$$

## Parallel Vectors

Collinear Vectors are those vectors that act either along the same line or along parallel lines. These vectors may act either in the same direction or in opposite directions.

If two collinear vectors $\mathrm{a}^{\overrightarrow{2}}$ and $\mathrm{b}{ }^{\overrightarrow{ }}$ act in the same direction, then the angle between them is $0^{\circ}$, as shown in the figure given below. When vectors act along the same direction, they are called Parallel Vectors.


Parallel Vectors
Anti-parallel Vectors: If collinear vector acts in the opposite directions, then the angle between them is $180^{\circ}$ or $\pi$ radians, as shown in the figure below


## Anti-Parallel Vectors

Vectors are said to be anti-parallel if they act in the opposite directions

## Vector Addition

Scalars can be added algebraically. However, vectors do not obey the ordinary laws of algebra. This is because vectors possess both magnitude and direction. Vectors are added geometrically. The process of adding two or more vectors is known as addition or composition of vectors. When two or more vectors are added, the result is a single vector called the resultant vector.

The resultant of two or more vectors is that single vector which alone produces the same effect as that produced by the two individual vectors.

Three laws have been evolved for the addition of vectors:

- Triangle law of vectors for addition of two vectors.
- Parallelogram law of vectors for addition of two vectors.
- Polygon law of vectors for addition of more than two vectors.

Triangle Law of Vectors : Let a particle be at the points A, B, C at three successive times $t$, $t^{\prime}$ and $t^{\prime \prime}$ respectively. $\mathrm{AB} \rightarrow$ is the displacement vector from time t to $\mathrm{t}^{\prime} . \mathrm{BC} \rightarrow$ is the displacement vector from time $\mathrm{t}^{\prime}$ to $\mathrm{t}^{\prime \prime}$. The total displacement vector $\mathrm{AC} \rightarrow$ is the sum or the resultant of individual displacement vector $\mathrm{AB} \rightarrow$ and $\mathrm{BC} \rightarrow$.


$$
\therefore \mathrm{AC} \rightarrow=\mathrm{AB} \rightarrow+\mathrm{BC} \rightarrow
$$

This leads to the statement of the law of triangle of vectors.
If two vectors can be represented both in magnitude and direction by the two sides of a triangle taken in the same order, then the resultant is represented completely, both in magnitude and direction, by the third side of the triangle taken in the opposite order.

Vector Subtraction : To subtract two vectors ,a and -b, add the first vector to the negative of the second vector: $\mathrm{a}-\mathrm{b}=\mathrm{a}+(-\mathrm{b})$. The negative of the second vector is obtained by reversing it direction.

## Vector Dot Product

Dot product is nothing but it is a product of vectors. The other name of Dot product is scalar product. Scalar product (or) dot product is commutative. When two vectors are dot product the answer obtained will be only number and no vectors. Below we can see some brief explanation about dot product.

The dot product of two same vectors is the value leaving the vector. Similarly the dot product of two different vectors the answer is zero. There are many different properties of dot product. Dot product are used in many cases.

Scalar product or dot product : Let $\mathrm{p}^{\overrightarrow{2}}$ and $\mathrm{q}^{\overrightarrow{ }}$ be two non zero vectors. Then the scalar product of $\mathrm{p}^{\vec{~}}$ and $\mathrm{q}^{\vec{~}}$ is denoted by $\mathrm{p}^{\vec{~}} \cdot \mathrm{p}^{\vec{~}}$ and is defined as the scalar $\left\|\mathrm{p}^{\vec{~}}\right\|\|\|$ $\rightarrow|\mid \cos \theta$.

$$
\text { Thus, } \mathrm{p}^{\overrightarrow{ } \cdot \mathrm{q}} \overrightarrow{\mathrm{q}}=\|\mathrm{p} \overrightarrow{ }\| \| \mathrm{q}| | \cos \theta=\mathrm{pq} \cos \theta
$$

The scalar product of two vectors is a scalar quantity. Therefore the product is called scalar product. Since we are putting dot between $\mathrm{p}^{\overrightarrow{2}}$ and $\mathrm{q} \overrightarrow{ }$, it is also called dot product.

## Vector Cross Product

The vector product of two vectors $\mathrm{a}^{\overrightarrow{2}}$ and $\mathrm{b}^{\overrightarrow{ }}$ is written as $\mathrm{a} \overrightarrow{ } \times \mathrm{b}^{\rightarrow}$ and is another vector $\mathrm{c}^{\vec{~}}$ where $\mathrm{c}^{\overrightarrow{2}}=\mathrm{a}^{\vec{~}} \times \mathrm{b}^{\vec{~}}$. The magnitude of $\mathrm{c}^{\vec{~}}$ is defined by $\left\|\mathrm{c}^{\vec{~}}\right\| \|=\mathrm{c}=$ $a b \sin \phi$


The direction of $\vec{c}$ is that in which a right handed screw advances when turned from $\vec{a}$ to $\vec{b}$


The direction of $\vec{c}$ can also be obtained from the right hand rule

(c) The vector product changes sign when the order of the

$$
\text { factors is reversed : } \vec{a} \times \vec{b}=-\vec{b} \times \vec{a}
$$

Where, $\phi$ is the angle between a and b , the direction of $\mathrm{c}^{\overrightarrow{ }}$, the vector product of $\mathrm{a} \overrightarrow{ }$ and $\mathrm{b}^{\overrightarrow{ }}$ is defined to be perpendicular to the plane containing $\mathrm{a}^{\rightarrow}$ and $\mathrm{b} \overrightarrow{ }$. To understand the direction of the vector $\mathrm{c}^{\vec{\prime}}$, let us refer to the figure. Imagine rotating a right handed screw whose axis is perpendicular to the plane formed by $a$ and $b$ so as to twist it from $a$ to $b$ through the angle $p$ between them. Then the direction of advance of the screw gives the direction of the vector product $\mathrm{a}^{\overrightarrow{2}} \times \mathrm{b} \overrightarrow{ }$. Another way of determining the direction of the vector product is the right hand rule. If the right hand is held so that the curled fingers follow the rotation of a $\overrightarrow{\text { into } b} \overrightarrow{ }$, the extended right thumb will point in the direction of $\mathrm{c} \rightarrow$.
$a^{\vec{~}} \times \mathbf{b}^{\vec{~}}$ is not the same vector as $b^{\vec{~}} \times \mathbf{a}^{\vec{~}}$ so that the orders of factors in a vector product is important. This is not true for scalars because, the order of factors in algebra or arithmetic does not affect the resulting product. Actually

$$
\mathbf{a}^{\overrightarrow{2}} \times \mathbf{b}^{\vec{~}}=-\mathbf{b} \overrightarrow{ } \times \mathbf{a}^{\vec{~}}
$$

