Winter 2014 ME 230 Kinematics and Dynamics
Homework # 1 Solution

12-10: The position of the particle when $t = 6s$:

$$S = 1.5(6)^3 - 13.5(t^2) + 22.5(6) = \underline{-27.0 \text{ ft}} \;\#$$

Total distance traveled:
(Hint: plot the path)

Determine velocity: $v = \frac{ds}{dt} = 4.5t^2 - 27t + 22.5$

The times when the particle stops are:

$$4.5t^2 - 27t + 22.5 = 0$$

$$t = 1s \text{ and } t = 5s$$

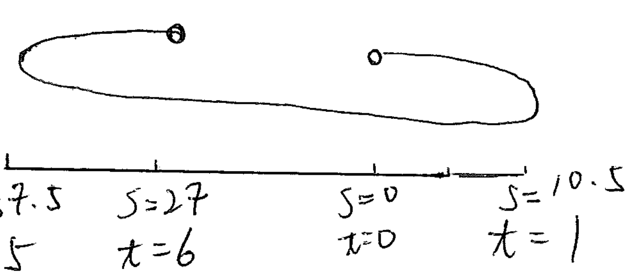
Find the position when $t = 0s$, $1s$ and $5s$

$$S(t=0) = 0$$

$$S(t=1) = 1.5 - 13.5 + 22.5 = 10.5 \text{ ft}$$

$$S(t=5) = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

plot the path:



$$S_{total} = 10.5 + 48.0 + 10.5 = \underline{69.0 \text{ ft}} \;\#$$

12-22

Determine the position when $t = 6s$

$$S(t=6) = 6^3 - 9(6^2) + 15(6) = \underline{-18 \text{ ft}} \;\#$$

Determine total distance:

$$v = \frac{ds}{dt} = 3t^2 - 18t + 15 = 3(t-1)(t-5)$$

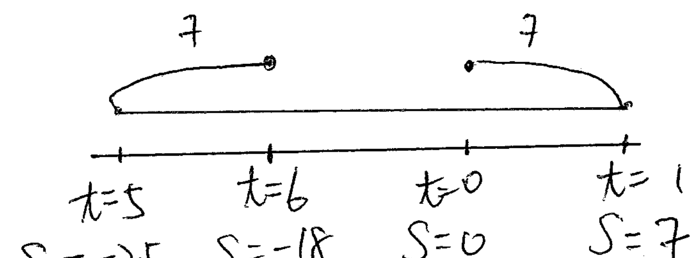
$\Rightarrow v = 0$ when $t = 1s$ and $5s$

when $t=0$, $S = 0$

$t = 1$, $S = 7$ ft

$t = 5$, $S = -25$ ft

plot the path:



So $S_{total} = 7 + (7+25) + (-18-(-25))$

$= 7 + 32 + 7 = \underline{46 \text{ ft}} \;\#$

12-24

$$v = -4s^2 = \frac{ds}{dt}$$

$$-4 = s^{-2}\frac{ds}{dt}$$

$$\int_0^t -4\,dt = \int_2^s s^{-2}\,ds$$

$(t=0,\ s=2)$

$$\Rightarrow -4\Big|_0^t = -\frac{1}{s}\Big|_2^s$$

$$\Rightarrow -4t = -\frac{1}{s} + \frac{1}{2}$$

$$\Rightarrow \frac{1}{s} = \frac{1+8t}{2}$$

$$s = \frac{2}{1+8t}$$

$$V = -4s^2 = -4\cdot\left(\frac{2}{1+8t}\right)^2 = \frac{16}{(8t+1)^2}\ \frac{m}{s}$$

#

$$a = \frac{dv}{dt} = \frac{256}{(8t+1)^3}\ \frac{m}{s^2}$$

#

12-26

Motion of Car A:

$$v = v_A - a_A t$$

at the instant A stops: $0 = v_A - a_A t$

$$t = \frac{v_A}{a_A}$$

$$v^2 = v_0^2 + 2a_c (s - s_0)$$

$$0 = v_A^2 + 2(-a_A)(s_A - 0)$$

$$s_A = \frac{v_A^2}{2a_A}$$

Motion of car B:

$$s_B = v_B t = v_B \left(\frac{v_A}{a_A}\right) = \frac{v_A v_B}{a_A}$$

distance between cars A and B is:

$$s_{BA} = |s_B - s_A| = \left| \frac{v_A v_B}{a_A} - \frac{v_A^2}{2a_A} \right|$$

$$= \left| \frac{2 v_A v_B - v_A^2}{2a_A} \right|$$

#

12-28

$$v = \frac{5}{4+s}$$

$$v\,dv = a\,ds \Rightarrow dv = \frac{-5\,ds}{(4+s)^2}$$

$$\frac{5}{(4+s)}\left(\frac{-5\,ds}{(4+s)^2}\right) = a\,ds$$

$$a = \frac{-25}{(4+s)^3}$$

when $s = 2m$, $\underline{a = -0.116\ m/s^2}$ #

12-32 Initial condition: $v=0,\ s=1$

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_1^s \frac{1}{4} s^{\frac{1}{2}}\,ds$$

$$\frac{v^2}{2} = \frac{1}{6} s^{3/2}\Big|_1^s \Rightarrow v = \frac{1}{\sqrt{3}}\left(s^{\frac{3}{2}}-1\right)^{\frac{1}{2}}\ \frac{m}{s}$$

when $s=2m$, $\underline{v = 0.781\ \frac{m}{s}}$ #

12-37 $v\,dv = a\,dy$

$$\int_v^0 v\,dv = -g_0 R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$v = \sqrt{2 g_0 R} = \sqrt{2(9.81)(6356)(10)^3}$$

$$= \underline{11.2\ \frac{km}{s}}$$ #

12-62

s-t graph:

$0 \le t < 5\,s$. initial condition: $s=0$ when $t=0$

$$\frac{ds}{dt} = v \Rightarrow ds = v\,dt$$

$$\Rightarrow \int_0^s ds = \int_0^t 2t^2\,dt$$

$$\Rightarrow s = \frac{2}{3}t^3 \Big|_0^t = \frac{2}{3}t^3 \ (ft)$$

when $t = 5\,s$.

$$s = \frac{2}{3}(5)^3 = 83.33\ ft$$

$$a = \frac{dv}{dt} = 4t = 20\ \left(\frac{ft}{s^2}\right)$$

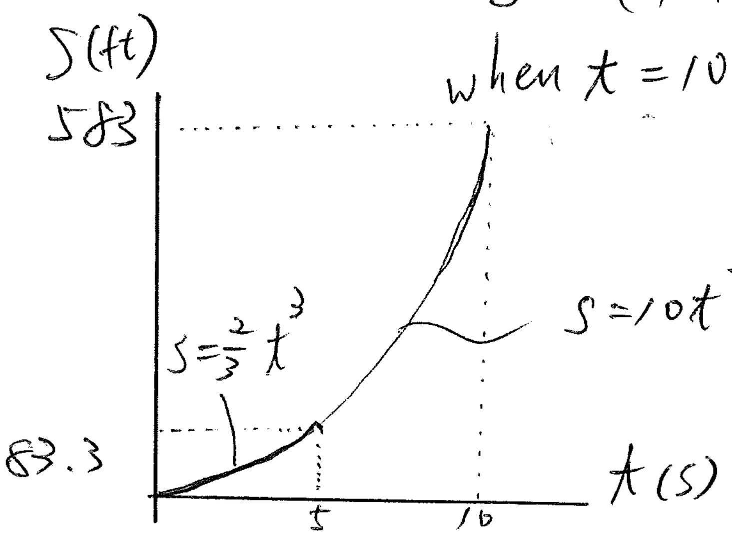
For the time interval $5\,s < t \le 10\,s$,

initial condition: $s = 83.33\,ft$, $t = 5\,s$

$$ds = v\,dt \Rightarrow \int_{83.33}^{s} ds = \int_5^t (20t - 50)\,dt$$

$$\Rightarrow s = (10t^2 - 50t + 83.33)\ ft$$

when $t = 10\,s$, $s(t=10) = 583\ ft$.



12-62 continued

a-t graph

For the time interval $0 \le t < 5s$

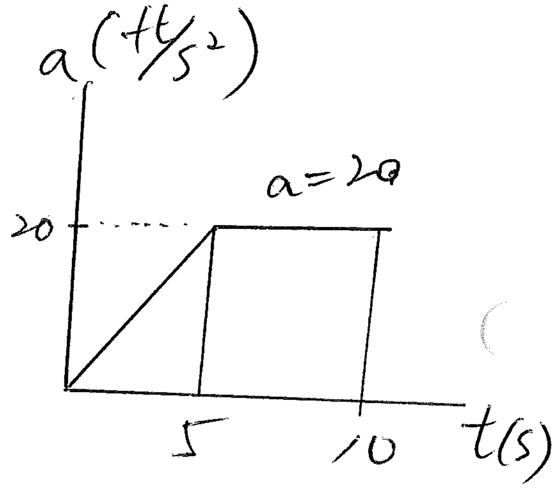
$$a = \frac{dv}{dt} = 4t \left(\frac{ft}{s^2}\right)$$

For the time interval $5s < t \le 10s$

$$a = \frac{dv}{dt} = 20\ ft/s^2$$

$t = 5s, \quad a = 20\ ft/s^2$

a-t graph is shown below:



12-71 For $0 \le s < 100$ ft,

$$a = v\frac{dv}{ds} = (0.1s + 5)(0.1) = 0.01s + 0.5 \left(\frac{ft}{s^2}\right)$$

Thus as $s = 0$ and 100 ft

$$a(s=0) = 0.5 \left(\frac{ft}{s^2}\right)$$

$$a(s=100) = 1.5 \left(ft/s^2\right)$$

For $100\ ft < s < 350\ ft$

$$a = v\frac{dv}{ds} = (-0.04s + 19)(-0.04)$$

$$= (0.0016s - 0.76)\frac{ft}{s^2}$$

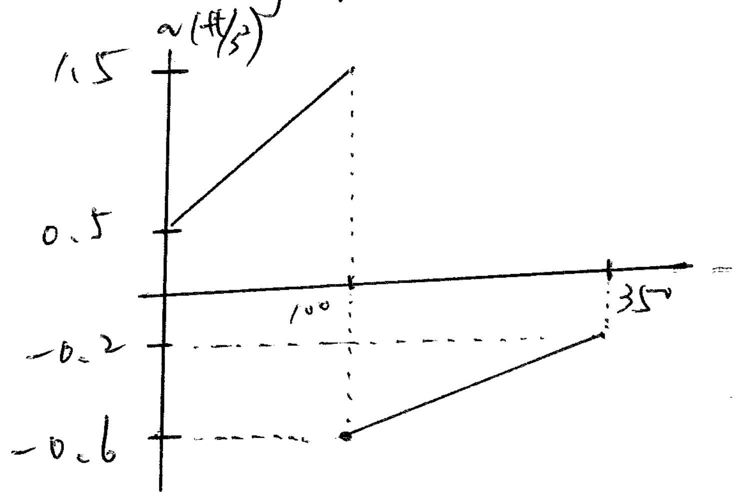
12-71 continued

Thus at $S = 100$ ft and 350 ft

$$a(S=100) = -0.6 \ \frac{ft}{s^2}$$

$$a(S=350) = -0.2 \ ft/s^2$$

The a-S graph is shown below



12-92 To strike B:

$(\xrightarrow{+})\ S = S_0 + V_0 t \Rightarrow 2.5 = 0 + V_A \cos 30^\circ t$

$(+\uparrow)\ S = S_0 + V_0 t + \frac{1}{2} a t^2 \Rightarrow 0.25 = 1 + V_A \sin 30^\circ t - \frac{1}{2} g t^2$

Solving $\Rightarrow \begin{cases} t = 0.6687 \text{ s.} \\ \underline{V_A = 4.32 \text{ m/s}} \end{cases}$ #

To strike C:

$(\xrightarrow{+})\ S = S_0 + V_0 t \Rightarrow 4 = 0 + V_A \cos 30^\circ t$

$(+\uparrow)\ S = S_0 + V_0 t + \frac{1}{2} a t^2 \Rightarrow 0.25 = 1 + V_A \sin 30^\circ t - \frac{1}{2} g t^2$

Solving: $\begin{cases} t = 0.79 \text{ s} \\ \underline{V_A = 5.85 \text{ m/s}} \end{cases}$ #

12-92 continued

Time between throws

$\Delta t = 0.79 - 0.6687 = \underline{0.121\,s}$ #

12-98 $(\xrightarrow{+})$ $s = s_0 + v_0 t \Rightarrow 30 = 0 + 48\cos\theta\, t$

$\Rightarrow t = \dfrac{30}{48\cos\theta}$ ..... (1)

$(+\uparrow)$ $s = s_0 + v_0 t + \frac{1}{2} a t^2$

$\Rightarrow 0 = 3 + 48\sin\theta\, t + \frac{1}{2}(-32.2)t^2$

Insert (1) $\Rightarrow$ $0 = 3 + \dfrac{48\sin\theta\,(30)}{48\cos\theta} - 16.1\left(\dfrac{30}{48\cos\theta}\right)^2$

$0 = 3\cos^2\theta + 30\sin\theta\cos\theta - 6.2891$

$3\cos^2\theta + 15\sin 2\theta = 6.2891$

Solving: $\Rightarrow \underline{\theta = 6.41^\circ \text{ or } 77.9^\circ}$ #

12-112 Vertical motion:

$(+\uparrow)$ $s_y = (s_0)_y + (v_0)_y t + \frac{1}{2} a t^2$

$0 = 0 + 40\sin 60^\circ t + \frac{1}{2}(-32.2)t^2$

$t = 2.152\,s$

Horizontal motion:

$(\xrightarrow{+})$ $s_x = (s_0)_x + (v_0)_x t$

12-112 continued

$$S_x = 0 + 40\cos 60^\circ (2.152) = 43.03\ ft$$

The distance for which player B must travel in order to catch the baseball is

$$43.03 - 15 = \underline{28.0\ ft}\ \#$$

player B is required to run at the same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 40\cos 60^\circ = \underline{20\ ft/s}\ \#$$

12-120 The car is on the curved path

$$a_t = 0.05t^2 .$$

$$\int_0^v dv = \int_0^t 0.05t^2\,dt$$

$$v = 0.0167t^3 \text{ ——— } (1)$$

$$\int_0^s ds = \int_0^t 0.0167t^3\,dt$$

$$s = 4.167\times10^{-3}\,t^4 = 550$$

$$t = 19.06\ s$$

Insert (1) $\Rightarrow$ $v = 0.0167\times(19)^3 = \underline{115\ ft/s}\ \#$

$$a_n = \frac{v^2}{R} = \frac{(115.4)^2}{240} = 55.51\ \frac{ft}{s^2}$$

$$a = \sqrt{55.51^2 + 18.17^2} = 58.4\ ft/s^2\ \#$$

12-122

$$v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_0^s (6-0.06s)\,ds$$

$$v = \sqrt{12s - 0.06s^2} \text{ m/s}$$

$$v_B = \sqrt{12(40) - 0.06(40)^2} = 19.60 \text{ m/s}$$

Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1+\left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{m}} = 79.30 \text{ m}$$

Acceleration:

$$a_t = \dot{v} = 6 - 0.06(40) = 3.6 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.60^2}{79.30} = 4.842 \text{ m/s}^2$$

The magnitude of the roller coaster's acceleration at B is

$$a = \sqrt{a_t^2 + a_n^2} = 6.03 \text{ m/s}^2$$

#

12-144

$$\int_0^v dv = \int_0^t 4t\,dt$$

$$v = 2t^2$$

$$\int_0^s ds = \int_0^t 2t^2\,dt$$

$$s = \frac{2}{3}t^3$$

when $s = \frac{\pi}{6}(40)$ ft, $\frac{\pi}{6}(40) = \frac{2}{3}t^3$, $t = 3.1554$ s

$$v = 2(3.1554)^2 = \underline{19.91\left(\frac{ft}{s}\right)}$$

#

$$a_t = \dot{v} = 4t = 12.62\ ft/s^2$$

$$a_n = \frac{v^2}{\rho} = \frac{19.91^2}{40} = 9.91\ \frac{ft}{s^2}$$

$$a = \sqrt{a_n^2 + a_t^2} = \underline{16.0\ \frac{ft}{s^2}}$$

#

12-163

Velocity:

Applying Eq. 12-25

$$v_r = \dot{r} = 0 \quad . \quad v_\theta = r\dot{\theta} = 300(0.4) = 120\ \frac{ft}{s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \underline{120\ ft/s}$$

#

Acceleration: Applying Eq 12-29,

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 300(0.4)^2 = -48\ \frac{ft}{s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 300(0.2) + 0 = 60\ ft/s^2$$

12-163 continued

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-48)^2 + 60^2} = 76.8 \ \frac{ft}{s^2}$$

#

12-175

$$r = \frac{\theta}{10}$$

$$\dot{r} = -\left(\frac{10}{\theta^2}\right)\dot{\theta}$$

since $v^2 = \dot{r}^2 + (r\dot{\theta})^2$

$$(20)^2 = \left(\frac{10^2}{\theta^4}\right)\dot{\theta}^2 + \left(\frac{10^2}{\theta^2}\right)\dot{\theta}^2$$

$$20^2 = \left(\frac{10^2}{\theta^4}\right)(1+\theta^2)\dot{\theta}^2$$

$$\dot{\theta} = \frac{2\theta^2}{\sqrt{1+\theta^2}}$$

$$v_r = \dot{r} = -\left(\frac{10}{\theta^2}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = -\frac{20}{\sqrt{1+\theta^2}}$$

#

$$v_\theta = r\dot{\theta} = \left(\frac{10}{\theta}\right)\left(\frac{2\theta^2}{\sqrt{1+\theta^2}}\right) = \frac{20\theta}{\sqrt{1+\theta^2}}$$

#

when $\theta = 1$ rad,

$$\begin{cases} v_r = -\frac{20}{\sqrt{2}} = -14.1 \ ft/s \\ v_\theta = \left(\frac{20}{\sqrt{2}}\right) = 14.1 \ ft/s \end{cases}$$

#

12-179

$\dot{r} = 4t$ m/s

$\dot{r}(t=1) = 4.$

$\ddot{r} = 4$

$\dot{\theta} = 6, \quad \ddot{\theta} = 0$

$\int_0^1 dr = \int_0^1 4t \, dt$

$r = 2t^2 \Big|_0^1 = 2$

$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} = 12.6 \text{ m/s}$ #

$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} = 83.2 \frac{m}{s^2}$ #