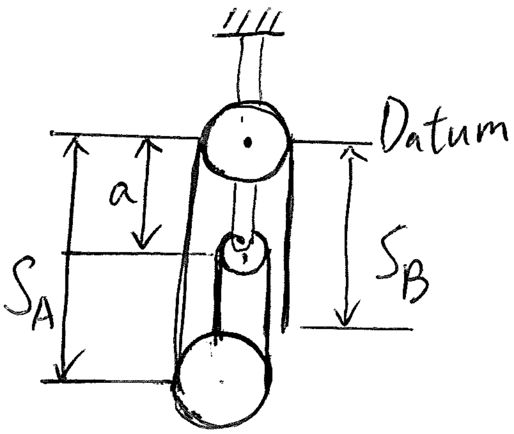
Winter 2014 ME 230 Kinematics and Dynamics

HW # 2 Solution

12 - 211



position Coordinates:

$$S_B + S_A + 2(S_A - a) = l$$

$$\Rightarrow S_B + 3S_A = l + 2a$$

Take time derivitive:

$$(+\downarrow) \quad v_B + 3v_A = 0$$

$v_B = 4\ m/s$, Thus $\underline{V_A = 1.33\ m/s\ (\uparrow)}$ #

12-232

Normal acceleration of car B is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{50^2}{0.7} = 3571.43\ \frac{mi}{h^2}$$

Applying $\bar{a}_B = \bar{a}_A + \bar{a}_{B/A}$ (eq. 12-35),

$$(3571.43 \cos 30^\circ - 1400 \sin 30^\circ)\bar{i} + (-1400 \cos 30^\circ - 3571 \sin 30^\circ)\bar{j}$$

$$= 800\bar{j} + \bar{a}_{B/A}$$

$$\therefore \bar{a}_{B/A} = (2392.95\bar{i} - 3798.15\bar{j})\ \frac{mi}{h^2}$$

12-238

$(\xrightarrow{+})$ Ball: $s = s_0 + v_0 t$

$s_C = 0 + 20\cos 60^\circ t$

$(+\uparrow)$ $v = v_0 + a_c t$

$-20\sin 60^\circ = 20\sin 60^\circ - 9.81t$

$\begin{cases} t = 3.53\text{ s} \\ s_C = 35.31\text{ m} \end{cases}$

Player B: $(\xrightarrow{+})$ $s_B = s_0 + v_B t$

$5.31 = 15 + v_B(3.53)$

$\underline{v_B = 5.75\ \frac{m}{s}}$ #

At the time of the catch

$(v_C)_x = 20\cos 60^\circ = 10\text{ m/s } (\rightarrow)$

$(v_C)_y = 20\sin 60^\circ = 17.32\ \frac{m}{s}\ (\downarrow)$

$\overline{v_C} = \overline{v_B} + \overline{v_{C/B}}$

$\Rightarrow 10\bar{i} - 17.32\bar{j} = 5.751\bar{i} + (v_{C/B})_x\bar{i} + (v_{C/B})_y\bar{j}$

$(\xrightarrow{+})$ $10 = 5.75 + (v_{C/B})_x$

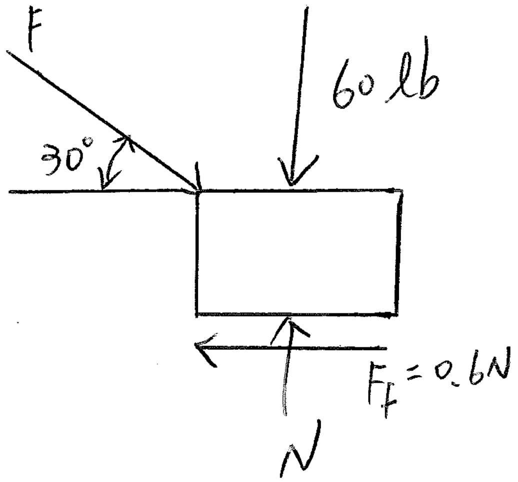
$(+\uparrow)$ $-17.32 = (v_{C/B})_y$

$\Rightarrow \begin{cases} (v_{C/B})_x = 4.25\text{ m/s } (\rightarrow) \\ (v_{C/B})_y = 17.32\text{ m/s } (\downarrow) \end{cases}$

$v_{C/B} = \sqrt{(4.25)^2 + (17.32)^2} = 17.8\text{ m/s}$ Ans

$\theta = \tan^{-1}\left(\frac{17.32}{4.25}\right) = 76.2^\circ$

13-16



Force to produce motion

$(\xrightarrow{+})\ \Sigma F_x = 0: \quad F\cos 30° - 0.6N = 0$

$(+\uparrow)\ \Sigma F_y = 0; \quad N - 60 - F\sin 30° = 0$

$\Rightarrow \begin{cases} N = 91.8 \text{ lb} \\ F = 63.6 \text{ lb} \end{cases}$

Since $N = 91.8$ lb

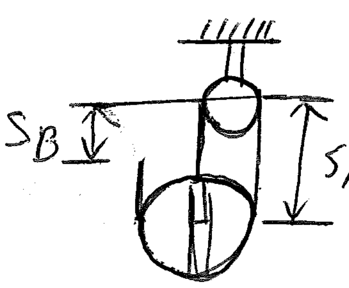
$\xrightarrow{+} \Sigma F_x = m a_x ; \quad 63.6 \cos 30° - 0.3(91.8)$

$= \left(\frac{60}{32.2}\right) a$

$a = 14.8 \text{ ft}/s^2$

#

13-22



Kinematic: $S = S_0 + V_0 t + \frac{1}{2} a_c t^2$

$(\nwarrow +)\ 0.75 = 0 + 0 + \frac{1}{2} a_B (2^2), \quad a_B = 0.375 \text{ m}/s^2$

Position - Coordinate equation: $2S_A + (S_A - S_B) = l$

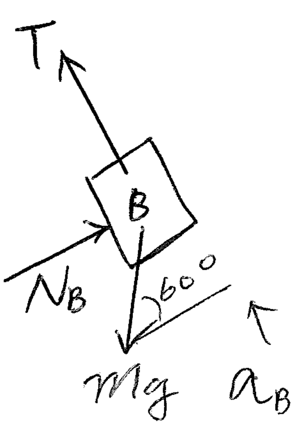
$3S_A - S_B = l$

Time derivitive

$3V_A - V_B = 0$

$3a_A - a_B = 0$

$3a_A - 0.375 = 0, \quad a_A = 0.125 \frac{m}{s^2}$

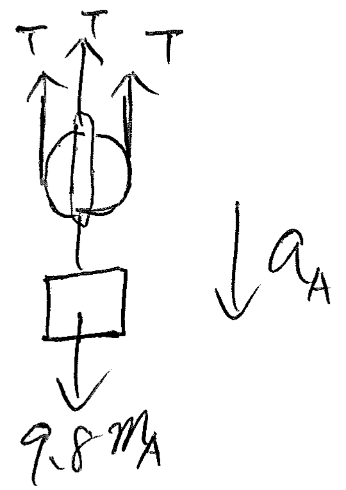


Equation of motion:

$\nwarrow + \Sigma F_y = m a_y ; \quad T - 5(9.81)\sin 60° = 5(0.375)$

$T = 44.35 N$

13-22 continued

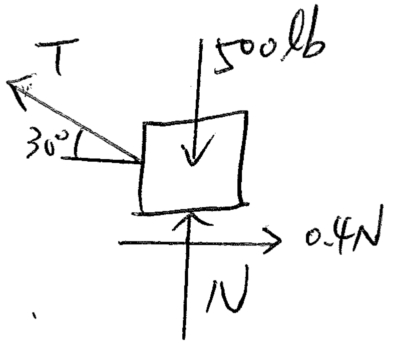


$+\uparrow \Sigma F_y = m a_y$

$3(44.35 - 9.81\, m_A = m_A(-0.125)$

$m_A = 13.7\ kg$ #

13-28

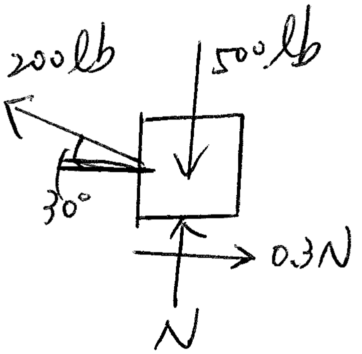


Equilibrium: $\overset{+}{\rightarrow}\Sigma F_x = 0;\ -T\cos 30° + 0.4N = 0$

$+\uparrow \Sigma F_y = 0;\ N + T\sin 30° = 500$

$\Rightarrow \begin{cases} T = 187.6\ lb \\ N = 406.2\ lb \end{cases}$

T<200 lb, the cord will not break at the moment the crate slides.



After the crate begins to slide:

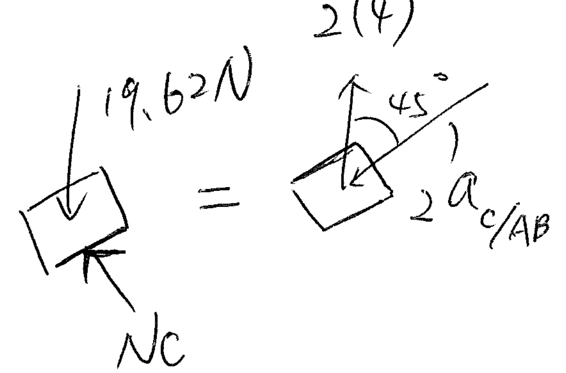
$+\uparrow \Sigma F_y = m a_y;\ N + 200\sin 30° = 500$

$N = 400\ lb$

$\overset{+}{\rightarrow}\Sigma F_x = m a_x:\ 200\cos 30° - 0.3(400) = \frac{500}{32.2} a$

$a = 3.43\ \frac{ft}{s^2}$ #

13-42



$\overset{+}{\leftarrow} \Sigma F_x = m a_x;\ N \sin 45° = 2 a_{c/AB} \sin 45°$

$N = 2 a_{c/AB}$

$+\uparrow \Sigma F_y = m a_y;\ N \cos 45° - 19.62 = 2(4) - 2 a_{c/AB} \cos 45°$

$a_{c/AB} = 9.76514$

$a_c = a_{AB} + a_{c/AB}$

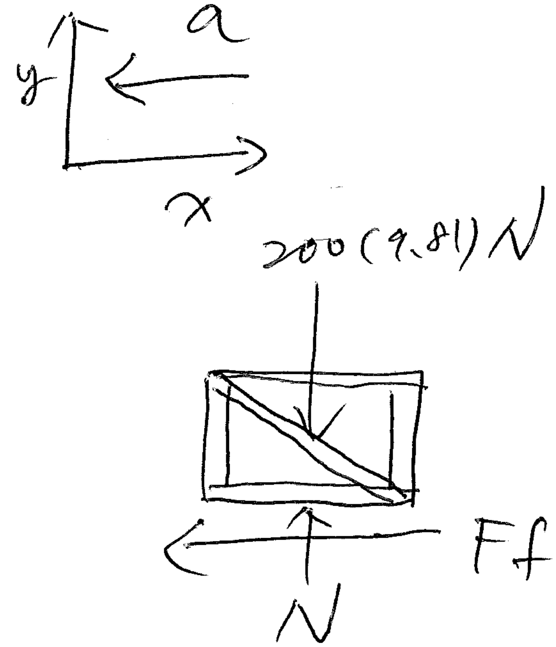
$(a_c)_x = 0 + 9.76514 \sin 45° = 6.905 \leftarrow$

$(a_c)_y = 4 - 9.76514 \cos 45° = 2.905 \downarrow$

$a_c = \sqrt{(a_c)_x^2 + (a_c)_y^2} = 7.49\ m/s^2$ #

$\theta = \tan^{-1}\left(\frac{2.905}{6.905}\right) = 22.8°$ #

13-43



Equation of Motion:

$+\uparrow \Sigma F_y = m a_y:\ N - 200(9.81) = 0$

$N = 1962\ N$

$\overset{+}{\rightarrow} \Sigma F_x = m a_x:\ -0.3(1962) = 200(-a)$

$a = 2.943\ m/s^2$

13-43 continued.

Kinematics

$$v = 60\,\frac{km}{h} = 16.67\,\frac{m}{s}$$

a is constant

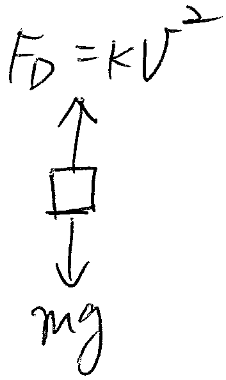
$$(\overset{+}{\leftarrow})\quad v = v_0 + a_c t$$

$$16.67 = 0 + 2.943\,t$$

$$\underline{t = 5.66\,s}\ \#$$

13-48

$$+\downarrow \Sigma F_z = m a_z: \quad mg - kv^2 = m\frac{dv}{dt}$$



$$dt = \left(\frac{m}{mg - kv^2}\right) dv$$

$$\int_0^t dt = \int_0^v \left(\frac{m}{mg - kv^2}\right) dv$$

$$\int_0^t dt = \frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2}$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right) \ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m} t \left(2\sqrt{\frac{mg}{k}}\right) = \ln \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

13-48 continued

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}\, e^{2t\sqrt{\frac{mg}{k}}} - v e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \quad \#$$

when $t \to \infty$, $v_t = \sqrt{\frac{mg}{k}}$ #