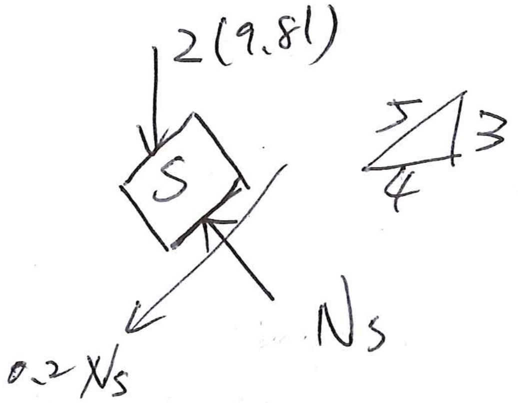
Winter 2014 ME230 Kinematics and Dynamics

HW #3 Solution

Ch 13: 59, 65, 66, 75, 91, 93, 97, 107

Ch 14: 3, 11, 14, 21

13-59



$\rho = 0.25 \times \frac{4}{5} = 0.2m$

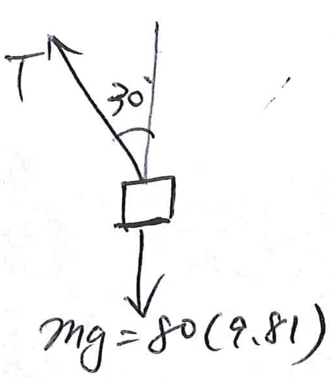
$\overset{+}{\leftarrow}\Sigma F_n = m a_n: \; N_s\left(\frac{3}{5}\right) + 0.2 N_s\left(\frac{4}{5}\right) = 2\left(\frac{v^2}{0.2}\right)$

$+\uparrow \Sigma F_b = m a_b: \; N_s\left(\frac{4}{5}\right) - 0.2 N_s\left(\frac{3}{5}\right) - 2g = 0$

$N_s = 28.85 N$

$\underline{v = 1.48 \frac{m}{s}}$ #

13-65



$\overset{+}{\leftarrow}\Sigma F_n = m a_n; \; T\sin 30° = 80\left(\frac{v^2}{4 + 6\sin 30°}\right)$

$+\uparrow \Sigma F_b = 0; \; T\cos 30° = mg = 80(9.81)$

$T = 906.2N, \quad \underline{v = 6.3 \; m/s}$ #

13-65 continued

50 kg passenger

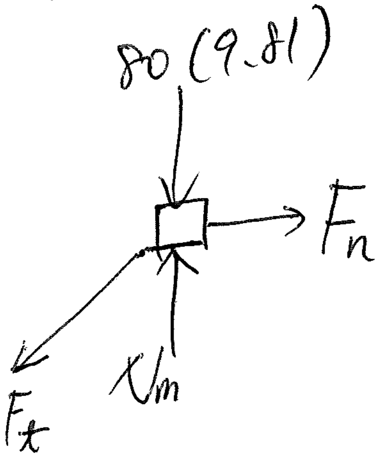
$$a_n = \frac{v^2}{r} = \frac{6.3^2}{(4+6\sin 30^\circ)} = \frac{6.3^2}{7}$$

$$\Sigma F_n = ma_n = 50\left(\frac{6.3^2}{7}\right) = \underline{283\,N}\ \#$$

$$\Sigma F_t = ma_t;\quad \underline{F_t = 0}\ \#$$

$$\Sigma F_b = ma_b;\quad \underline{F_b = 490\,N}\ \#$$

13-66



$$\Sigma F_t = ma_t;\quad F_t = 80(0.4) = 32\,N$$

$$\Sigma F_n = ma_n;\quad F_n = 80\left(\frac{v^2}{3}\right)$$

$$F = \mu_s N_m = \sqrt{F_t^2 + F_n^2}$$

$$0.3(80)(9.81) = \sqrt{32^2 + 80\left(\frac{v^2}{3}\right)^2}$$

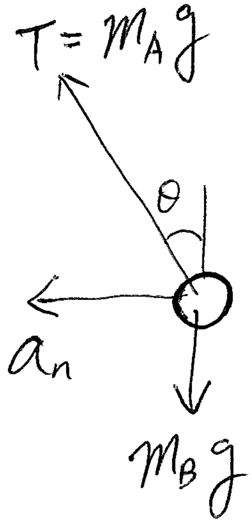
$$v = 2.9575\,\frac{m}{s}$$

$$a_t = \frac{dv}{dt} = 0.4$$

$$\int_0^v dv = \int_0^t 0.4\,dt$$

$$v = 0.4t = 2.9575.\quad \underline{t = 7.395}\ \#$$

13-75

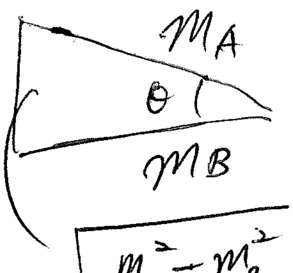


The radius of the horizontal circular path is $r = (l-h)\sin\theta$

$$\Rightarrow a_n = \frac{v^2}{\rho} = \frac{v_B^2}{(l-h)\sin\theta}$$

$$+\uparrow \Sigma F_b = 0; \quad m_A g\cos\theta = m_B g$$

$$\theta = \cos^{-1}\frac{m_B}{m_A}$$



$$\overset{+}{\leftarrow}\Sigma F_n = m a_n: \quad m_A g\sin\theta = m_B\left[\frac{v_B^2}{(l-h)\sin\theta}\right]$$

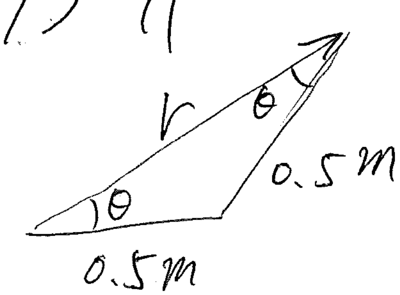
$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\sin\theta \quad \text{—— (1)}$$

$$\sin\theta = \frac{\sqrt{m_A^2 - m_B^2}}{m_A}, \quad \text{substituting into (1)}$$

$$v_B = \sqrt{\frac{m_A g(l-h)}{m_B}}\left(\frac{\sqrt{m_A^2 - m_B^2}}{m_A}\right)$$

$$= \sqrt{\frac{g(l-h)(m_A^2 - m_B^2)}{m_A m_B}}$$

13-91



$r = 2(0.5\cos\theta) = 1\cos\theta$

$\dot{r} = -\sin\theta\,\dot{\theta}$

$\ddot{r} = -\cos\theta\,\dot{\theta}^2 - \sin\theta\,\ddot{\theta}$

$\theta = 30^\circ;\ \dot{\theta} = 4\ \frac{rad}{s},\ \ddot{\theta} = 8\ \frac{rad}{s^2}$

$r = 1\cos 30^\circ = 0.866\ ft$

$\dot{r} = -\sin 30^\circ \times 4 = -2\ ft/s$

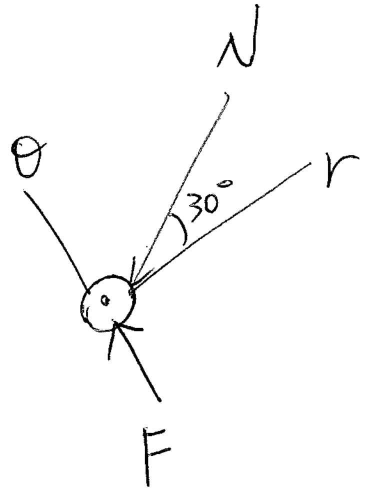
$\ddot{r} = -\cos 30^\circ(4^2) - \sin 30^\circ(8) = -17.856\ \frac{ft}{s^2}$

$a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.866(4^2)$

$= -31.713\ \frac{ft}{s^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.866(8) + 2(-2)4$

$= -9.072\ ft/s^2$



$\nearrow + \Sigma F_r = ma_r;\ -N\cos 30^\circ = \frac{0.5}{32.2}(-31.713)$

$N = 0.5686\ lb$

$+\nwarrow \Sigma F_\theta = ma_\theta;\ F - 0.5686\sin 30^\circ = \frac{0.5}{32.2} \times (-9.072)$

$\underline{F = 0.143\ lb}$ #

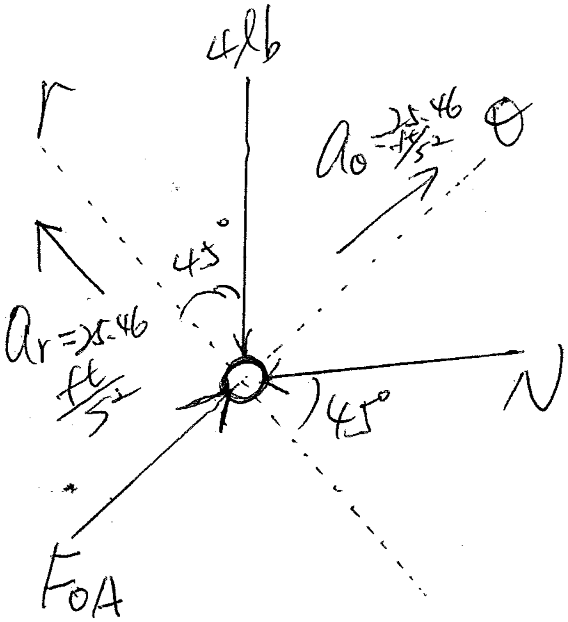
13-93

Kinematics: $\frac{4}{r} = \cos\theta$

at the instant $\theta = 45°$,

$r = 4\sec\theta\Big|_{\theta=45°} = 4\sec 45° = 5.657\,ft$

$\dot{r} = 4\sec\theta(\tan\theta)\dot{\theta}\Big|_{\theta=45°} = 8.485\,\frac{ft}{s}$



$\ddot{r} = 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \dot{\theta}(\sec\theta\sec^2\theta\dot{\theta} + \tan\theta\sec\theta\tan\theta\dot{\theta})\right]$

$= 4\left[\sec\theta(\tan\theta)\ddot{\theta} + \sec^3\theta\dot{\theta}^2 + \sec\theta\tan^2\theta\dot{\theta}^2\right]_{\theta=45°}$

$= 38.18\,\frac{ft}{s^2}$

So $a_r = \ddot{r} - r\dot{\theta}^2 = 38.18 - 5.657(1.5^2) = 25.46\,\frac{ft}{s^2}$

$a_\theta = r\ddot{\theta} - 2\dot{r}\dot{\theta} = 2.657(0) + 2(8.485)(1.5)$

$= 25.46\ ft/s^2$

Equation of Motion:

$\Sigma F_r = ma_r;\quad N\cos 45° - 4\cos 45°$

$= \frac{4}{32.2}(25.46)$

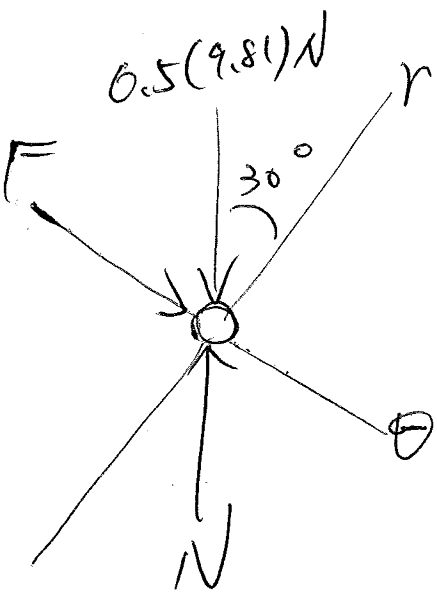
$N = 8.472\,lb$

$\Sigma F_\theta = ma_\theta;\quad F_{OA} - 8.472\sin 45° - 4\sin 45°$

$= \frac{4}{32.2}(25.46)$

$\underline{F_{OA} = 12\ lb}$ #

13-97



$$r = \frac{0.5}{\cos\theta} = 0.5\sec\theta$$

$$\dot{r} = 0.5\sec\theta\tan\theta\,\dot{\theta}$$

$$\ddot{r} = 0.5\{[(\sec\theta\tan\theta\,\dot{\theta})\tan\theta + \sec\theta(\sec^2\theta\,\dot{\theta})]\dot{\theta} + \sec\theta\tan\theta\,\ddot{\theta}\}$$

$$= 0.5[\sec\theta\tan^2\theta\,\dot{\theta}^2 + \sec^3\theta\,\dot{\theta}^2 + \sec\theta\tan\theta\,\ddot{\theta}]$$

when $\theta = 30°$, $\dot{\theta} = 2\,\frac{rad}{s}$, $\ddot{\theta} = 3\,\frac{rad}{s^2}$

$$r = 0.5\sec 30° = 0.5774\,m$$

$$\dot{r} = 0.5\sec 30°\tan 30°(2) = 0.6667\,\frac{m}{s}$$

$$\ddot{r} = 4.849\,\frac{m}{s^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4.849 - 0.5774(2^2) = 2.539\,\frac{m}{s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.3987\,\frac{m}{s^2}$$

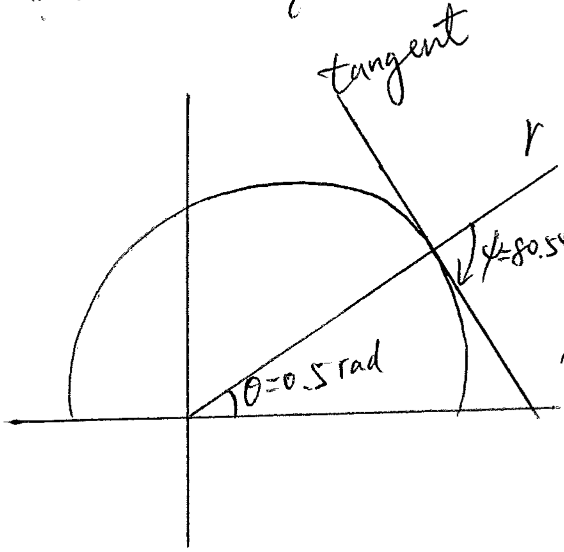
$\nearrow + \Sigma F_r = ma_r;\ N\cos 30° - 0.5(9.81)\cos 30° = 0.5(2.5396)$

$$\underline{N = 6.3712\,N}\ \#$$

$+\searrow \Sigma F_\theta = ma_\theta;\ F + 0.5(9.81)\sin 30° - 6.37\sin 30° = 0.5(4.3987)$

$$\underline{F = 2.93\,N}\ \#$$

13-107



$r = 2 + \cos\theta \qquad \theta = 0.5t^2$

$\dot{r} = -\sin\theta\dot{\theta} \qquad \dot{\theta} = t$

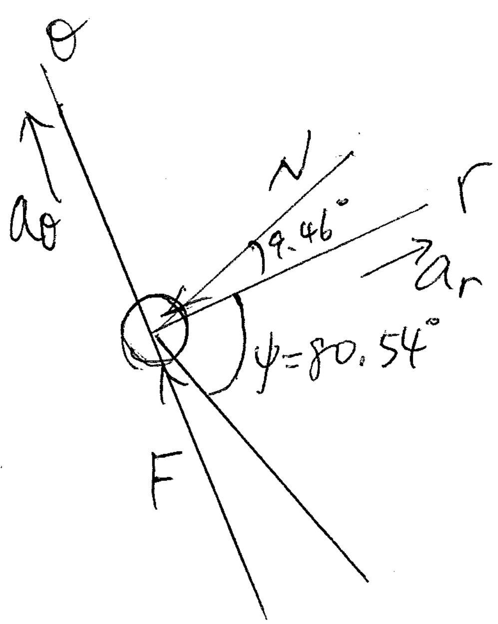
$\ddot{r} = -\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta} \qquad \ddot{\theta} = 1\ \frac{rad}{s^2}$

At $t = 1s$, $\theta = 0.5$ rad, $\dot{\theta} = 1\ \frac{rad}{s}$, $\ddot{\theta} = 1\ \frac{rad}{s^2}$

$r = 2 + \cos 0.5 = 2.8776\ ft$

$\dot{r} = -\sin 0.5(1) = -0.4794\ ft/s^2$

$\ddot{r} = -\cos 0.5(1)^2 - \sin 0.5(1) = -1.357\ \frac{ft}{s^2}$



$a_r = \ddot{r} - r\dot{\theta}^2 = -1.375 - 2.8776(1)^2$

$= -4.2346\ \frac{ft}{s^2}$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2.8776(1) + 2(-0.4794)(1)$

$= 1.9187\ \frac{ft}{s^2}$

$\tan\psi = \frac{r}{dr/d\theta} = \left.\frac{2+\cos\theta}{-\sin\theta}\right|_{\theta = 0.5\,rad} = -6.002$

$\psi = -80.54°$

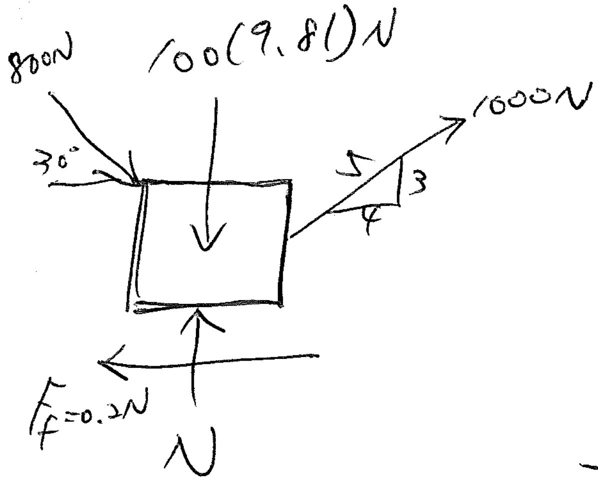
$+\nearrow \Sigma F_r = ma_r; \ -N\cos 9.46° = \frac{2}{32.2}(-4.2346)$

$N = 0.2666\ lb$

$+\nwarrow \Sigma F_\theta = ma_\theta; \ F - 0.2666\sin 9.46° = \frac{2}{32.2}(1.9187)$

$\underline{F = 0.163\ lb}$ #

14-3



$+\uparrow \Sigma F_y = m a_y$:

$N + 1000\left(\frac{3}{5}\right) - 800 \sin 30° - 100(9.81) = 100(0)$

$N = 781\,N$

Principle of work and energy:

$T_1 + \Sigma U_{1-2} = T_2$

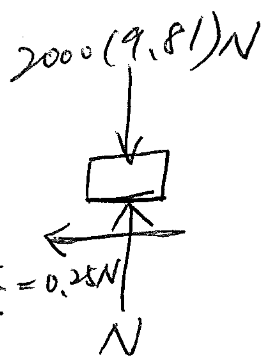
$800(\cos 30°)S + 1000\left(\frac{4}{5}\right)S - 781(0.2)$

$= \frac{1}{2}(100)(6^2)$

$S = 1.35\,m$

\#

14-11



Free dody diagram:

$+\uparrow \Sigma F_y = m a_y$; $N - 2000(9.81) = 2000 \times 0$

$N = 19620\,N$

friction force: $F_f = \mu_k N = \mu_k (19620)$

Principle of Work and Energy:

only F_f does work (negative)

Initial speed of the car: $V_1 = 100 \frac{km}{h} = 27.78 \frac{m}{s}$

14-11 continued

The skidding distance of the car is s'

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}(2000)(27.78^2) + [-\mu_k(19620)s'] = 0$$

$$s' = \frac{39.327}{\mu_k}$$

The distance traveled by the car during reaction time:

$$s'' = v_1 t = 27.78(0.75) = 20.83\ m$$

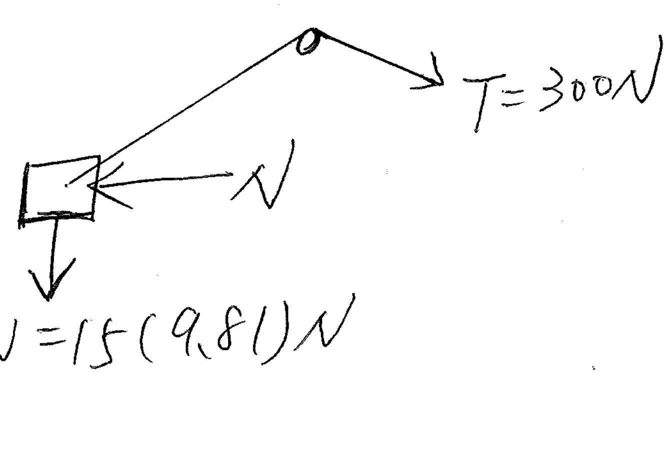
Total distance: $s = s' + s''$

$$175 = \frac{39.327}{\mu_k} + 20.83$$

$$\underline{\mu_k = 0.255}\ \#$$

14-14

Free body diagram:



14-14 continued

N doesn't work (always act perpendicular to the motion).

When collar moves from A to B,

W: vertically upward a distance $h = (0.3+0.2) = 0.5\,m$

F: distance $S = AC - AB = \sqrt{0.7^2+0.4^2} - \sqrt{0.2^2+0.2^2}$

$= 0.5234\,m$

(F: positive. W: negative)

$$T_A + \sum U_{A-B} = T_B$$

$$0 + 300(0.5234) + [-15(9.81)(0.5)] = \frac{1}{2}(15)\,v_B^2$$

$$v_B = 3.335\,\frac{m}{s} = \underline{3.34\,\frac{m}{s}}$$ #

14-21

$$T_1 + \sum U_{1-2} = T_2$$

$$\frac{1}{2}(1800)\,0.5^2 - \frac{1}{2}(5000)(0.5-0.3)^2 - \frac{1}{2}k_B(0.45-0.3)^2 = 0$$

$$\underline{k_B = 11.1\,\frac{kN}{m}}$$ #