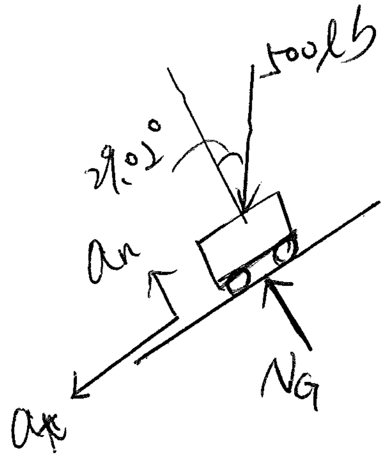
14 Winter ME 230 Kinematics and Dynamics

HW #4 Solution

Ch 14: 71, 77, 79, 91, 92,

Ch 15: 6, 11, 21, 42, 54, 57

14-71



$$y = \frac{1}{260}x^2$$

$$\frac{dy}{dx} = \frac{1}{130}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{130}$$

At $y = 20$ ft, $x = 72.11$ ft

$$\tan\theta = \frac{dy}{dx} = 0.555, \quad \theta = 29.02^\circ$$

$$\rho = \frac{[1+(0.555)^2]^{3/2}}{\frac{1}{130}} = 194.40 \text{ ft}$$

$$+\nwarrow \Sigma F_n = ma_n; \quad N_G - 500\cos 29.02^\circ = \frac{500}{32.2}\left(\frac{v^2}{194.40}\right) \qquad (1)$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}\left(\frac{500}{32.2}\right)v^2 - 500(100)$$

$v^2 = 6440, \quad \underline{v = 80.2 \text{ ft/s}}$ #

Sub. into (1), $N_G = 952$ lb #

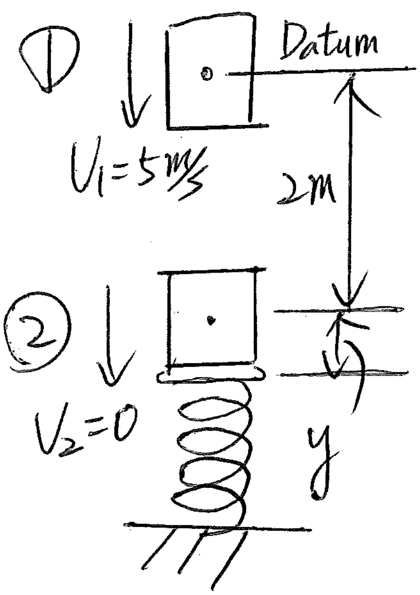
14-77

Potential Energy of the block:

$$(V_g)_1 = mgh_1 = 40(9.81)\times 0 = 0$$

$$(V_g)_2 = mgh_2 = 40(9.81)[-(2+y)]$$

Potential Energy of the spring:



$$(V_e)_1 = \frac{1}{2}ks_1^2 = \frac{1}{2}(25)(10^3)(0.15^2) = 281.25\,J$$

$$(V_e)_2 = \frac{1}{2}ks_2^2 = \frac{1}{2}(25\times10^3)(0.15+y)^2$$

Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}mV_1^2 + V_{g1} + V_{e1} = \frac{1}{2}mV_2^2 + V_{g2} + V_{e2}$$

$$\frac{1}{2}\times40\times5^2 + 0 + 281.25 = 0 + [-392.4(2+y)] + \frac{1}{2}(25\times10^3)(0.15+y)^2$$

$$12500y^2 + 3357.6y - 1284.8 = 0$$

Solving for the above equation:

$$y = 0.2133\,m = 213\,mm \;\#$$

(positive only)

14-79

Datum at B:

$$T_A + V_A = T_B + V_B$$

$$0 + 1.5 \times 10 = \frac{1}{2}\left(\frac{15}{32.2}\right) v_B^2 + 0$$

$$v_B = 25.4 \ ft/s \quad \#$$

14-91

Datum at ground:

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}(120)\,4^2 + 120(9.81) \times 12 = \frac{1}{2}(120)\, v_1^2 + 120(9.81) \cdot 10$$

$$v_1 = 7.432 \ m/s \quad \#$$

$$+\downarrow \Sigma F_n = m a_n; \quad 70(9.81) + N = 70\left(\frac{(7.432)^2}{5}\right)$$

$$N = 86.7 \ N \quad \#$$

14-92

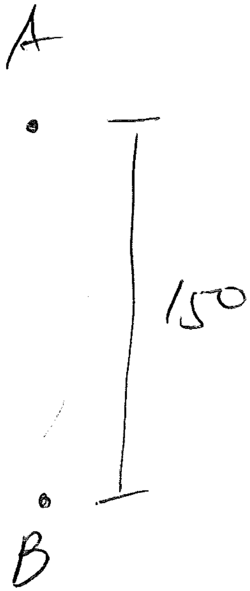
Potential Energy

Datum: surface of the water

$$(V_g)_A = mg h_A = 75(9.81)(150) = 110362.5 \ J$$

$$(V_g)_B = mg h_B = 75(9.81) \times 0 = 0$$

14-92 continued



potential energy of the elastic cord:

position A: unstratched ($S_A = 0$)

position B: $S_B = 150 - l_0$, l_0: unstretched length

$$(V_e)_A = \frac{1}{2} k S_A^2 = 0$$

$$(V_e)_B = \frac{1}{2} k S_B^2 = \frac{1}{2}(80)(150 - l_0)^2$$

Conservation of Energy:

$$T_A + V_A = T_B + V_B$$

$$\frac{1}{2} m V_A^2 + \left[(V_g)_A + (V_e)_A\right] = \frac{1}{2} m V_B^2 + \left[(V_g)_B + (V_e)_B\right]$$

$$\frac{1}{2}(75)(1.5)^2 + (110362.5 + 0)$$

$$= 0 + 40(150 - l_0)^2\Big]$$

$$\underline{l_0 = 97.5 \text{ m}} \quad \#$$