14 Winter ME230 Kinematics and Dynamics

HW #5 Solution

Ch 15 - 6, 11, 21, 42, 54, 57

15-6

$$(V_x)_2 = 40\ \frac{km}{h} = 11.11\ \frac{m}{s}$$

Entire train:

$$(\pm\rightarrow)\quad m(V_x)_1 + \Sigma\int F_x\,dt = m(V_x)_2$$

$$0 + F(80) = [50 + 3\times 30]\times 10^3 \times 11.11$$

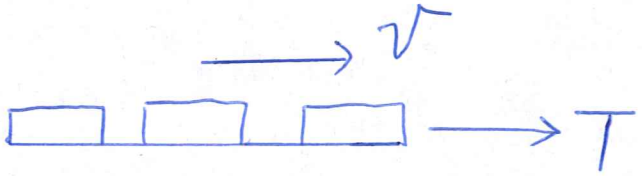
$$F = 19.4\ kN \quad \#$$

Three cars:

$$(\pm\rightarrow)\quad m(V_x)_1 + \Sigma\int F_x\,dt = m(V_x)_2$$

$$0 + T(80) = 3\times 30\times 10^3 \times 11.11$$

$$T = 12.5\ kN \quad \#$$



15-6 continued

Three cars:

$$(\pm\rightarrow)\ m(v_x)_1 + \Sigma\int F_x\,dt = m(v_x)_2$$

$$0 + T(80) = 3(30\times10^3)(11.11)$$

$$T = 12.5\ kN \quad \#$$

15-11

Equation of Motion

$$+\uparrow \Sigma F_y = m a_y;\quad N - 5(9.81) = 0,$$

$$N = 49.05\ N$$

$$\pm\rightarrow \Sigma F_x = m a_x;\quad -0.3(49.05) = -5a$$

$$a = 2.943\ m/s^2$$

Kinematics: Applying $v^2 = v_0^2 + 2a_c(s - s_0)$

$$(\pm\rightarrow)\ v^2 = 14^2 + 2(-2.943)(6-0),$$

$$v = 12.68\ m/s$$

Principle of Linear Impulse and Momentum

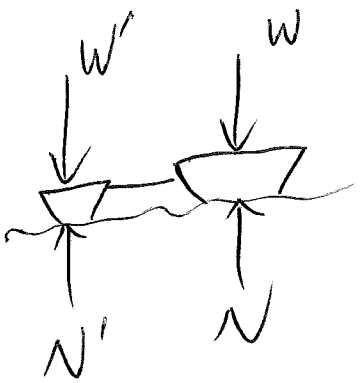
$$m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$(\pm\rightarrow)\ 5(12.68) - I = 5\times0$$

$$I = 63.4\ N\cdot s \quad \#$$

15-21

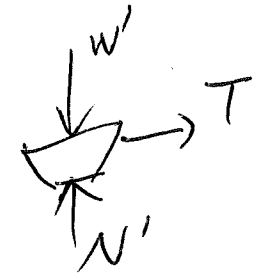
$25\frac{km}{h} = 6.944\frac{m}{s}$



System: $(\pm)\ mV_1 + \Sigma\int F\,dt = mV_2$

$[0+0] + F(35) = (50+75)\times10^3(6.944)$

$F = 24.8\ kN$ #

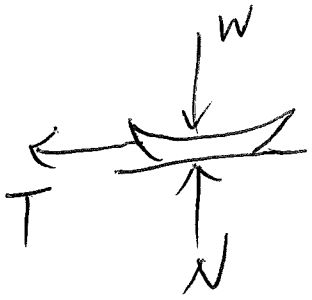


Barge:

$(\pm)\ mV_1 + \Sigma\int F\,dt = mV_2$

$0 + T(35) = (75\times10^3)(6.944)$

$T = 14.9\ kN$ #



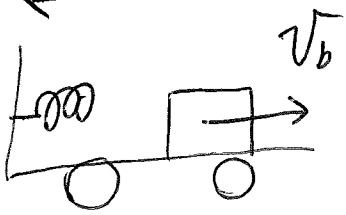
Tugboat:

$(\pm)\ mV_1 + \Sigma\int F\,dt = mV_2$

$0 + F(35) - 14.881(35) = 50\times10^3 \times 6.944$

$F = 24.8\ kN$ #

15-42



$T_1 + V_1 = T_2 + V_2$

$0+0+\frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(V_b)^2 + \frac{1}{2}(75)(V_c)^2$

$12 = 50\,V_b^2 + 75\,V_c^2$

$(\pm)\ \Sigma mV_1 = \Sigma mV_2$

$0+0 = 50\,V_b - 75\,V_c$

15-42 continued

$v_b = 1.5\, v_c$

$v_c = 0.253 \frac{m}{s} \leftarrow$

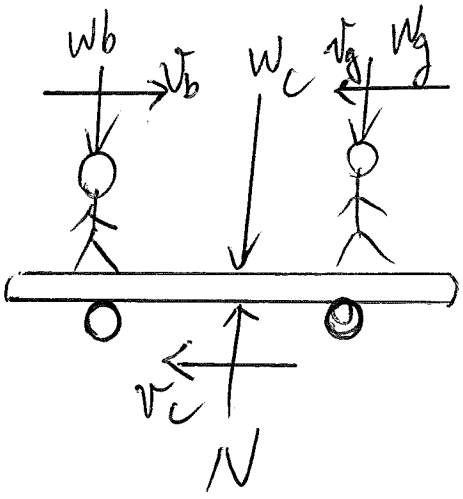
$v_b = 0.379 \frac{m}{s} \rightarrow$

$\overline{v_b} = \overline{v_c} + \overline{v_{b/c}}$

$(\pm\rightarrow)\ 0.379 = -0.253 + v_{b/c}$

$v_{b/c} = 0.632 \frac{m}{s} \rightarrow$ //

15-54



Conservation of Linear Momentum:

$(\pm\rightarrow)\ \sum m v_1 = \sum m v_2$

$0 + 0 + 0 = \frac{80}{32.2} v_b - \frac{60}{32.2}(v_g) - \frac{300}{32.2} v_c$

$80 v_b - 60 v_g - 300 v_c = 0$ — (1)

Kinematics:

the boy: $\overline{v_b} = \overline{v_c} + \overline{v_{b/c}}$

$v_b = -v_c + 3$ — (2)

the girl: $\overline{v_g} = \overline{v_c} + \overline{v_{g/c}}$

$v_g = v_c + 2$ — (3)

Solving equation (1), (2), and (3)

$v_b = 2.727$ ft/s ($\rightarrow$), $v_g = 2.273 \frac{ft}{s}$ ($\leftarrow$), $v_c = 0.273 \frac{ft}{s}$ ($\leftarrow$)

15-57

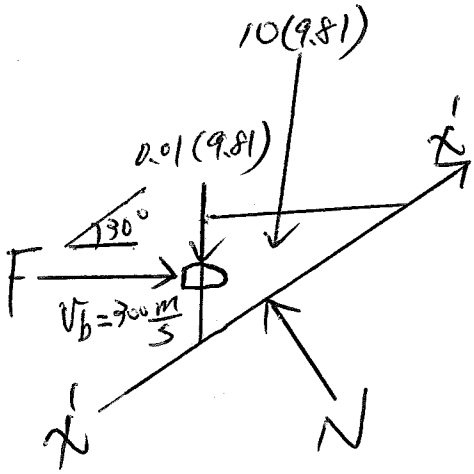
Conservation of Linear Momentum:

$$m_b(v_b)_{x'} = (m_b + m_B)v_{x'}$$

$$0.01(300\cos 30^\circ) = (0.01+10)v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy:



The datum is set at the blocks initial position

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10+0.01)(0.2595^2) = 0 + (10+0.01)gh$$

(at their highest point)

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = \frac{3.43}{\sin 30^\circ} = 6.87 \text{ mm}$$

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14 Winter ME230 Kinematics and Dynamics

HW #5 Solution

Ch 15 - 63, 80, 88, 92, 105, 108

15-63

$$(\pm\rightarrow)\ \Sigma m v_1 = \Sigma m v_2$$

$$mv - mv = mv_A + mv_B$$

$$v_A = -v_B$$

$$(\pm\rightarrow)\ e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{v_B - v_A}{v - (-v)}$$

$$2ve = 2v_B$$

$$v_B = ve \rightarrow \#$$

$$v_A = -ve = ve \leftarrow \#$$

At maximum deformation $v_A = v_B = v'$

$$(\pm\rightarrow)\ \Sigma m v_1 = \Sigma m v_2$$

$$mv - mv = (2m)v'$$

$$v' = 0 \quad \#$$

15-80

Before impact, $T_1 + V_1 = T_2 + V_2$

$$mgh = \frac{1}{2} m v_A^2$$

$$v_A = \sqrt{2gh}$$

$$(+\downarrow) \quad e = \frac{(v_B)_2 - (v_A)_2}{\sqrt{2gh}}$$

$$e\sqrt{2gh} = (v_B)_2 - (v_A)_2 \quad \text{——(1)}$$

$$(+\downarrow) \quad \Sigma m v_1 = \Sigma m v_2$$

$$m v_A + 0 = m(v_A)_2 + 2m(v_B)_2 \quad \text{——(2)}$$

Solving (1) and (2)

$$(v_B)_2 = \frac{1}{3}\sqrt{2gh}\,(1+e)$$

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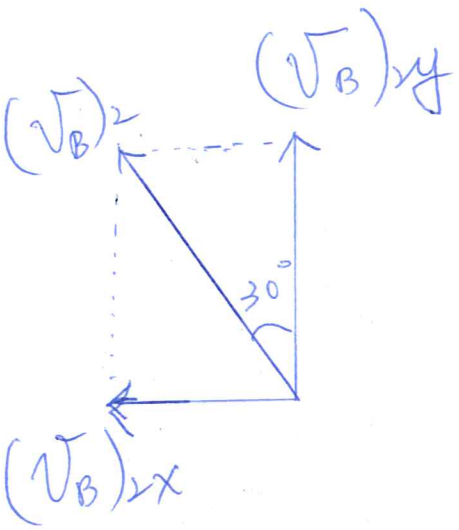
15-88

$$\sum m v_1 = \sum m v_2$$

$$(\pm\rightarrow)\quad 0.5\times 4\times\frac{3}{5} - 0.5\times 6 = -0.5(v_B)_{2x} + 0.5(v_A)_{2x}$$

$$-3.6 = -(v_B)_{2x} + (v_A)_{2x}$$



$$(+\uparrow)\quad 0.5\times 4\times\frac{4}{5} = 0.5(v_B)_{2y}$$

$$(v_B)_{2y} = 3.2\ \text{m/s}\ \uparrow$$

$$(v_B)_{2x} = 3.2\tan 30^\circ = 1.8475\ \frac{\text{m}}{\text{s}}\ (\leftarrow)$$

$$(v_A)_{2x} = -1.752\ \text{m/s}$$

$$= 1.752\ \text{m/s}\ (\leftarrow)$$

$$(\pm\rightarrow)\quad e = \frac{(v_A)_2 - (v_B)_2}{(v_B)_1 - (v_A)_1}$$

$$e = \frac{-1.752 - (-1.8475)}{4\left(\frac{3}{5}\right) - (-6)} = 0.0113$$

#

15-92

$$\Sigma m v_1 = \Sigma m v_2$$

$$(\pm\rightarrow) \quad -m \times 0.8 \sin 30^\circ - m \times 0.5 \left(\tfrac{4}{5}\right) = -m (v_A)_2 \sin 45^\circ - m (v_B)_2 \cos 30^\circ$$

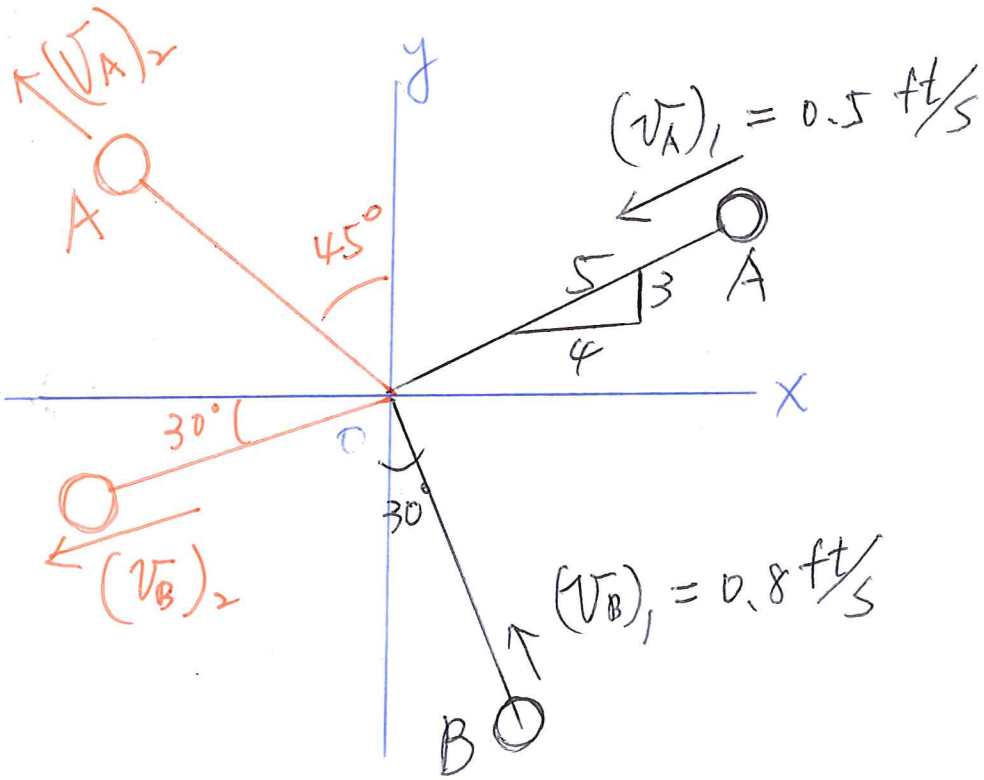
$$0.8 = 0.707 (v_A)_2 + 0.866 (v_B)_2$$

$$(+\uparrow) \quad m(0.8) \cos 30^\circ - m(0.5) \tfrac{3}{5} = m (v_A)_2 \cos 45^\circ - m (v_B)_2 \sin 30^\circ$$

$$-0.3928 = -0.707 (v_A)_2 + 0.5 (v_B)_2$$

Solving: $(v_B)_2 = 0.298$ ft/s #

$(v_A)_2 = 0.766$ ft/s #



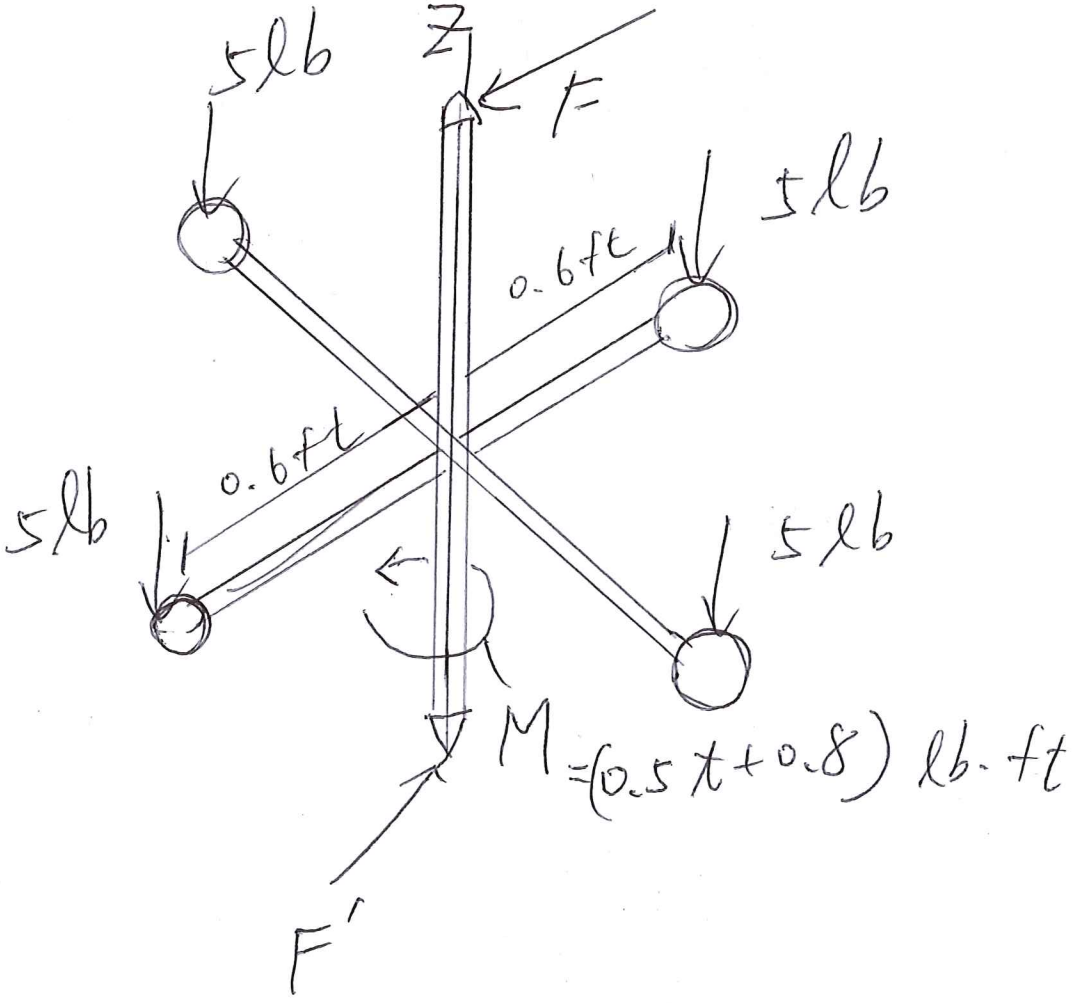
15-105

$$(H_z)_1 + \sum \int M_z \, dt = (H_z)_2$$

$$0 + \int_0^4 (0.5t + 0.8)\, dt = 4\left[\left(\frac{5}{32.2}\right)(0.6\, v_2)\right]$$

$$7.2 = 0.37267\, v_2$$

$$v_2 = 19.3 \text{ ft/s}$$



15-108

Before $\theta = 30^\circ$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2.8(1 - \cos 30^\circ)50 \times 9.81 = \frac{1}{2} \times 50 \times V_1^2 + 0$$

$$V_1 = 2.713 \text{ m/s}$$

$$H_1 = H_2$$

$$50(2.713)(2.80) = 50(V_2) \times 3$$

$$V_2 = 2.532 \text{ m/s}$$

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After $\theta = 0^\circ$

$$T_2 + V_2 = T_3 + V_3$$

$$\frac{1}{2}(50) \times 2.532^2 + 0 = 0 + 50(9.81) \times 3(1 - \cos\theta_2)$$

$$0.1089 = 1 - \cos\theta_2$$

$$\theta_2 = 27^\circ$$

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