Winter 2014 ME 230 Kinematics and Dynamics

HW #6 Solution

ch 16 - 13, 18, 34, 43, 49, 50, 65, 73, 91, 105, 111, 114, 125

16-13

$$\alpha_A = 4t^3$$

$$d\omega = \alpha\, dt$$

$$\int_{20}^{\omega_A} d\omega_A = \int_0^t \alpha_A\, dt = \int_0^t 4t^3\, dt$$

$$\omega_A = t^4 + 20$$

$t = 2s$,

$$\omega_A = 36 \text{ rad/s}$$

$$\omega_A r_A = \omega_B r_B$$

$$36(0.05) = \omega_B (0.15)$$

$$\omega_B = 12 \text{ rad/s}$$

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16-18

$$\alpha_A r_A = \alpha_c r_c$$

$$6(50) = \alpha_c (150)$$

$$\alpha_c = 2 \text{ rad/s}^2$$

$$a_B = \alpha_c r_B = 2(0.075) = 0.15 \frac{m}{s^2}$$

$$(+\uparrow)\ v^2 = v_0^2 + 2a_c(s - s_0)$$

$$v^2 = 0 + 2(0.15)(6-0)$$

$$v = 1.34 \text{ m/s}$$

#

16-34

$$v = r\omega$$

0, ω = constant

$$a = \frac{d(r\omega)}{dt} = \cancel{\frac{d\omega}{dt}} r + \omega \frac{dr}{dt}$$

$$= \omega\left(\frac{dr}{dt}\right)$$

$$r = r_1 + \left(\frac{r_2 - r_1}{L}\right)x$$

$$dr = \left(\frac{r_2 - r_1}{L}\right)dx$$

$$dx = \frac{d\theta}{2\pi} \cdot d$$

Thus $\frac{dr}{dt} = \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right) d\left(\frac{d\theta}{dt}\right) = \frac{1}{2\pi}\left(\frac{r_2 - r_1}{L}\right) d\omega$

Thus. $a = \frac{\omega^2}{2\pi}\left(\frac{r_2 - r_1}{L}\right) d$ #

16-43

$$x = 0.2\cos\theta + \sqrt{(0.75)^2 - (0.2\sin\theta)^2}$$

$$\dot{x} = -0.2\sin\theta\,\dot{\theta} + \frac{1}{2}\left[(0.75)^2 - (0.2\sin\theta)^2\right]^{-\frac{1}{2}}$$

$$\times(-2)(0.2\sin\theta)(0.2\cos\theta)\dot{\theta}$$

$$v_p = -0.2\omega\sin\theta - \left(\frac{1}{2}\right)\frac{(0.2)^2\omega\sin 2\theta}{\sqrt{(0.75)^2 - (0.2\sin\theta)^2}}$$

At $\theta = 30°$; $\omega = 150$ rad/s

$$v_p = -0.2(150)\sin 30° - \left(\frac{1}{2}\right)\frac{(0.2)^2 \times 150 \times \sin 60°}{\sqrt{(0.75)^2 - (0.2\sin 30°)^2}}$$

$$v_p = -18.5 \text{ ft/s} = 18.5 \text{ ft/s } (\leftarrow)$$ #

16-49

$$L\cos\phi + L\cos\theta = L$$

$$\cos\phi + \cos\theta = 1$$

$$\sin\theta\dot{\theta} + \sin\phi\dot{\phi} = 0 \quad \text{———— (1)}$$

$$\cos\theta(\dot{\theta})^2 + \sin\theta\ddot{\theta} + \sin\phi\ddot{\phi} + \cos\phi(\dot{\phi})^2 = 0 \text{ —(2)}$$

when $\theta = 60°$, $\phi = 60°$,

thus, $\dot{\theta} = -\dot{\phi} = \omega$ (from (1))

$$\ddot{\theta} = 0$$

$$\ddot{\phi} = -1.155\,\omega^2 \quad \text{(from (2))}$$

Also, $S_c = L\sin\phi - L\sin\theta$

$$v_c = L\cos\phi\dot{\phi} - L\cos\theta\dot{\theta}$$

$$a_c = -L\sin\phi(\dot{\phi})^2 + L\cos\phi(\ddot{\phi})$$
$$-L\cos\theta(\ddot{\theta}) + L\sin\theta(\dot{\theta})^2$$

At $\theta = 60°$, $\phi = 60°$

$S_c = 0$,

$$v_c = L(\cos 60°)(-\omega) - L\cos 60°(\omega) = -L\omega$$
$$= L\omega \uparrow$$

\#

$$a_c = -0.577L\omega^2 = 0.577L\omega^2 \uparrow$$

\#

16-50

$$x = \frac{a}{\tan\theta} = a\cot\theta$$

$$\frac{dx}{dt} = -a\csc^2\theta \underbrace{\frac{d\theta}{dt}}_{\omega} = -V_0$$

$$\Rightarrow -V_0 = -a\csc^2\theta(\omega)$$

$$\omega = \frac{V_0}{a\csc^2\theta} = \frac{V_0}{a}\sin^2\theta$$

#

$$\alpha = \frac{d\omega}{dt} = \frac{V_0}{a}(2\sin\theta\cos\theta)\frac{d\theta}{dt}$$

$$2\sin\theta\cos\theta = \sin 2\theta$$

$$\Rightarrow \alpha = \frac{V_0}{a}\sin 2\theta(\omega) = \frac{V_0}{a}\sin 2\theta\left(\frac{V_0}{a}\sin^2\theta\right)$$

$$= \left(\frac{V_0}{a}\right)^2 \sin 2\theta \sin^2\theta$$

#

16-65

$$v_C = v_B + v_{C/B}$$

$$(\overset{+}{\rightarrow}) \ -4 = 8 - 0.6\omega$$

$$\omega = 20 \ \text{rad/s} \ \#$$

$$v_A = v_B + v_{A/B}$$

$$(\overset{+}{\rightarrow}) \ v_A = 8 - 20(0.3)$$

$$v_A = 2 \ \text{ft/s} \rightarrow \ \#$$

$$v_C = v_B + \omega \times \bar{r}_{C/B}$$

$$-4\bar{i} = 8\bar{i} + (\omega\bar{k}) \times (0.6\bar{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \ \text{rad/s} \ \#$$

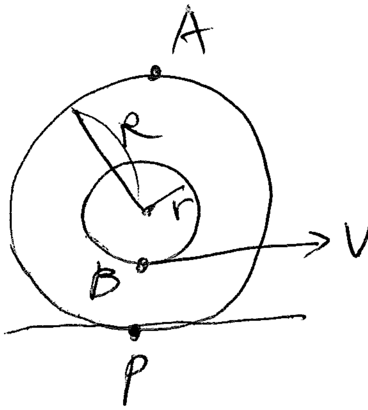
$$v_A = v_B + \omega \times \bar{r}_{A/B}$$

$$v_A\bar{i} = 8\bar{i} + 20\bar{k} \times (0.3\bar{j})$$

$$v_A = 2 \ \text{ft/s} \rightarrow$$

$$\#$$

16-73



$$v_B = v = (R-r)\omega$$

$$\omega = \frac{v}{R-r}$$

$$v_A = 2R\left(\frac{v}{R-r}\right)$$

#

(Spool rolls without slipping, the velocity of the contact point P is zero)

16-91

$$\frac{1.6-x}{5} = \frac{x}{10}$$

$$5x = 16-10x, \quad x = 1.0667 \text{ ft}$$

$$\omega = \frac{10}{1.0667} = 9.375 \text{ rad/s}$$

$$v_C = \omega(r_{IC-C})$$

$$= 9.375(1.0667-0.8)$$

$$= 2.5 \text{ ft/s}$$

#

$$v_E = \omega(r_{IC-E})$$

$$= 9.375\sqrt{(0.8)^2+(0.2667)^2}$$

$$= 7.91 \text{ ft/s}$$

#

16-105

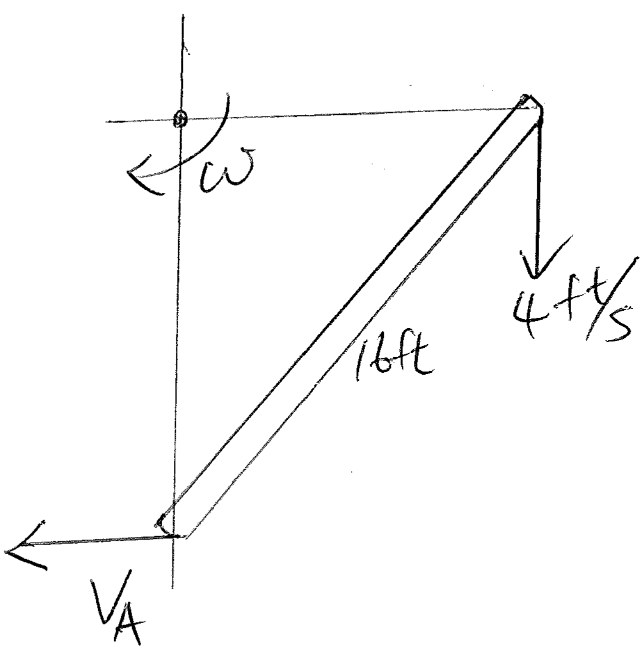
$$\omega = \frac{4}{16\cos 30^\circ} = 0.288675 \text{ rad/s}$$

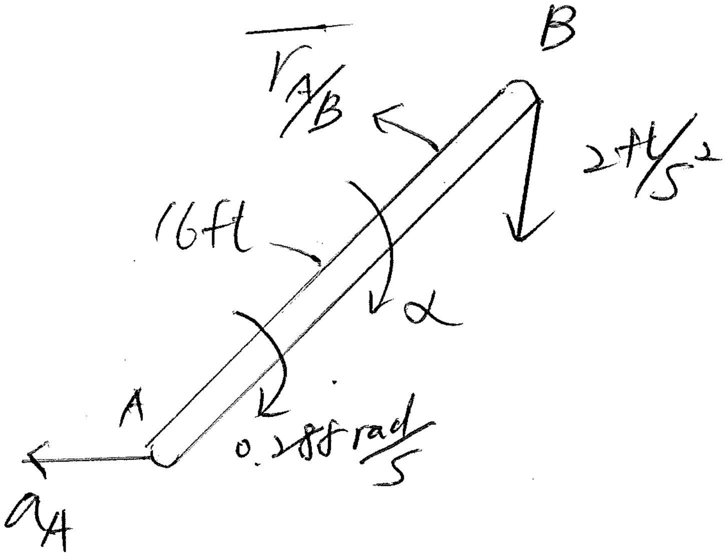
$$\overline{a_A} = \overline{a_B} + \alpha \times \overline{r_{A/B}} - \omega^2 \overline{r_{A/B}}$$

$$-a_A \bar{i} = -2\bar{j} + (\alpha \bar{k}) \times (-16\cos 30^\circ \bar{i} - 16\sin 30^\circ \bar{j})$$
$$-(0.288675)^2(-16\cos 30^\circ \bar{i} - 16\sin 30^\circ \bar{j})$$

$$-a_A = 8\alpha + 1.1547$$
$$0 = -2 - 13.856\alpha + 0.6667$$
$$\alpha = -0.0962 \text{ rad/s}^2 = \underline{0.0962 \text{ rad/s}^2 \circlearrowright}$$

$$a_A = -0.385 \text{ ft/s}^2 = \underline{0.385 \text{ ft/s}^2 \rightarrow}$$

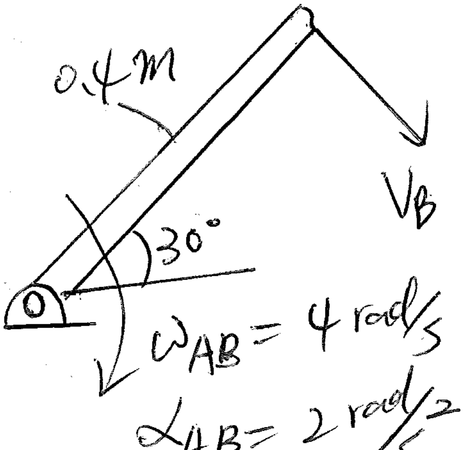




16-111

Solution:

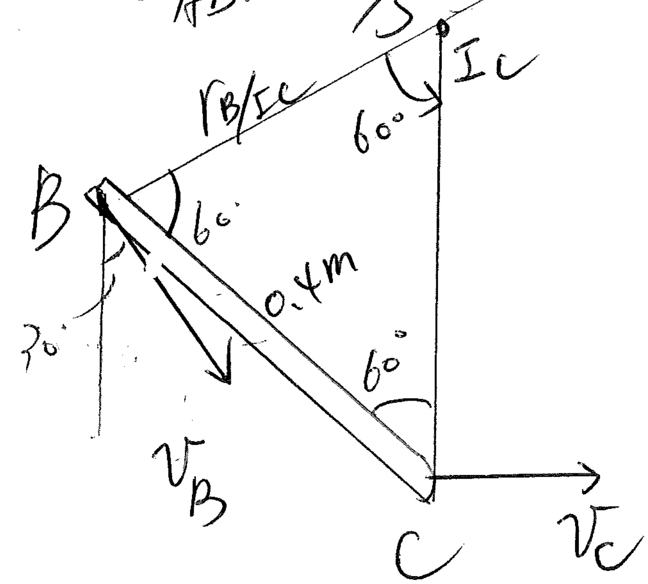
Angular velocity: AB rotates about a fixed axi



$$v_B = \omega_{AB} r_B = 4 \times 0.4 = 1.6 \text{ m/s}$$

From the geometry, $r_{B/IC} = 0.4 \text{ m}$

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{16}{0.4} = 4 \text{ rad/s}$$



$$a_B = \alpha_{AB} \times \bar{r}_B - \omega_{AB}^2 \bar{r}_B$$

$$= (-2\bar{k}) \times (0.4\cos30°\,\bar{i} + 0.4\sin30°\,\bar{j})$$

$$- 4^2(0.4\cos30°\,\bar{i} + 0.4\sin30°\,\bar{j})$$

$$= \left[-5.143\bar{i} - 3.893\bar{j}\right] \text{ m/s}^2$$

$$\bar{a}_C = \bar{a}_B + \alpha_{BC} \times r_{C/B} - \omega_{BC}^2 \bar{r}_{C/B}$$

$$a_C\bar{i} = (-5.143\bar{i} - 3.893\bar{j}) + (\alpha_{BC}\bar{k}) \times$$

$$(0.4\cos30°\,\bar{i} - 0.4\sin30°\,\bar{j}) - 4^2(0.4\cos30°\,\bar{i}$$

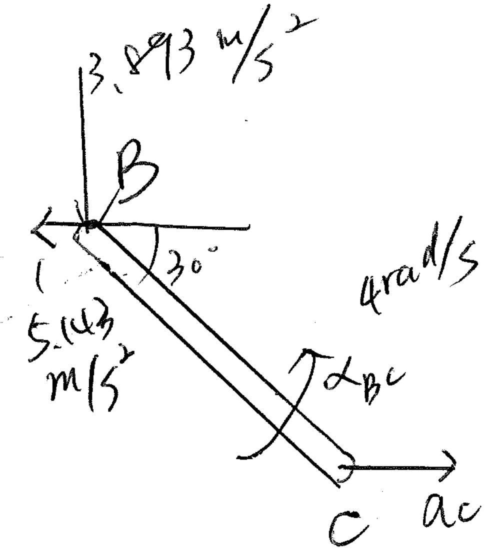
$$-0.4\sin30°\,\bar{j})$$

$$a_C\bar{i} = (0.2\alpha_{BC} - 10.69)\bar{i}$$

$$+ (0.3464\alpha_{BC} - 0.6928)\bar{j}$$

Equating $\bar{i}$ and $\bar{j}$ components,

3



$$a_C = 0.2\alpha_{BC} - 10.69 \quad (1)$$

$$0 = 0.3464\alpha_{BC} - 0.6928 \quad (2)$$

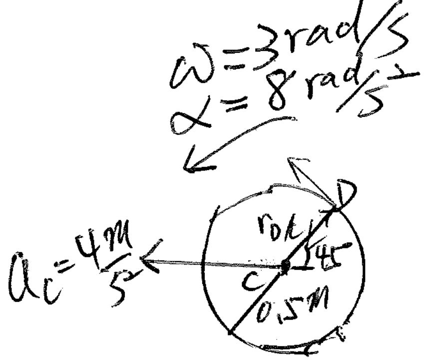
Solving (1) and (2),

$$\alpha_{BC} = 2 \text{ rad/s}^2$$

$$a_C = -10.29 \text{ m/s}^2 = 10.3 \text{ m/s}^2 \leftarrow$$

\#

16-114



$$a_C = 8 \times 0.5 = 4 \text{ m/s}^2$$

$$\bar{a}_D = \bar{a}_C + \bar{a}_{D/C}$$

$$= 4(\leftarrow) + 3^2 \times 0.5(\swarrow 45^\circ) + (8 \times 0.5)(\nwarrow 45^\circ)$$

$$(a_D)_x = -10.01 \text{ m/s}^2 \quad (\overset{+}{\rightarrow})$$

$$(a_D)_y = -0.3536 \text{ m/s}^2 \quad (+\uparrow)$$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ$$

\#

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10 \text{ m/s}^2$$

\#

$$\bar{a}_D = \bar{a}_C + \alpha \times \bar{r}_{D/C} - \omega^2 \bar{r}_{D/C}$$

$$(a_D)_x \bar{i} + (a_D)_y \bar{j} = -4\bar{i} + (8\bar{k}) \times (0.5 \cos 45^\circ \bar{i} + 0.5 \sin 45^\circ \bar{j})$$

$$- 3^2 (0.5 \cos 45^\circ \bar{i} + 0.5 \sin 45^\circ \bar{j})$$

$$(\overset{+}{\rightarrow}) \ (a_D)_x = -4 - 8(0.5 \sin 45^\circ) - 3^2 (0.5 \cos 45^\circ)$$

$$= -10.01 \ m/s^2$$

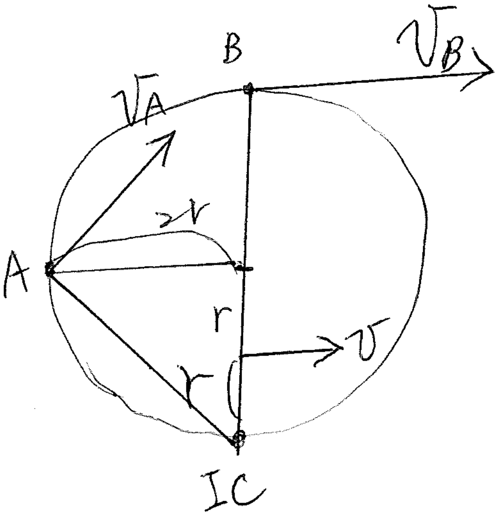
$$(+\uparrow) \ (a_D)_y = 8(0.5 \cos 45^\circ) - 3^2 (0.5 \sin 45^\circ)$$

$$= -0.3536 \ m/s^2$$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = \underline{2.02^\circ \ \swarrow} \ \#$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = \underline{10.0 \ m/s^2} \ \#$$

16-115



$$\omega = \frac{v}{r}$$

$$v_B = \omega r_{B/IC} = \frac{v}{r}(4r) = 4v \rightarrow$$

$$v_A = \omega r_{A/IC} = \frac{v}{r}\left(\sqrt{(2r)^2+(2r)^2}\right) = 2\sqrt{2}v$$

$\angle 45°$

Acceleration equation:

$a_G = 0, \ \alpha = 0.$

$\bar{r}_{B/G} = 2r\bar{j}, \ r_{A/G} = -2r\bar{i}$

$$\bar{a}_B = \bar{a}_G + \alpha \times \bar{r}_{B/G} - \omega^2 \bar{r}_{B/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2(2r\bar{j}) = -\frac{2v^2}{r}\bar{j}$$

$$a_B = \frac{2v^2}{r} \downarrow$$

$$\bar{a}_A = \bar{a}_G + \alpha \times r_{A/G} - \omega^2 r_{A/G}$$

$$= 0 + 0 - \left(\frac{v}{r}\right)^2(-2r\bar{i}) = \frac{2v^2}{r}\bar{i}$$

$$a_A = \frac{2v^2}{r} \rightarrow$$